Researching and Modeling of Discrete Linear Time-Invariant Systems with Difference Equations in Matlab

Angel Hr. Chekichev[1], Daniela A. Shehova[2], Slavi Y. Lyubomirov[3], Stanislav M. Asenov[4] and Katya K. Asparuhova[5]
[1,2,3,4] Faculty of Physics and Engineering Technologies, Plovdiv University "Paisii Hilendarski"
4700 Smolyan, Bulgaria
ma8721@abv.bg, shehovadaniela@gmail.com, slavilubomirov@gmail.com, st.assenow@gmail.com
[5]Department of Electronics, Faculty of Electronic Engineering and Technologies
Technical University of Sofia
8 Kliment Ohridski blvd., 1000 Sofia, Bulgaria
k_asparuhova@tu-sofia.bg

Abstract – **The paper examines the opportunities for developing the skills and competencies of engineering students to understand the fundamental concepts embedded in digital signal processing. The Matlab-Simulink development software environment was used for research and teaching of Discrete Linear Time-Invariant (LTI) Systems through Difference Equations (DE), for which purpose a general engineering algorithm and a simulation model have been created, implementing the recursive Discrete LTI systems. The results of the conducted analytical and simulation researches are presented.**

Keywords – **Discrete Linear Time-Invariant Systems, Difference Equation, Matlab.**

I. Introduction

The systems for Digital signal processing (DSP) are increasingly used in the largest telecommunications systems, systematized for sound and image processing, digital television, Internet, etc.

Knowledge and understanding of Digital Signal Processing are important for students studying in higher technical engineering schools. They must receive good engineering training to have the necessary competencies for the development of the algorithm related to the processing of recursive or non-recursive difference equations, their program, and model-simulation implementation. These competencies are necessary because they correspond directly to Digital Signal Processing [1, 2].

In the mathematics differential equation is research from different aspects, intending to create and simplify methods for solving them. The complexity of differential equations, determined by the application of integrals, derivatives, and other more complex functions, requires the practical use of simpler methods and approaches for calculation, i.e. the use of difference equations, realized with the simplified mathematical operators – addition, subtraction, and multiplication [3,4].

The difference equations can be considered as a discrete analog of the differential equations [5].

Matlab /Simulink/ is a suitable environment in engineering education for code writing, construction, and simulation research of Discrete LTI Systems with recursive or non-recursive difference equations [6,7,8].

The combination of analytical and simulation research of Discrete LTI Systems provides students with the opportunity to analyze and compare the results.

The creation of a simulation model of Discrete Linear Time-Invariant Systems with difference equations provides an opportunity to study various conceptual and fundamental features of their construction and the processes taking place in them.

II. Analytical Investigation of Discrete Linear Time-Invariant Systems

A. Algorithm for engineering research of Discrete Linear Time-Invariant Systems

To perform the analysis and synthesis of the main characteristics of Discrete Linear Time-Invariant Systems [3], an algorithm has been created that contains the following procedures:

- determining the type and order of the difference equation;
- drawing up the structural scheme of the system, corresponding to the given difference equation;
- determining and drawing of the impulse response of the system;
- determining the stability of the system from the obtained impulse response;
- recording the equation for determining the transfer function, corresponding to the set difference equation;
- determining the poles and zeros, and constructing the Pole-zero diagram;
- determining the stability of the system, according to the obtained Pole-zero diagram;
- calculation of the transmission coefficient and construction of the Amplitude-frequency response.

To determine the output signal, if no difference equation is set, but the pulse characteristic and the input signal are set, it is necessary to use the convolution method [9].

B. Implementation of the algorithm in the Matlab Environment

To show the application of the specified algorithm, we use Discrete Linear Time-Invariant Systems, described by the following difference equation:

$$y(n)=2x_{(n-2)}-x_{(n-4)}+8.8306x_{(n-6)}-0.6714y_{(n-3)} \\ -2.25063y_{(n-5)}+0.25470y_{(n-7)} \quad (1)$$

where: n is the number of discrete samples for which the calculations will be performed.

Perform the procedures of the described algorithm:

- Determining the type and order of the difference equation.

The given difference equation has a recursive character because the output signal is determined not only by the input signal given at the input and its previous states but also by the previous states of the output signal.

