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NONLINEAR ADAPTIVE OBSERVER DESIGN WITH EXPONENTIAL STATE AND PARAMETER ESTIMATION

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Abstract: The paper presents a new approach for nonlinear adaptive state observer design for nonlinear systems in adaptive observer canonical form. The adaptive observer and parameter estimator presented in the paper are globally uniformly exponentially stable. This is achieved by introducing a data accumulation process for the unknown parameters of the objective nonlinear system. This process is exponentially stable conditioned by the defined parametric identifiability criterion. The performance specifications of the nonlinear adaptive observer, parameter estimator, and data accumulation dynamics introduced are controllable. The approach is applied to a single-link flexible joint robot arm for illustration.

Keywords: nonlinear systems, nonlinear adaptive observers, parameter estimators, filtered transformation, exponential stability

INTRODUCTION

One of the earlier results solving the problem for joint state and parameter estimation for nonlinear systems is presented in [1] by defining adaptive observer canonical form (AOCF) for nonlinear systems with time-varying parameters. The same idea is utilized in [3] where conditions for transformation of nonlinear systems with constant parameters into AOCF are given. The filtered transformation [5] transforms linearly reparameterized systems in nonlinear observer canonical form into AOCF. The AOCF structure allows asymptotic nonlinear adaptive observer design by applying the Mayer-Kalman-Yakubovic lemma [9]. The asymptotic convergence of this observer is improved in [4] to exponential convergence with arbitrary rate. For systems with unmeasured but Lipschitz nonlinearities an adaptive observer design task is investigated in the works [17, 12] and solutions are given. All the above results are unified in [2] by defining a new more general AOCF. Recently the task for exact parameter estimation is solved in [10, 11, 18] but these approaches use unstable dynamics for some matrices. Exponential forgetting is utilized in [13, 14], and [15] to deal with this problem. A disadvantage of all methods introduced is the requirement for persistent excitation of the objective systems in order to achieve asymptotic state or parameter estimation which contradicts with the control goals.

The paper considers a method for nonlinear adaptive observer design in adaptive observer canonical form. The adaptive observer and the parameter estimator are globally uniformly exponentially stable without the unacceptable requirement for persistency of excitation. The method is illustrated by a dynamic simulation.

PROBLEM STATEMENT

The general multi-input multi-output nonlinear systems considered are described by the equations

$$\dot{\boldsymbol{\eta}} = \mathbf{f}_{\eta}(\boldsymbol{\eta},\mathbf{u}),\ \boldsymbol{\eta}(0)=\boldsymbol{\eta}_0, \tag{1a}$$

$$\mathbf{y} = \mathbf{h}(\boldsymbol{\eta},\mathbf{u}), \tag{1b}$$

where $\boldsymbol{\eta} \in B_{\eta} \subset R^n$, $\mathbf{u} \in B_u \subset R^r$, $\mathbf{y} \in B_y \subset R^m$ are the state, the control and the output vectors. The vector functions $\mathbf{f}(\boldsymbol{\eta},\mathbf{u}) \in B_f \subset R^n$, $\mathbf{h}(\boldsymbol{\eta},\mathbf{u}) \in B_h \subset R^m$ are sufficiently smooth in the considered domains of $\boldsymbol{\eta}$ and $\mathbf{u}$. It is assumed that system (1) is locally observable with observability indices $n_s > 0$, $s = 1,2,\ldots,m$, such that $n_1 + n_2 + \cdots + n_m = n$ and transformable into RGOCF [16]. The linearly reparameterized RGOCF (RRGOCF) for system (1) reads

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{g}(\mathbf{y},\bar{\mathbf{u}}) + \mathbf{G}(\mathbf{y},\bar{\mathbf{u}})\boldsymbol{\theta},\ \mathbf{x}(0)=\mathbf{x}_0 \tag{2a}$$

