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# SIMPLE TECHNIQUE AND DEVICE FOR SLURRIES VISCOSITY MEASUREMENT

S. ALACI[a*], ZH. KALITCHIN[b], M. KANDEVA[c,d], F. C. CIORNEI[a], C. BUJOREANU[e], J. MACHADO[f]

[a]*Mechanics and Technologies Department, Stefan cel Mare University of Suceava, 13 Universitatii Street, Corp B, 720 229 Suceava, Romania*
*E-mail: stelian.alaci@usm.ro*
[b]*SciBuCom 2 Ltd., P.O.Box 249, 1113 Sofia, Bulgaria*
[c]*Faculty of Industrial Engineering, Tribology Centre, Technical University – Sofia, 8 Kl. Ohridski Blvd., 1756 Sofia, Bulgaria*
[d]*South Ural State University, 76 Prospekt Lenina, Chelyabinsk, Russia*
[e]*Gheorghe Asachi Technical University, Iasi, Romania*
[f]*University of Minho, MEtRICs Research Centre, Guimarães, Portugal*

**Abstract.** *In situ* and real time measurements of physical parameters, like viscosity, are frequently necessary in the environment protection procedures. Unknown mixtures of liquids and solid particles are frequently met and being potential dangerous (corrosive, radioactive) the problem of safe transportation and evacuation occurs. The flow parameters depend on a large number of liquid characteristics varying in wide ranges and a method and devices for estimation for rapid, safe and inexpensive measurements are required. Viscosity is one of these parameters and the starting from the idea of improving a vertical falling sphere viscometer, a model of rotational viscometer is proposed. The viscosity is determined when the resultant torque is zero. From the equation of dynamic equilibrium for rotational motion two values for viscosity are obtained, both probable in equal manner. The concern of the paper is to find a criterion for the selection of the correct solution and also to explain the meaning of the other root.

*Keywords*: viscosity, experimental device, numerical method.

## AIMS AND BACKGROUND

Both engineering applications and everyday life is confronted with the problem of residues amid which the liquid or semisolid ones may create environmental damages. Slurries can contain particles of different sizes and can be settling or non-settling fluids. A large number of parameters influence the rheological behaviour of the slurry: particle size, particle density, volume fraction, particle drag coefficient, temperature[1,2], particle shape, particle interaction, aggregation structure of suspended particles, the structure of fluid flow[3]. Even if the liquid is

[*] For correspondence.

non-homogenous it can exhibit Newtonian, non-Newtonian, shear thinning, shear thickening, viscoplastic behaviours as function of the shear rate. The viscosity measurements vary for the same liquid mixture over wide ranges of deformation rates and require different equipments for measurements, according to the shear-rate domain of the instrument. In engineering processes (transportation, dewatering, wet grinding[4]) during which the flow parameters of the slurry (coal, bauxite, cement, limestone, biomass[5], etc.) are controlled, one can select the appropriate viscometer. Engineering fluids frequently experience local conditions that change their bulk rheological properties and *in situ* measurement techniques have been developed[6,7] which necessitate complex and cutting edge equipment. There are also other situations when a rapid *in situ*, real-time estimation, using a simple device, inexpensive, eventually with removable single use parts, is required.

The study of a natural phenomenon assumes recognition, showing it up and after that, elaboration of a mathematical model. The validation of the model is done when there is concordance between the results given by the model for a series of parameters, chosen for well defined running conditions of the phenomenon, and the actual values of the same parameters. A good correlation between theoretical and experimental parameters allows the prognosis of the evolution of the considered phenomenon using the established model, and thus, for cases when there is an unfavourable evolution, actions can be taken in time. In the situations when the concordance is not satisfactory, the model requires improvements by considering more profound aspects of the phenomenon, which in a first state were ignored.

One of the most concluding examples is the wheatear forecast[8]. Actually, using it, one can know, for a few days in advance, the meteorological evolution from a certain region of the globe and the necessary decisions can be made when the progress is abnormal. The comfort emerging from identifying the exact weather evolution is the result of tens of years of research work for obtaining the actual model[9,10]. The elaboration of the model describing a certain phenomenon assumes often intellectual and financial effort without the certitude of a concluding result, despite all efforts. The plainest example is the earthquake phenomenon. It is obvious what an achievement would be the possibility of announcing it at least with tens of minutes before happening.