The order of the difference equation is 7th based on the last previous state of the input and output signal.

- Drawing up the structural scheme of the system.

Based on the given difference equation and the first procedure of the algorithm, we can compile the block diagram (Fig.1) for the realization of equation (1). The block diagram of the researched difference equation contains adder, amplifiers, and memory cells z^{-1} (blocks with which the previous state of a discrete retained).

- Determining the Impulse response of the system.

The determination of the Impulse response and its coefficients is done by the following code written in the Matlab environment:

```
b = input('b = ');   % b = [0 0 2 0 -1 0 .8306]
a = input('a = ');   % a = [1 0 0 .6714 0 .25063 0 -.25470]
n = input('n = ');   % n = 150
h = impz(b,a,n)
impz(b,a,n), grid on; xlabel('n'); ylabel('h(n)');
title('Impulse response');
```

The Impulse response is shown in Fig. 2.

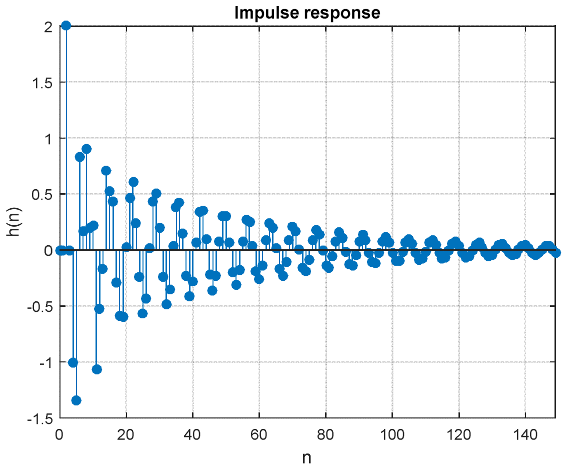

Fig. 2. Impulse response

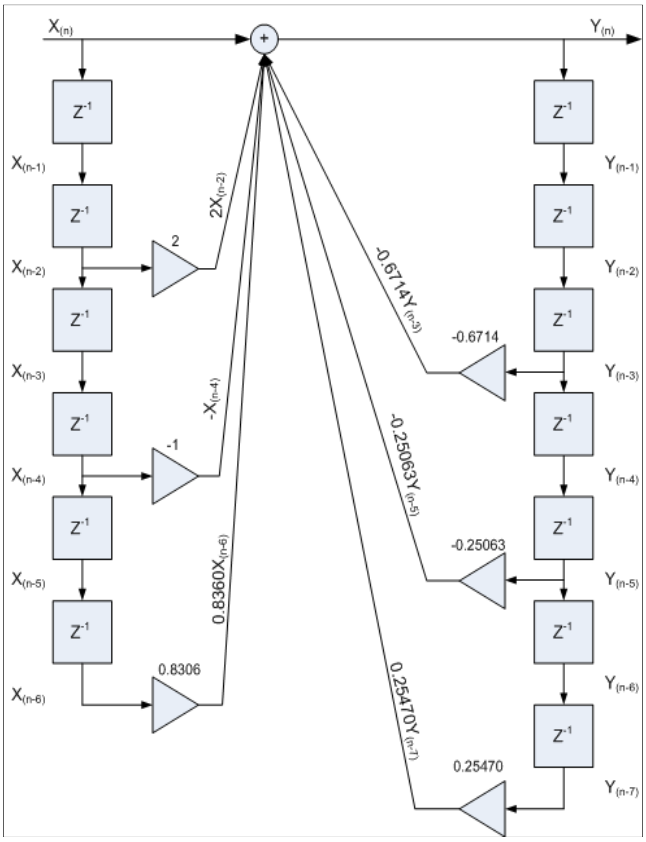

Fig. 1. The structural scheme of the studying difference equation

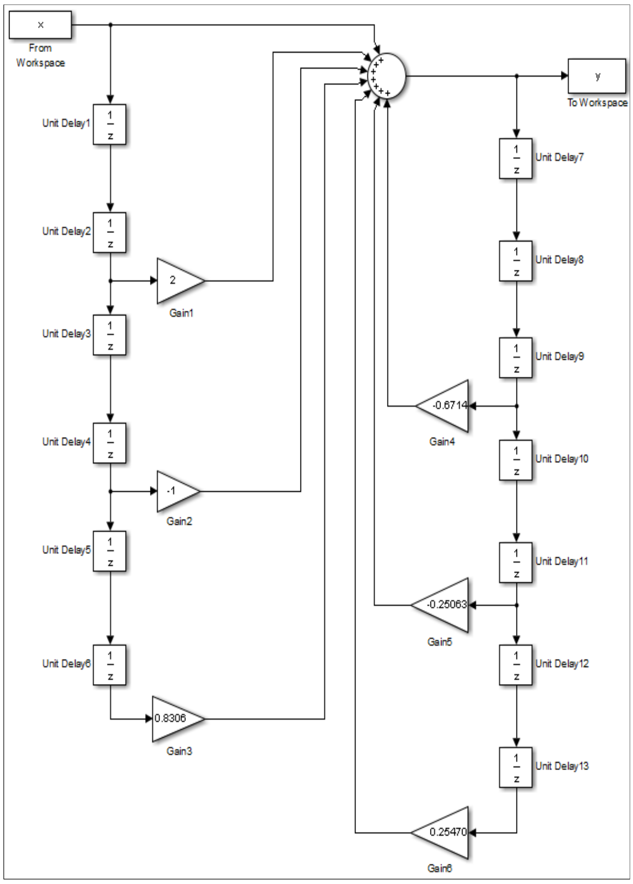

Fig. 3. A simulation model of Discrete LTI Systems for a certain difference equation

Fig. 3 presents the simulation model for research of Discrete LTI Systems with a difference equation in Matlab/Simulink.

