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Post Pareto Optimization –a Case

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Abstract. Simulation performance may be evaluated according to multiple quality measures that are in competition and their simultaneous consideration poses a conflict. In the current study we propose a practical framework for investigating such simulation performance criteria, exploring the inherent conflicts amongst them and identifying the best available tradeoffs, based upon multi-objective Pareto optimization. This approach necessitates the rigorous derivation of performance criteria to serve as objective functions and undergo vector optimization. We demonstrate the effectiveness of our proposed approach by applying it with multiple stochastic quality measures. We formulate performance criteria of this use-case, pose an optimization problem, and solve it by means of a simulation-based Pareto approach. Upon attainment of the underlying Pareto Frontier, we analyze it and prescribe preference-dependent configurations for the optimal simulation training.

INTRODUCTION

Multiple objective optimization involves the simultaneous optimization of more than one objectives. Such problems arise in a variety of real-world applications. In multi-objective optimization there is a set of equally good alternatives with different trade-offs, also known as Pareto-optimal solutions. [10]

There are two general approaches to solve multiple objective optimization problems: [1], [5]

- Mathematicalmethods;
- Meta-heuristic methods.

The first approach involves the aggregation of the attributes into a linear combination of the objective functions.

Once a Pareto-optimal set has been obtained, the decision-maker faces potentially large set of solutions, and selects one solution over.

There are several approaches to realize the post-Pareto analysis:There exists a need for efficient methods that can reduce the size of the Pareto-optimal set. This decision-making stage is usually known as the post-Pareto analysis stage and is the focus of this work. There are several approaches to investigate the post Pareto optimization. Some of them are:

- A first method is the generalization of a method known as the non-numerical ranking preferences method. This method helps to reduce the number of design possibilities to small subsets that clearly reflect the decision maker's objective function preferences without having to provide specific weight values.
- A second method uses a non-uniform weight generator method to reduce the size of the Pareto-optimal set.
- A third method, called sweeping cones technique reduces the size of the Pareto set projecting all of the objective function values and weights into the space over a unit radius sphere and then using sweeping cones to capture desirable Pareto points.
- A fourth method called Orthogonal Search for post-Pareto optimality generates a decreasing succession of mesh points guided by what is called an ideal direction.

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This work presents a case to perform post-Pareto analysis using the second method namely a non-uniform weight generator method.

In the case of a linear multiple objective problem, it makes sense about extreme effective (Pareto-optimal) solutions. There are a finite number of such solutions, which substantially simplifiessolution of the problem of choice. [10]

DESCRIPTION OF THE PROBLEM

Let X denote the set of admissible solutions in some problem. $x \in X$ is an acceptable solution. Suppose that each solution $x \in X$ is estimated by n criteria ($n \geq 2$).

Let $H_i(x)$, $x \in X$, is a real function whose values are estimates of the solution $x \in X$ by criterion $i, i = 1,2,..., n$. The vector $H(x) = (H_1(x), H_2(x),..., H_n(x))$ is the set of solution estimates by all criteria. Suppose that the larger value $H_i(x)$ gives the better the solution x by criterion $i, i = 1,2,..., n$.

Solution $x^* \in X$ called Pareto-optimal, if there is no other solution $x \in X$ for which $H_i(x) \geq H_i(x^*)$, $i, i = 1,2,..., n$, $\exists i_0 : H_{i_0}(x) > H_{i_0}(x^*)$.

A solution $x \in X$ is Pareto optimal in the multiple objective problem, if and only if when it is a solution of the problem: [6], [7], [9], [10]

$$\max_{x \in X} \sum_{i=1}^{n} \lambda_i . H_i(x), \tag{1}$$

where the parameters λ_i satisfy the equation:

$$\sum_{i=1}^{n} \lambda_i = 1, \lambda_i \in (0,1). \tag{2}$$

The basis of the method of targeted programming for solving multiple objective problems is the ordering of criteria inimportance. The initial problem is solved by successively solving a number of problems with one objective function.The solution of the problem with a less important aim cannot worsen the optimal value of the objective function with a higher priority. As a result, we get a satisfactory solutionfor the problem in question.