$$\mathbf{y} = \mathbf{C}\mathbf{x}, \tag{2b}$$

where $\mathbf{x} \in B_x \subset R^n$ is the state vector, $\dot{\mathbf{u}} \in B_{\dot{u}} \subset R^r$ is the control input time derivative, $\bar{\mathbf{u}} = [\mathbf{u}^T, \dot{\mathbf{u}}^T]^T$, $\bar{\mathbf{u}} \in R^{r^2}$, $\boldsymbol{\theta} \in R^p$ is the unknown parameter vector, $\mathbf{g}(\mathbf{y},\bar{\mathbf{u}}) \in B_g \subset R^n$, $\mathbf{G}(\mathbf{y},\bar{\mathbf{u}}) \in B_G \subset R^{n\times p}$, the matrix pairs $(\mathbf{A}_s, \mathbf{C}_s)$ $\mathbf{A}_s \in R^{n_s \times n_s}$, $\mathbf{C}_s \in R^{1\times n_s}$, $s = 1,2,\ldots,m$ are in single-output Brunovski form, and $\mathbf{A} = \mathrm{diag}[\mathbf{A}_1, \mathbf{A}_2, \ldots, \mathbf{A}_m]$, $\mathbf{C} = \mathrm{diag}[\mathbf{C}_1, \mathbf{C}_2, \ldots, \mathbf{C}_m]$. The model (2) is linear with respect to the unknown parameter vector $\boldsymbol{\theta}$ which is necessary condition for transformation into AOCF. The filtered transformation for multi-input multi-output systems in the form (2) is defined as

$$\mathbf{z} = \mathbf{x} - [\mathbf{T}_1^T, \mathbf{T}_2^T, \ldots \mathbf{T}_m^T]^T \boldsymbol{\theta}^T, \tag{3a}$$

$$\dot{\mathbf{T}}_s = \begin{bmatrix} \mathbf{0}_{1\times p} \\ \dot{\mathbf{M}}_s \end{bmatrix},\ s = 1,2,\ldots,m, \tag{3b}$$

$$\dot{\mathbf{M}}_s = \mathbf{A}_{b_s}\mathbf{M}_s + \mathbf{B}_{b_s}\mathbf{G}_s(\mathbf{y},\bar{\mathbf{u}}),\ \mathbf{M}_s(0) = \mathbf{0}_{(n_s-1)\times p}, \tag{3c}$$

where $\mathbf{z} \in B_z \subset R^n$ is the new state vector and the matrices of the filtered transformation dynamics are

$\mathbf{G}(\mathbf{y},\bar{\mathbf{u}}) = [\mathbf{G}_1(\mathbf{y},\bar{\mathbf{u}})^T, \ldots, \mathbf{G}_m(\mathbf{y},\bar{\mathbf{u}})^T]^T$, $\mathbf{G}_s \in R^{n_s \times p}$,

$\mathbf{A}_{b_s} = \left[-\bar{\mathbf{b}}_s \;\middle|\; \begin{array}{c} \mathbf{I}_{(n_s-2)\times(n_s-2)} \\ \hline \mathbf{0} \end{array} \right]$, $\mathbf{B}_{b_s} = [-\bar{\mathbf{b}}_s, \mathbf{I}_{(n_s-1)\times(n_s-1)}]$,

$s = 1,2,\ldots,m$ with Hurwitz polynomial coefficients vector $\mathbf{b}_s = [1, \bar{\mathbf{b}}_s^T]^T$, $\bar{\mathbf{b}}_s = [b_{s_1}, b_{s_2} \ldots, b_{s_{(n_s-1)}}]^T$. The transformation (3) transforms the RRGOCF (2) into AOCF reading

$$\dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \mathbf{g}(\mathbf{y},\bar{\mathbf{u}}) + \mathbf{B}\boldsymbol{\omega}^T\boldsymbol{\theta},\ \mathbf{z}(0) = \mathbf{z}_0, \tag{4a}$$

$$\mathbf{y} = \mathbf{C}\mathbf{z}, \tag{4b}$$

where the vector $\boldsymbol{\omega}^T = \mathbf{CAT} + \mathbf{CG}(\mathbf{y},\bar{\mathbf{u}})$, $\boldsymbol{\omega}^T \in B_{\omega} \subset R^{m\times p}$

and the matrix $\mathbf{B} = \mathrm{diag}(\mathbf{b}_1,\ldots,\mathbf{b}_m)$, $\mathbf{B} \in R^{n\times m}$. The state of system (4) is represented by the equation

$$\mathbf{z} = \mathbf{F}\mathbf{z}_0 + \mathbf{\Omega}\mathbf{\theta} + \xi \quad (5)$$