The functional purpose of the block *Unit Delay* is to hold the signal by reversing a state. This simulates the use of previous states of the given signal. This block is equivalent to the discrete-time operator z^{-1}. The block has one input and one output. Each signal can be a number or a vector. If the input is a vector, the block is holding and delay all elements of the vector for the same sample period.

The block *From Workspace* reads the data from the workspace and outputs it as a signal.

The block *To Workspace* connects to the input of the signal and the recording of the signal data in the Matlab workspace. During the simulation, the block writes data to an internal buffer. When the simulation is complete or paused, this data is saved to the workspace.

To visualize the results of the model's work, it is necessary to write a program (code), built into the simulation model. Figure 4 shows the results of the simulations with the created model.

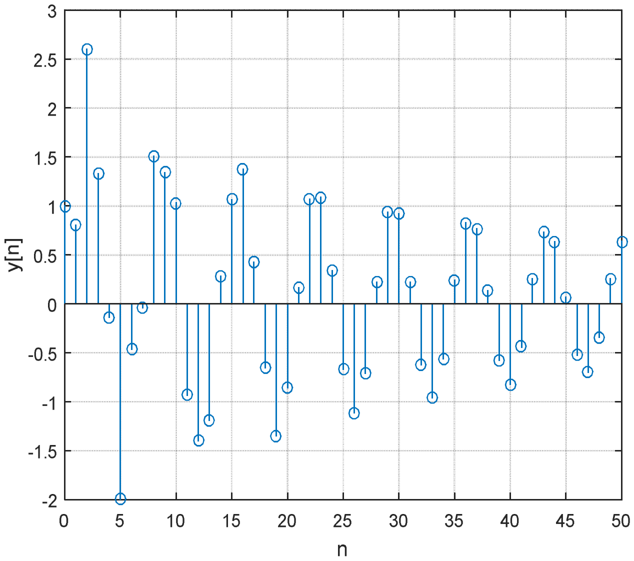

Fig.4. Impulse response

III. Stability Research of Discrete LTI Systems in the Matlab Environment

The study of the stability of the system is done using the following algorithm:

- Record the equation for determining the transfer function:

$$H(z)=\frac{\sum_{i=0}^{M} b_i z^i}{1+\sum_{i=1}^{N} a_i z^i}=\frac{b_2 z^{-2}+b_4 z^{-4}+b_6 z^{-6}}{1+b_3 z^{-3}+b_5 z^{-5}+b_7 z^{-7}}=$$

$$=\frac{2z^{-2}-z^{4}+0.8306z^{6}}{1+0.6714z^{-3}+0.25063z^{-5}-0.25470z^{-7}}*\frac{z^7}{z^7}= \qquad (2)$$

$$=\frac{2z^{5}-z^{3}+0.8306z^{1}}{z^{7}+0.6714z^{4}+0.25063z^{2}-0.25470}$$

- Determining the poles and zeros to build a Pole-zero diagram.