The problem we solve here is two stage multi objective Pareto optimization with post optimal analysis.At every stage we solve knap-sack and assignment problems.[6], [7], [9], [10]

The knap-sack problem in Pareto optimization is presented in the following way: [2], [3], [4]

$$\begin{aligned} &\max_{x \in X} Z(X) = \sum_{i=1}^{n} \sum_{t=1}^{n} \frac{Q_{it}}{n} . x_{it} \\ &subject \;\; to : \begin{cases} \sum_{i=1}^{n} c_i . x_i \leq S - \frac{C.S}{100} \\ x_i = \{0,1\}, i = 1,2,..., n \end{cases} \end{aligned} \tag{3}$$

The assignment problem in Pareto optimization is presented in the following way:[2], [3], [4]

$$\max_{y \in Y} F(Y) = \sum_{i=1}^{n} \sum_{l=1}^{s} Q_{il} . y_{il}$$

$$subject\ \ to: \begin{cases} \sum_{i=1}^{n} y_{il} = 1, l = 1,2,..., s, \\ \sum_{l=1}^{s} y_{il} \le 1, i = 1,2,..., n, \\ y_{il} = \{0,1\}, i = 1,2,..., n, l = 1,2,..., s \end{cases} . \qquad (4)$$

Our main idea is to compare the results by changing the priorities of the criteria. Our problem, apart from being multiple objectives, is also two-step, a multiple objective solution is sought at each stage.

A Case

A company is financially impeded and has to do optimization. [8]

A set of abilities presents every employee: $a_1, a_2, ..., a_m$.

Some measures are available for every person and every criterion $p_{ij}, i = 1,2,..., n, j = 1,2,..., m$ and

$$P_{nxm} = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1m} \\ p_{21} & p_{22} & \cdots & p_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ p_{n1} & p_{n2} & \cdots & p_{nm} \end{pmatrix} . \qquad (5)$$

Every activity needs some abilities:

$$\begin{cases} d_1 = (a_{11}, a_{21}, ..., a_{m1}) \\ d_2 = (a_{12}, a_{22}, ..., a_{m2}) \\ \cdots \\ d_k = (a_{1k}, a_{2k}, ..., a_{mk}) \end{cases}, d_r = (a_{1r}, a_{2r}, ..., a_{mr}), r = 1,2,..., k, \qquad (6)$$

$$a_{jr} = \begin{cases} 0, ability \notin d_r \\ 1,, ability \in d_r \end{cases}, j = 1,2,..., m, r = 1,2,..., k.$$

In the matrix form:

$$A_{mxk} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1k} \\ a_{21} & a_{22} & \cdots & a_{2k} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mk} \end{pmatrix} . \qquad (7)$$

The general criterion giving relationship between abilities and positions is

$$D = P.A = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1m} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ p_{n1} & p_{n2} & \cdots & p_{nm} \end{pmatrix} . \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1k} \\ a_{21} & a_{22} & \cdots & a_{2k} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mk} \end{pmatrix} = \begin{pmatrix} d_{11} & d_{12} & \cdots & d_{1k} \\ d_{21} & d_{22} & \cdots & d_{2k} \\ \cdots & \cdots & \cdots & \cdots \\ d_{n1} & d_{n2} & \cdots & d_{nk} \end{pmatrix} . \qquad (8)$$

The weight matrix defines meaning of every action for every position:

$$W = \begin{pmatrix} w_{11} & w_{12} & \dots & w_{1n} \\ w_{21} & w_{22} & \dots & w_{2n} \\ \dots & \dots & \dots & \dots \\ w_{k1} & w_{k2} & \dots & w_{kn} \end{pmatrix}. \tag{9}$$

The new general criterion is obtained as linear combination of measures of success of every person and gives the efficiency: Ability→Activity→Position: [8]

$$Q = D.W = \begin{pmatrix} d_{11} & d_{12} & \dots & d_{1k} \\ d_{21} & d_{22} & \dots & d_{2k} \\ \dots & \dots & \dots & \dots \\ d_{n1} & d_{n2} & \dots & d_{nk} \end{pmatrix} . \begin{pmatrix} w_{11} & w_{12} & \dots & w_{1n} \\ w_{21} & w_{22} & \dots & w_{2n} \\ \dots & \dots & \dots & \dots \\ w_{k1} & w_{k2} & \dots & w_{kn} \end{pmatrix} = \begin{pmatrix} Q_{11} & Q_{12} & \dots & Q_{1n} \\ Q_{21} & Q_{22} & \dots & Q_{2n} \\ \dots & \dots & \dots & \dots \\ Q_{n1} & Q_{n2} & \dots & Q_{nn} \end{pmatrix}. \tag{10}$$

We solved two-stage problem.

At the first stage reduce number of positions to reduce costs. At this stage we solved a knapsack problem in post Pareto optimization.

At the second stage increase efficiency in reduced positions, we reassign employees. We solved an assignment problem in post Pareto optimization.

To post Pareto optimization we look same problem exchanging the places in the stages of the first stage. We are looking for efficiency to reduce the posts then redefining the employees.

Finally, we compare the results.