where $\mathbf{F}$, $\mathbf{\Omega}$ and ξ have the dynamics

$$\dot{\mathbf{F}} = (\mathbf{A}-\mathbf{K}\mathbf{C})\mathbf{F},\ \mathbf{F}(0) = \mathbf{I}_{n\times n}, \quad (6a)$$

$$\dot{\mathbf{\Omega}} = (\mathbf{A}-\mathbf{K}\mathbf{C})\mathbf{\Omega} + \mathbf{B}\mathbf{\omega}^T,\ \mathbf{\Omega}(0) = \mathbf{0}, \quad (6b)$$

$$\dot{\xi} = (\mathbf{A}-\mathbf{K}\mathbf{C})\xi + \mathbf{K}\mathbf{y} + \mathbf{g}(\mathbf{y},\overline{\mathbf{u}}),\ \xi(0) = \mathbf{0}. \quad (6c)$$

Since the signals of the vector $\mathbf{\omega}$, $\mathbf{y}$, and $\mathbf{g}(\mathbf{y},\overline{\mathbf{u}})$ are bounded by definition then the stability of the these dynamics is provided by stabilization of the matrix $(\mathbf{A}-\mathbf{K}\mathbf{C})$ via the design matrix $\mathbf{K}$. Considering equation (5) the output equation (4b) reads

$$\mathbf{y} = \mathbf{C}(\mathbf{\Omega}\mathbf{\theta} + \mathbf{F}\mathbf{z}_0 + \xi)$$

By defining the vector $\mathbf{\varphi} = \mathbf{y} - \mathbf{C}\xi$, the generalized parameter vector $\mathbf{a} = [\mathbf{\theta}^T, \mathbf{z}_0^T]^T$ and the regression matrix $\mathbf{W} = \mathbf{C}[\mathbf{\Omega},\mathbf{F}]$ the last equation is transformed into regression form

$$\mathbf{\varphi} = \mathbf{W}\mathbf{a}. \quad (7)$$

Base on this form the following dynamic equations are defined

$$\dot{\mathbf{Q}} = \mathbf{\Lambda}_q(\mathbf{Q}_r - \mathbf{Q})(\mathbf{W}^T\mathbf{R}_w\mathbf{W} + \mathbf{Q}^T\mathbf{R}_q\mathbf{Q}),\ \mathbf{Q}(0) = \mathbf{0}, \quad (8a)$$

$$\dot{\mathbf{\psi}} = \mathbf{\Lambda}_q(\mathbf{Q}_r - \mathbf{Q})(\mathbf{W}^T\mathbf{R}_w\mathbf{\varphi} + \mathbf{Q}^T\mathbf{R}_q\mathbf{\psi}),\ \mathbf{\psi}(0) = \mathbf{0}. \quad (8b)$$

where $\mathbf{Q} \in R^{p_a\times p_a}$, $p_a = p + n$ is data accumulation matrix, $\mathbf{\Lambda}_q \in \mathbf{R}^{p_a\times p_a}$, $\mathbf{\Lambda}_q > 0$ is an eigenvalues matrix, $\mathbf{Q}_r \in R^{p_a\times p_a}$ is full rank reference matrix, and $\mathbf{R}_w \in R^{m\times m}$, $\mathbf{R}_q \in R^{p_a\times p_a}$ are positive definite weighting matrices. Equations (8a) and (8b) are related by the equation $\mathbf{\psi} = \mathbf{Q}\mathbf{a}$, $\mathbf{\psi} \in R^{p_a}$ hence, algebraic prediction error can be defined as

$$\widetilde{\mathbf{\psi}} = \mathbf{\psi} - \mathbf{Q}\hat{\mathbf{a}}, \quad (9a)$$

$$\widetilde{\mathbf{\psi}} = \mathbf{Q}\widetilde{\mathbf{a}} \quad (9b)$$

where $\hat{\mathbf{a}} = [\hat{\mathbf{\theta}}^T, \hat{\mathbf{z}}_0^T]^T$ is the generalized parameter estimation vector with estimation error $\widetilde{\mathbf{a}} = \mathbf{a} - \hat{\mathbf{a}}$. The nonlinear adaptive observer in the AOCF (4) has the structure