To implement this procedure, the following code is written in the Matlab environment:

```
a=input('a = '); %a = [1 0 0 .6714 0 .25063 0 -.25470]
b=input('b = ');  % b = [0 0 2 0 -1 0 .8306]
p = roots(a)
q = roots(b)
figure(2), zplane(b,a)
```

After starting the code in the development environment, a Pole-zero diagram is obtained from Fig. 5:

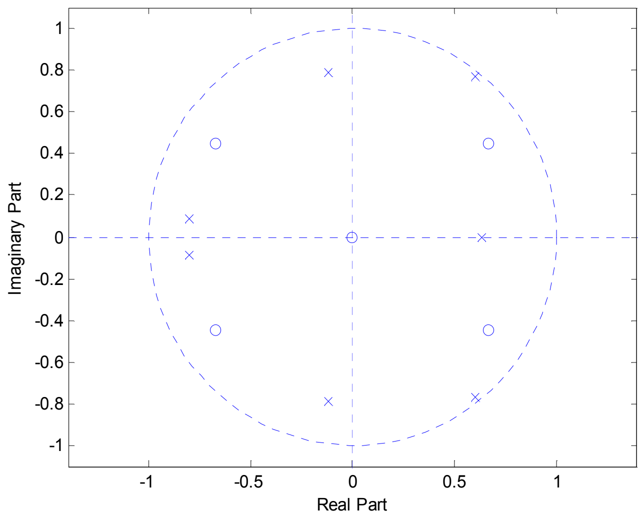

Fig. 5. Pole-zero diagram

- Determining the stability of the system from the Pole-zero diagrams.

From the Pole-zero diagram (Fig. 5) the students determine the stable character of the system, since all the poles lie or are located in the unit circle. If the poles weren't surrounded by a unit circle i.e. lying on it or outside it, the system would be unstable.

In this way, learners can consolidate their knowledge that the stability of the system is determined only by the location of the poles in the unit circle and not by the location of the zeros.

- Determining the transmission coefficient and plotting the amplitude-frequency response.

The transmission coefficient is determined by the following dependence:

$$\dot{K}(\omega)=\frac{\sum_{i=0}^{M} b_i e^{-j\omega i}}{1+\sum_{i=1}^{N} a_i e^{-j\omega i}}=\frac{b_2 e^{-j2\omega}+b_4 e^{-j4\omega}+b_6 e^{-j6\omega}}{1+a_3 e^{-j3\omega}+a_5 e^{-j5\omega}+a_7 e^{-j7\omega}} \qquad (3)$$

It can be optimized using the values of the coefficients before *x* and *y* of difference equation described by Eq. 1. $a_3=0,6714;\ a_5=0,25063;\ a_7=-0,25470;\ b_2=2;\ b_4=-1;\ b_6=0,8306$ and five arbitrary values of the frequencies are selected - $\omega = 0, 2\pi;\ 4\pi;\ 0.39597$ *and* 1.75973.

The analytical expression for the transmission coefficient yields the form:

$$\left|\dot{K}(0)\right| = \left|\dot{K}(2\pi)\right| = \left|\dot{K}(4\pi)\right| = 1.0979\ rad,$$

$$\left|\dot{K}(0.39597)\right| = 0.8195\ rad$$

$$\left|\dot{K}(1.75973)\right| = 3.7387\ rad \qquad (4)$$

The change in the Amplitude frequency response is shown in Fig. 6.