EXAMPLE

There are given following table: [8]

TABLE 1. Employees data

Positions	Salaries (lv./month)	Efficiency
Manager printer	1200	13504
Operational bookkeeper	760	5434
Operator	610	5878.5
Operator	610	4584
Operator	610	5924.5
Operator	610	4796
Operator	610	3393.5
Operator	610	4967
Programmer	1000	4586.4
Programmer	1000	3505.5
Driver	550	2278
Driver	550	2551
Driver	550	2964.7
Technician	800	2813.5
Technician	800	2950
Doorman	280	702.5
Doorman	280	812.6
Hygienist	400	1581
Hygienist	400	1856
Hygienist	400	1834

Solutions

Data is processed as described above andmade a post Paretto optimization: Salary optimization and Efficiency optimization, by appropriate software – MathLab. This is shown in following figures.

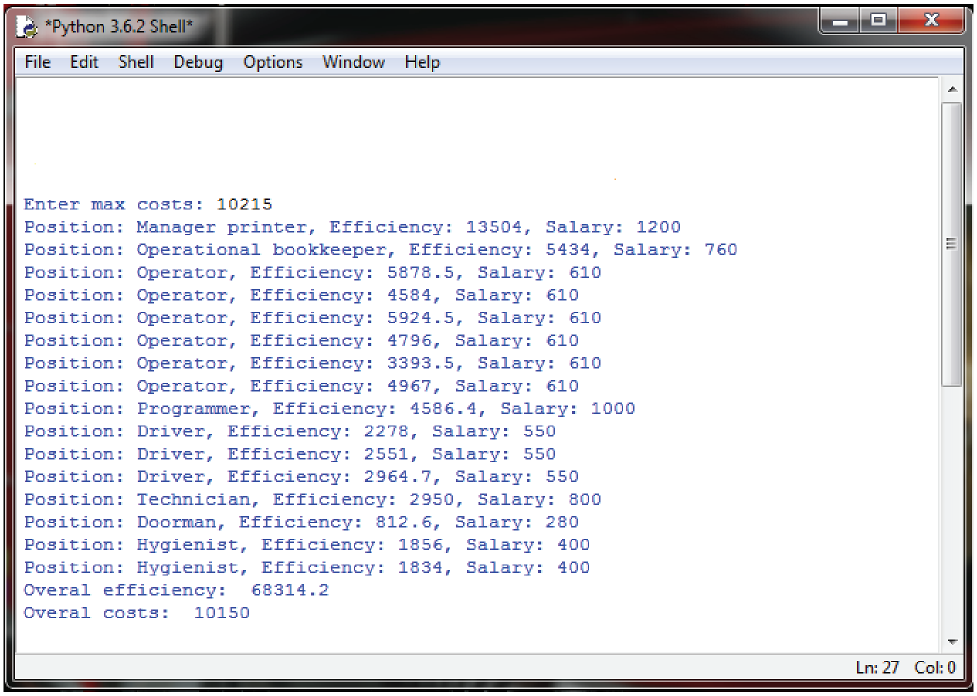

FIGURE 1. Salary optimization

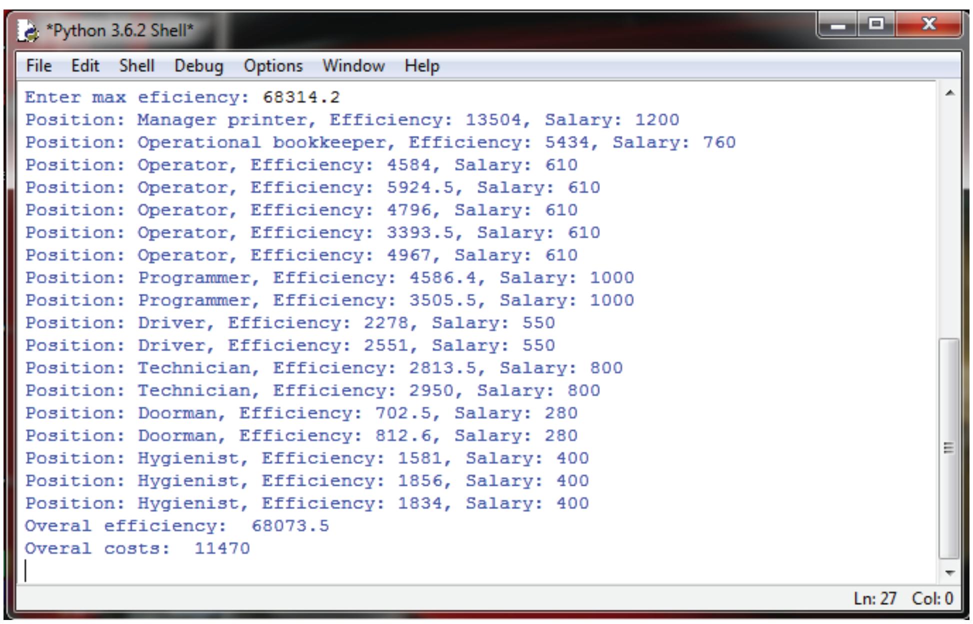

FIGURE 2. Efficiency optimization

Results

Salary optimization:

- Overall efficiency is 68 314.4.
- Overall costs are 10 150lv./month.