$$\dot{\hat{\mathbf{z}}} = \mathbf{A}\hat{\mathbf{z}} + \mathbf{g}(\mathbf{y},\overline{\mathbf{u}}) + \mathbf{B}\mathbf{\omega}^T\hat{\mathbf{\theta}} + \mathbf{N}(\mathbf{y} - \hat{\mathbf{y}}),\ \hat{\mathbf{z}}(0) = \hat{\mathbf{z}}_0, \quad (10a)$$

$$\hat{\mathbf{y}} = \mathbf{C}\hat{\mathbf{z}}. \quad (11b)$$

where $\mathbf{N} \in R^{n\times m}$ is the observer gain matrix and the parameter estimates $\hat{\mathbf{\theta}}$ are generated by the adaptive gradient parameter estimator

$$\dot{\hat{\mathbf{a}}} = \mathbf{\Gamma}([\mathbf{\omega}^T, \mathbf{0}_{m\times n}]^T\widetilde{\mathbf{y}} + \mathbf{Q}^T\mathbf{\Gamma}_\psi\widetilde{\mathbf{\psi}}),\ \hat{\mathbf{a}}(0) = \mathbf{0}, \quad (12)$$

with matrix $\mathbf{\Gamma} = \mathbf{\Gamma}^T > 0$ and weighting matrix $\mathbf{\Gamma}_\psi = \mathbf{\Gamma}_\psi^T > 0$.

STABILITY ANALYSIS

This section is devoted to stability analysis of the data accumulation process, the nonlinear adaptive observer, and the parameter estimator. In this context a special property of the objective system is defined by the following definition.

Definition 1 – Parametric identifiability (PI)
The dynamic system (4) is parametrically identifiable if there exist a finite time interval $[t_0, t_1]$, $t_1 < \infty$ in which

$$\mathbf{\Phi}(t_0,t_1) = \mathbf{\Phi}_0 + \int_{t_0}^{t_1} \mathbf{W}^T(\tau)\mathbf{W}(\tau)d\tau > 0,\ \mathbf{\Phi}_0 = \mathbf{W}_0^T\mathbf{W}_0 \ge 0 \quad (13)$$

where $\mathbf{W}(t) = \mathbf{C}[\mathbf{\Omega}(t),\mathbf{F}(t)]$ is the system regression matrix defined by the dynamics (6). □

This property characterizes the possibility to exactly estimate all the parameters of a given nonlinear system.

Theorem 1 – Global uniform exponential stability of the matrix $\mathbf{Q}(t)$
If the criterion (13) is met then the data accumulation matrix $\mathbf{Q}(t)$ will be globally uniformly exponentially stable with respect to the error $\mathbf{E}(t) = \mathbf{Q}_r - \mathbf{Q}(t)$ and $\mathrm{rank}\mathbf{Q}(t \ge t_1) = p_a$, otherwise, $\mathbf{Q}(t) \in L_\infty$ and $\mathrm{rank}\mathbf{Q}(t) < p_a$. □

Proof. See theorems 3.1 and 3.2 in [6].

The dynamics of the algebraic observer error $\widetilde{\mathbf{z}} = \mathbf{z} - \hat{\mathbf{z}}$ is

$$\dot{\widetilde{\mathbf{z}}} = (\mathbf{A}-\mathbf{N}\mathbf{C})\widetilde{\mathbf{z}} + \mathbf{B}\mathbf{\omega}^T\widetilde{\mathbf{\theta}}. \quad (14a)$$

$$\widetilde{\mathbf{y}} = \mathbf{C}\widetilde{\mathbf{z}} \quad (14b)$$

Stabilizing the matrix $(\mathbf{A}-\mathbf{N}\mathbf{C})$ is not enough for asymptotic stability of the error $\widetilde{\mathbf{z}}$ since the parameter estimation errors $\widetilde{\mathbf{\theta}}$ are unknown. The stability of the adaptive observer is investigated with the Lyapunov function candidate

$$V(\widetilde{\mathbf{z}},\widetilde{\mathbf{a}},t) = \frac{1}{2}\widetilde{\mathbf{z}}^T\mathbf{P}\widetilde{\mathbf{z}} + \frac{1}{2}\widetilde{\mathbf{a}}^T\mathbf{\Gamma}^{-1}\widetilde{\mathbf{a}}, \quad (15)$$