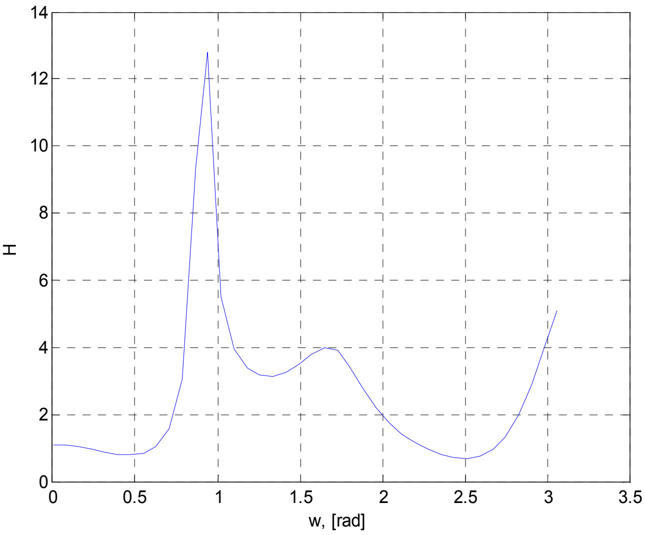

Fig. 6. Amplitude frequency response

Figures 7 and 8 show the pole-zero diagram and the impulse response of a Unstable Discrete LTU system, described by the following differential equation (5):

$$y(n) = x(n) - x(n-2) + 2.3y(n-1) - 1.7y(n-3)$$

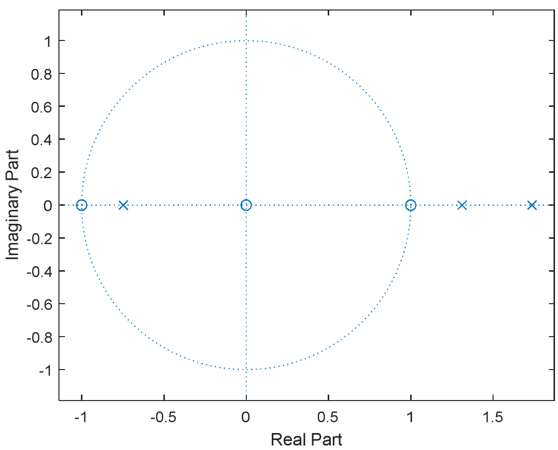

Fig. 7. Pole-zero diagram

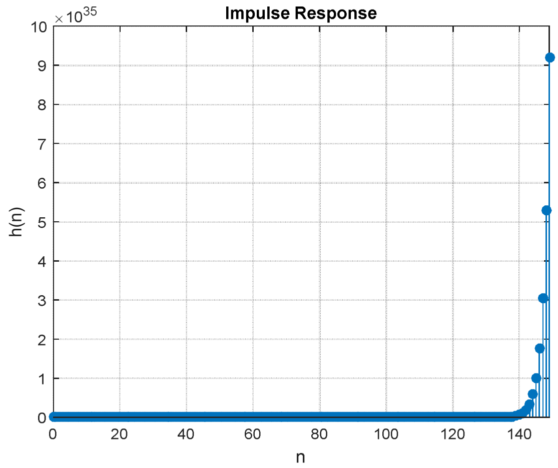

Fig. 8. Impulse response

VI. Conclusion

The article presents the use of Matlab for simulation research of Discrete Linear Time-Iinvariant System with difference equations, to support the teaching in digital signal processing.

The combination of analytical and simulation research of Discrete LTI Systems provides an opportunity for students to analyze and compare the results.

Acknowledgment

The authors would like to acknowledge the support of the "Research & Development" division of UNIVERSITY OF PLOVDIV PAISII HILENDARSKI in the project ФП19/ФТФ-012.

References

[1] S. M. Kuo, B. H. Lee, W. Tian, Real-Time Digital Signal processing. Implementations and Applications, Second Edition, John Wiley & Sons, Ltd, 2006.

[2] https://brilliant.org/wiki/linear-time-invariant-systems/

[3] https://www.student-circuit.com/learning/year2/signals-and-systems-intermediate/properties-of-lti-system/

[4] A. Benzaouia, M. Mohamed, F. Benhayoun, Saturated Control of Linear Systems, 2017.

[5] B. P. Lathi, R. Green, Linear Systems and Signals, Oxford University Press, 2017.

[6] T. K. Steven, Signals and Systems with MATLAB Computing and Simulink Modeling, 2007.

[7] A. Palamides, Signals and Systems Laboratory with MATLAB M-files, 2020.

[8] I. W. Forrester, DSP for Matlab and LabVIEW, Fundamentals of Discrete Signal Processing, 2008.

[9] S. Sotirov, D. Tokmakov, „Wireless Current Measurement System Based on Integrated Fluxgate Magnetic Sensor for Isolated Current Sensing", Participation (ELECTRONICA) 2019 X National Conference with International, 2019,pp. 1-4.