The models describing natural phenomena are complex due to nonlinear character of natural world. In numerous occasions, when a model is completed, the system may follow several alternatives and the duty of scientist is to complete unambiguous criteria to allow stipulation of univoque direction to be followed by the phenomenon at a certain time. The multiple variants of the model are caused by the nonlinear equations which describe the analysed phenomenon. To exemplify the affirmation, it is considered the case of a body in gravity field, launched vertically with initial velocity $v_0$ and seeking the time when the body reaches the height $h$. The height $z$ of the body at a time $t$ after launching is a quadratic function of time.

Imposing the condition that the altitude $z$ equals $h$ it is obtained an equation of second degree with the solutions giving the sought times. Even for this very simple case, much attention is required. When the solutions are complex due to negative discriminant, the problem has no solution, the launching velocity being too small the body does not reach the considered height. When the discriminant is positive, two real solutions exist, $t_1$ and $t_2$ corresponding to the moments of reaching the altitude in ascension and the other one corresponding to the falling body[11]. From this simple example it results that all alternatives generated by an adequate model of the evolution of the phenomenon are equally possible but at the same time with well specified physical signification.

Another relevant illustration is from spatial kinematics domain, more precise from spherical motions. It is known that any spherical motion can be described using an orthogonal matrix. To characterise the spherical motion, the axis of rotation (namely the versor of the rotation axis) and the rotation angle must be specified. Knowing these two elements, the expression of the rotation matrix can be immediately written[12]. In the case when the rotation matrix is known, the axis of rotation must be obtained: McCarthy[13] proposes the be sought after among the vectors that have the image collinear with the initial vector after applying the rotation operator. Thus, an algebra equation of third degree is obtained, its roots being the eigenvalues of the rotation matrix. McCarthy[13] shows that the characteristic equation always has three roots of unity modulus, a real root and two complex conjugate roots. The eigenvector corresponding to the real value is the versor of the axis of rotation. The other two roots should not be ignored since their argument is the angle of rotation. It is therefore confirmed that the equation obtained trying to describe a particular aspect of a phenomenon can establish facts about several aspects of it or even the complete description.

The objective of the present paper is to prove that one can find a new methodology for experimental tests for finding the viscosity, but the significance of the solutions of the model lead to a dilemma.

## EXPERIMENTAL

The device proposed in the present work begins with the test rig used in the fluid mechanics laboratory for finding the viscosity of a fluid. As principle, the device consists of a vertical tube, filled with the liquid to be studied of density $\rho_L$. A ball of radius $r_b$ made of a material with density $\rho_b$ falls free in the liquid. The method is based on the balance between the (Archimedes) buoyant force, the hydrodynamic drag force and the weight of the ball, assuming rectilinear uniform motion. The precision of the method depends essentially if the estimation of the velocity is made before or after accomplishment of uniform rectilinear motion of the ball.

To this purpose, it is useful to find the minimum fall distance required for the ball to attain the regime of uniform motion. The equation of motion of the ball is:

$$m\ddot{y} = G - F_A - F_D, \tag{1}$$

where $G$ is the weight of the ball:

$$G = \frac{4\pi}{3} r_b^3 \rho_b g. \tag{2}$$

$F_A$ is the buoyant force exerted by the liquid upon the sphere:

$$F_A = \frac{4\pi}{3} r_L^3 \rho_L g. \tag{3}$$

$F_D$ is the drag force, the hydraulic resistance opposed by the fluid[14]:

$$F_D = 1/2\, C_D \rho_L V^2 (\pi r_b^2) \tag{4}$$

where $C_D$ is the drag coefficient and $V$ – the velocity of the ball with respect to fluid.