whose time derivative considering the dynamics (14) reads

$$\dot{V}(\widetilde{\mathbf{z}},\widetilde{\mathbf{a}},t) = -\widetilde{\mathbf{z}}^T\mathbf{S}\widetilde{\mathbf{z}} + \widetilde{\mathbf{\theta}}^T\mathbf{\omega}\mathbf{B}^T\mathbf{P}\widetilde{\mathbf{z}} - \widetilde{\mathbf{a}}^T\mathbf{\Gamma}^{-1}\dot{\hat{\mathbf{a}}}. \quad (16)$$

where $\mathbf{S} = \mathbf{S}^T > 0$, and $(\mathbf{A}-\mathbf{N}\mathbf{C})^T\mathbf{P} + \mathbf{P}(\mathbf{A}-\mathbf{N}\mathbf{C}) = -\mathbf{S}$ is the Lyapunov equation. The matrix $\mathbf{B}$ consists of the Hurwitz polynomial coefficients vectors $\mathbf{b}_s$, $s = 1,2,\ldots,m$ then, the dynamics (14) has strictly positive real transfer function

$$W(s) = [\mathbf{C}(s\mathbf{I} - (\mathbf{A}-\mathbf{N}\mathbf{C}))^{-1}\mathbf{B}]$$

as the matrix triple $((\mathbf{A}-\mathbf{N}\mathbf{C}),\mathbf{C},\mathbf{B})$ is observable. For such systems the Mayer-Kalman-Yakubovic lemma states that there exists a positive definite solution $\mathbf{P}$ of the Lyapunov equation for which $\mathbf{B}^T\mathbf{P} = \mathbf{C}$. Applying this result and replacing the adaptive parameter estimator dynamics (12) in (16) yields

$$\dot{V}(\widetilde{\mathbf{z}},\widetilde{\mathbf{a}},t) = -\widetilde{\mathbf{z}}^T\mathbf{S}\widetilde{\mathbf{z}} - \widetilde{\mathbf{a}}^T\mathbf{Q}^T\mathbf{\Gamma}_\psi\mathbf{Q}\widetilde{\mathbf{a}} = -W_e(\widetilde{\mathbf{z}},\widetilde{\mathbf{a}}),\ V(t_0) = V_0$$

Depending on the parametric identifiability criterion (13) there are two cases. The first case is when the objective nonlinear system is not parametrically identifiable. In this case $\mathrm{rank}\,\mathbf{Q}(t) < p_a$ hence, the matrix $\mathbf{Q}^T\mathbf{\Gamma}_\psi\mathbf{Q} \ge 0$. This implies that $\dot{V}(\widetilde{\mathbf{z}},\widetilde{\mathbf{a}},t) \le 0$ thus, $\widetilde{\mathbf{z}} \in L_\infty$, $\widetilde{\mathbf{a}} \in L_\infty$ and $\dot{\widetilde{\mathbf{z}}} \in L_\infty$. Taking the derivative of the function $W_e(\widetilde{\mathbf{z}},\widetilde{\mathbf{a}})$ yields

$$\dot{W}_e(\widetilde{\mathbf{z}},\widetilde{\mathbf{a}}) = 2\widetilde{\mathbf{z}}^T\mathbf{S}\dot{\widetilde{\mathbf{z}}} + \widetilde{\mathbf{a}}\mathbf{Q}^T\mathbf{\Gamma}_\psi\dot{\mathbf{Q}}\widetilde{\mathbf{a}} + \widetilde{\mathbf{a}}\mathbf{Q}^T\mathbf{\Gamma}_\psi\mathbf{Q}\dot{\widetilde{\mathbf{a}}}$$