Efficiency optimization:

- Overall efficiency is 68 073.5.
- Overall costs are 11 470 lv./month.

With data set for this company it is better first to optimize by costs and then by efficiency, but this is not a rule. This is shown in following figure:

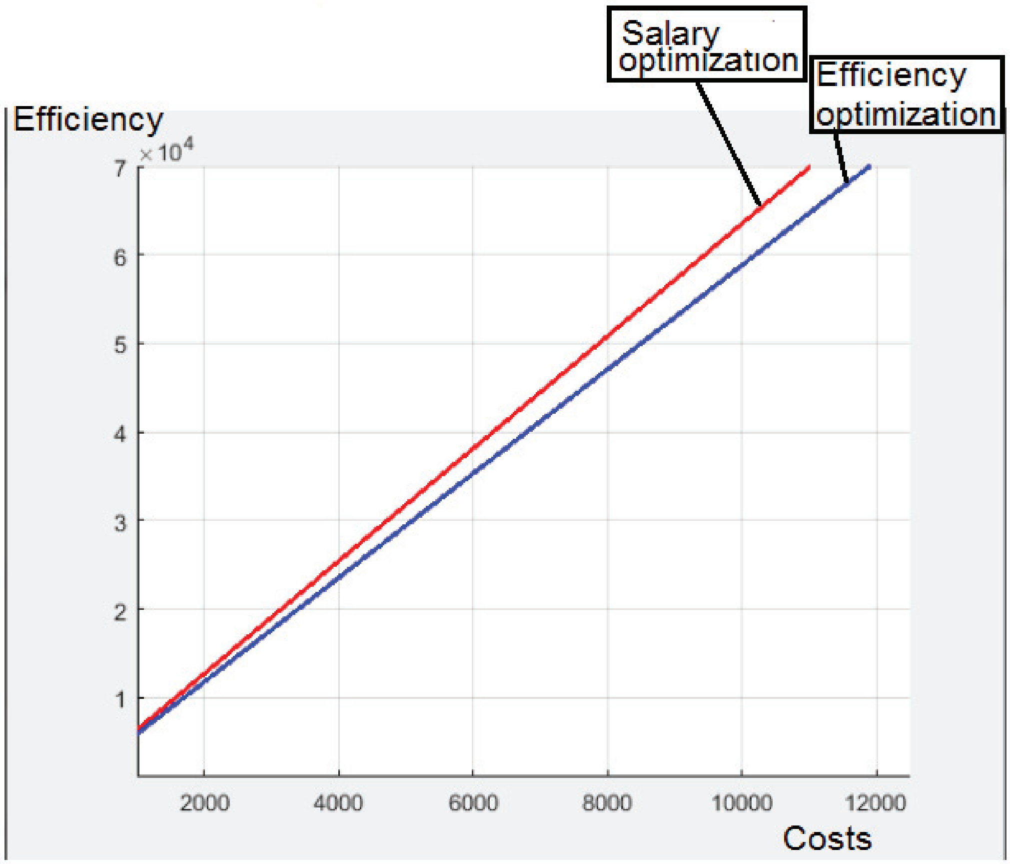

FIGURE 3. Graphic presentation ofResults from Salary and Efficiency optimization

We propose to the company exceeding efficiency for the account of costs.

CONCLUSION

Most of the modern research and applied optimization issues are multi-criteria and contradictory in principle. The choice of a system of evaluation criteria and their ranking by degree of significance does not lead to unambiguous interpretation and leads to subjective decisions. Another important feature of the multi-criteria problem is that they have no single solution. Overall, the result of these solutions is a series of so-called Pareto-optimal solutions resulting from Wilfried Pareto's proposal for optimal consistency. Since none of Pareto's optimized solutions are better than the other, it is necessary to find a unique solution that requires additional information and a subjective look at the compromise.[11], [12], [13], [14]

Multi-criteria optimization issues can be found in a variety of areas: products in the design process, financing, aircraft design, oil and gas industry, automotive design, or where optimal solutions must be made in the presence of compromises between two or more contradictory goals. Raising revenue and decreasing product value increasing productivity and minimizing fuel consumption of the vehicle; minimizing weight while increasing the maximum content of a component are examples of multi-purpose optimization problems.[11], [12], [13], [14]

If the multi-criterion problem is well-formed, there must be no separate solution that simultaneously reduces each goal to its greatest. In any case, one goal must have peaked so that when trying to optimize the goal further, the other goals "suffer" as a result. Finding such a solution and determining how this solution is better than many other such solutions is the goal in creating and solving a multi-criteria optimization problem. [11], [12], [13], [14]

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