In technical literature is presented that the drag coefficient depends on velocity via the Reynolds number:

$$\mathrm{Re}(\mu) = \frac{\rho_L V (2r_b)}{\mu}. \tag{5}$$

The manner the drag coefficient depends on Reynolds number is an open matter. There are numerous relations describing this dependency, starting with the relation of Stokes[14] valid for low Reynolds numbers (Re < 1):

$$C_D(\mathrm{Re}) = 24/\mathrm{Re} \tag{6}$$

and to very intricate formulae that try to interpolate optimum the actual experimentally found variation. Next, for the dependence $C_D = C_D(\mathrm{Re})$ is used the relation proposed by Clift and Gauvin[15], for $\mathrm{Re} < 3 \times 10^5$:

$$C_D(\mathrm{Re}) = \frac{24}{\mathrm{Re}} (1 + 0.15\, \mathrm{Re}^{0.687}) + \frac{0.42}{1 + 4.25 \times 10^4\, \mathrm{Re}^{-1.16}}. \tag{7}$$

The dependencies described by equations (6) and (7) are plotted in Fig. 1.

Considering that the Reynolds number is a function of velocity $V = \dot{y}$, (equation (5)), equation (1) becomes a differential nonlinear equation:

$$m\ddot{y} = f(\dot{y}). \tag{8}$$

Considering a steel ball $\rho_b = 7800$ kg/m$^3$ of radius $r_b = 0.01$ m, equation (8) was numerically integrated via Runge-Kutta[16] method, for glycerol ($\rho_L = 1260$ kg/m$^2$ and $\mu = 0.95$ Pa s) and for water ($\rho_L = 1000$ kg/m$^3$, $\mu = 0.001$ Pa s) the results being presented in Fig. 2 – the displacement, and in Fig. 3 – the velocity of the ball.

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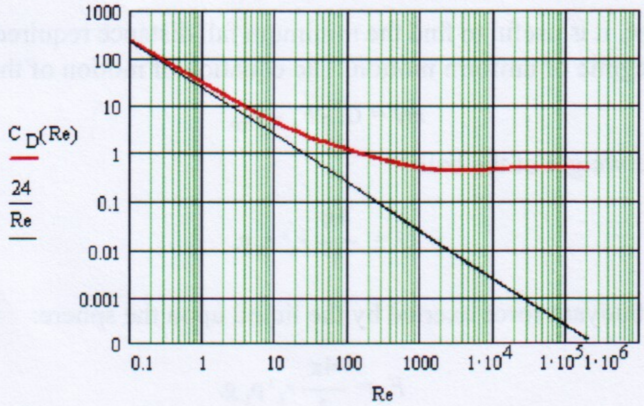



**Fig. 1.** Drag coefficient $C_D$ versus Reynolds number[8]

If in glycerol the ball reaches uniform motion after about ≈ 0.5 s, time during which the ball falls a distance of 0.12 m, in the case of falling in water, the constant velocity is reached after ≈ 1.4 s (Fig. 5) while the ball falls 1.2 m (Fig. 4). It can be concluded that for less viscous fluids, long tubes are necessary, with lengths[17] over 1 m.

Therefore, for liquids with low viscosity a longer time is necessary for attaining uniform motion of the ball, due to greater difference between weight and drag force and thus longer tubes are required. From here, the idea of replacing the vertical motion with a motion in a horizontal plane occurred.

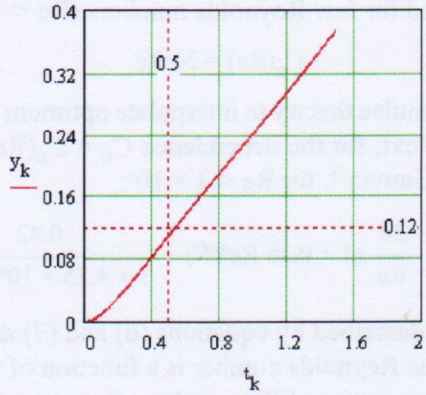



**Fig. 2.** Displacement of ball versus time in glycerol

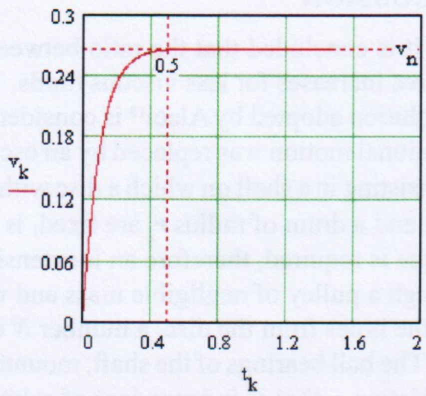



**Fig. 3**. Variation of ball velocity versus time in glycerol

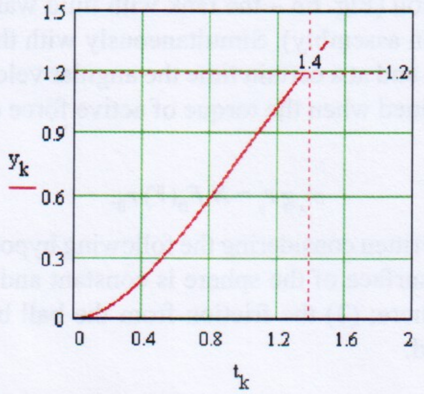



**Fig. 4**. Displacement of ball (*y*) versus time in water

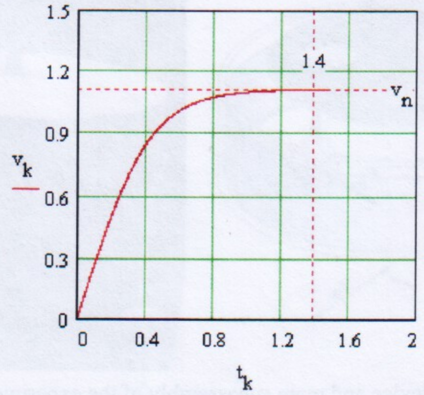



**Fig. 5**. Variation of ball velocity (*v*) versus time in water

## RESULTS AND DISCUSSION

From previous section it is concluded that the ratio between apparent weight of the ball and the drag force increases for less viscous fluids. To balance the effects of the two forces, the solution adopted by Alaci[18] is considered, where a vibratory viscometer with translational motion was replaced by an oscillatory viscometer. A rotation viscometer, consisting in a shaft on which a disc with two set of equidistant holes placed circularly, and a drum of radius $r_a$ are fixed, is designed (Fig. 6*a*). A constant operating torque is required, therefore an inextensible wire is wound on the drum, passing through a pulley of negligible mass and with an attached mass $m_a$ at the other end. In the holes from the disc, a number $N$ of spherical shells are fixed by threaded rods. The ball bearings of the shaft, mounted to the ground, were cleaned and thus dry friction – that is independent of relative velocity, between balls and races exists. One test implies wounding the wire on the drum, then the body is set in free motion (Fig. 6*b* – the tank with fluid was removed in order to take a photo of the main assembly). Simultaneously with the falling of the body, the shaft starts to rotate and at a certain time the angular velocity is ω. An uniform angular velocity is attained when the torque of active force equals the moment of drag forces:

$$m_a g r_a = N F_D(V) r_B. \tag{9}$$

Equation (9) was written considering the following hypothesis: (1) the velocity of any point from the surface of the sphere is constant and equal to the velocity of the centre of the sphere; (2) the friction from the ball bearings and from the pulley joint is neglected.

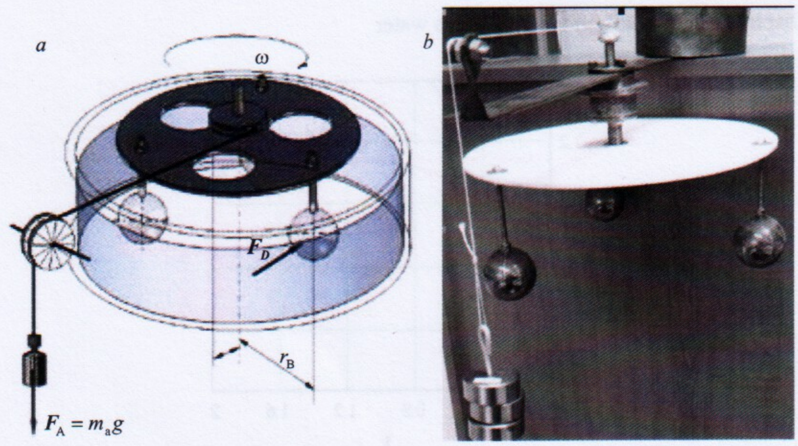



**Fig. 6**. Principle scheme of device and main subassembly of the experimental device

The surface of the disc was marked on radial direction in order to characterise the motion of the disc. The motion of the disc was filmed and the movie was split into frames in order to identify the instants when the disc attains complete rotations[19].

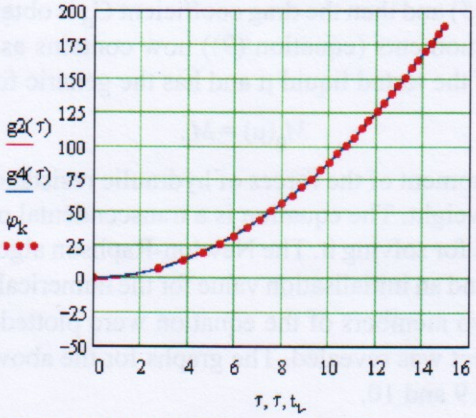



**Fig. 7**. Rotation angle of main shaft in air and interpolation of experimental data

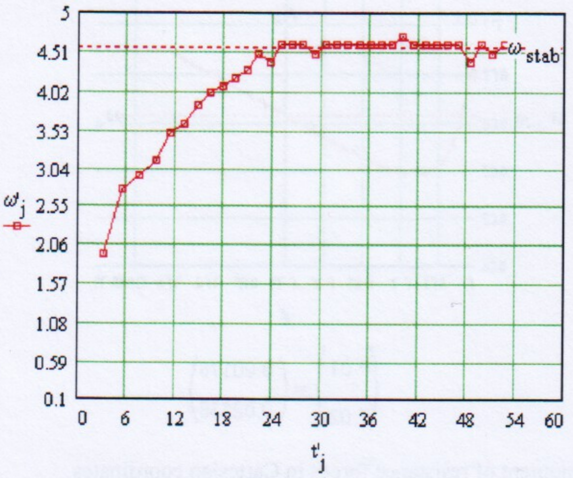
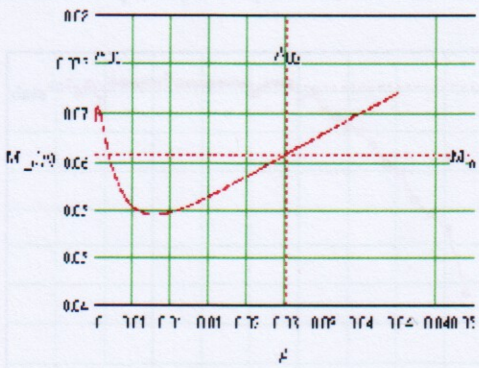



**Fig. 8**. Rotation angle of shaft in the presence of liquid

The variation with time of the shaft angle of rotation in absence and presence of fluid is presented in Figs 7 and 8, respectively. The liquid is water and the other parameters used for the results are: mass of the actuating body $m_a = 0.568$ kg; drum radius $r_a = 0.011$ m; spherical shells of radii $r_b = 0.025$ m mounted at a radius $r_B = 0.13$ m. The experimental data from Fig. 7 were interpolated with two polynomials[20], of second and of fourth degree, respectively. As noticed from Fig. 8, the two interpolation curves are practically identical and this leads to the conclusion that the second derivative of the polynomial of second degree, that represents the angular velocity of the disc, is a constant and therefore the torque is constant, independent of the angular velocity of the disc. In Fig. 8 it can be observed that the angular velocity of the disc reaches a constant, steady value, $\omega_{stab}$ = 4.582 rad/s, fact that validates equation (9). With known angular velocity $\omega_{stab}$ the Reynolds number is found with equation (5) and then the drag coefficient $C_D$ is obtained using equation (7). The balance of moments (equation (9)) now contains as only unknown the dynamic viscosity of the tested liquid μ and has the generic form:

$$M_D(\mu) = M_a, \tag{10}$$

where $M_D(\mu)$ is the moment of the forces of hydraulic resistance and $M_a$ – the moment of the driving weight. The equation is a transcendental one and a numerical procedure is required for solving it. The Newton-Raphson algorithm[21] was used in the present case. To find an initialisation value for the numerical method for solving equation (10), the two members of the equation were plotted on the same chart, and a surprising aspect was revealed. The graphs for the above given parameters are presented in Figs 9 and 10.



$$\begin{pmatrix} \mu 01 \\ \mu 02 \end{pmatrix} = \begin{pmatrix} 0.00176 \\ 0.02538 \end{pmatrix}$$

**Fig. 9**. Variation of moment of resistance forces in Cartesian coordinates

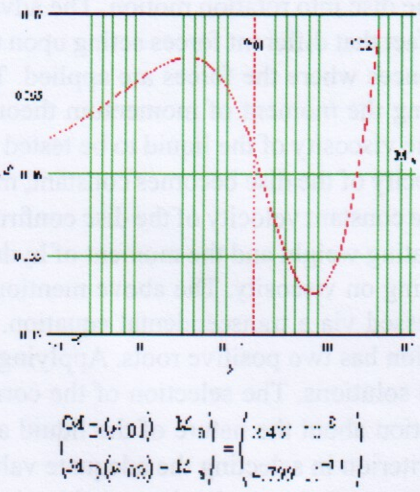


**Fig. 10**. Variation of moment of resistance forces in semi-logarithmic plot

As it can be observed, from both figures, the equation has two solutions: $\mu_{01}$ = 0.00176 Pa s and $\mu_{02}$ = 0.02538 Pa s. From Figs 9 and 10 and the manner by which equation (10) was obtained, there is no reason for preferring one of the two values. From experience and literature[22,23] we know that a value about $\mu$ = 0.001 Pa s is expected. But, the question emerging is: when we do not have experience or information concerning the nature of the liquid, which one of the two values is the actual one? And, other arising questions are: based on which criterion the correct solution should be chosen? After identifying the correct root, the other root does have significance? What is that? To support these matters, it is reminded the well known example of the Dirac equation, first elaborated for the energy levels of the hydrogen atom and afterwards with consequences on the discovery of the positrons, by Anderson[24].

## CONCLUSIONS

The paper shows that the mathematical modelling of a particular characteristic of a phenomenon may lead to a model which completely describes the phenomenon, with all its aspects and this statement is exemplified by two simple cases, first from material point dynamics and the second from spatial kinematics. Additionally, when the considered model generates more than one solution, every one of these can work and each solution has a particular physical significance. A simple device is presented, with a falling ball into a liquid, frequently used in viscometer of falling sphere in a tube type. Analysing the limitations of this device, an improved alternative is proposed concerning several spherical bodies placed equidistant on a rotating disc. A weight suspended on a wire, wounded on a drum fixed co-axially on the disc actuates the disc into rotation motion. The advantage of the new solution resides from the fact that different forces acting upon the disc can be changed by adjusting the distances where the forces are applied. The equation of motion is obtained by applying the moment of momentum theorem. The experiment is video-recorded and the viscosity of the liquid to be tested is found at the moment when the angular velocity of the disc becomes constant, moment chosen from the frames of the film. The constant velocity of the disc confirms the equality between the torque of the actuating weight and the moment of hydraulic resistance forces, these directly depending on viscosity. The above mentioned balance equation is mathematically expressed via a transcendental equation. A plot of the equation reveals that the equation has two positive roots. Applying a numerical procedure helps finding the two solutions. The selection of the correct solution was made based on the information about the nature of the liquid and on routine, thus the experience was the criterion in selecting the adequate value of viscosity. The occurring question presents two aspects: (1) does an objective criterion for selecting, from the two values, the correct one, for any liquid, exist? (2) what are the physical significations of the two solutions?

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