The boundedness of the matrix $\mathbf{Q}$ is proved by theorem 1, the regression matrix $\mathbf{W} \in L_\infty$ since, $\mathbf{F} \in L_\infty$ and $\mathbf{\Omega} \in L_\infty$ hence, the dynamics $\dot{\mathbf{Q}}$, and $\dot{\widetilde{\mathbf{a}}}$ are globally bounded. Therefore $\dot{W}_e(\widetilde{\mathbf{z}},\widetilde{\mathbf{a}})$ is bounded because all its signals are bounded, thus the function $W_e(\widetilde{\mathbf{z}},\widetilde{\mathbf{a}})$ is uniformly continuous. The Lyapunov function (15) is decrescent and bounded from below by zero hence it converges $V(\widetilde{\mathbf{z}}(\infty),\widetilde{\mathbf{a}}(\infty),\infty) = V_f$ as $t \to \infty$ therefore

$$\int_{t_0}^{t} W_e(\widetilde{\mathbf{z}}(\tau),\widetilde{\mathbf{a}}(\tau))\,d\tau = -\int_{t_0}^{t} \dot{V}(\widetilde{\mathbf{z}}(\tau),\widetilde{\mathbf{a}}(\tau),\tau)\,d\tau = V_0 - V_f,$$

and $\lim_{t\to\infty}\int_{t_0}^{t} W_e(\widetilde{\mathbf{z}}(\tau),\widetilde{\mathbf{a}}(\tau))\,d\tau$ exists and is finite. Applying the Barbalat's lemma $\lim_{t\to\infty} W_e(\widetilde{\mathbf{z}},\widetilde{\mathbf{a}}) = \mathbf{0}$ and as a consequence $\lim_{t\to\infty}\widetilde{\mathbf{z}} = \mathbf{0}$, and $\lim_{t\to\infty}\mathbf{Q}\widetilde{\mathbf{a}} \to \mathbf{0}$. This means that the nonlinear adaptive observer error $\widetilde{\mathbf{z}}$ will be globally asymptotically stable but the unknown parameters will be Lyapunov stable only conditioned by $\mathrm{rank}\mathbf{Q} < p_a$. The second case is when the objective nonlinear system is parametrically identifiable. In this case $\mathrm{rank}\,\mathbf{Q} = p_a$, $\forall t \ge t_1$, the matrix $\mathbf{Q}^T\mathbf{\Gamma}_\psi\mathbf{Q} > 0$ and the following inequalities hold

$$V(\mathbf{e},t) \le \frac{1}{2}\lambda_{max}[\mathbf{S}]\mathbf{e}^T\mathbf{e}, \ \mathbf{S} = \mathrm{diag}[\mathbf{P}, \mathbf{\Gamma}^{-1}]$$

$$\dot{V}(\mathbf{e},t) \le -\lambda_{min}[\mathbf{H}]\mathbf{e}^T\mathbf{e}, \ \mathbf{H} = \mathrm{diag}[\mathbf{S}, \mathbf{Q}^T\mathbf{\Gamma}_\psi\mathbf{Q}]$$

where $\mathbf{e} = [\tilde{\mathbf{z}}^T, \tilde{\mathbf{a}}^T]^T$ is generalized error vector. The time derivative $\dot{V}(\mathbf{e},t)$ can be expressed as a linear equation with respect to the Lyapunov function $V(\mathbf{e},t)$ in the form

$$\dot{V}(\mathbf{e},t) \le -2\sigma(t)V(\mathbf{e},t), \ V(\mathbf{e}(t_1),t_1) = V_0,$$

where $\sigma(t) = \lambda_{min}[\mathbf{H}(t)]/\lambda_{max}[\mathbf{S}]$, with solution

$$V(t) \le -e^{2\int_{t_1}^{t}\sigma(\tau)d\tau} V_0$$

Since $\sigma(t) > 0$, $\forall t \ge t_1$ then the integral in the above equation tends to infinity when $t \to \infty$. The eigenvalues of the matrices $\mathbf{H}$ and $\mathbf{S}$ can be chosen appropriately via the matrices $\mathbf{\Gamma}$ and $\mathbf{\Gamma}_\psi$. The convergence specifications of the data accumulation matrix $\mathbf{Q}(t)$ to the full rank reference matrix $\mathbf{Q}_r$ can also be controlled. Therefore, the lower bound of $\sigma(t)$ can be appropriately chosen. This implies that the nonlinear adaptive observer (10) and the parameter estimator (12) are globally uniformly exponentially stable.

APPLICATION OF THE APPROACH

The approach is applied to a single-link flexible joint robot arm driven by a permanent magnet synchronous motor controlled in current mode. The objective system model when the flexible joint is modeled as a linear torsional spring is

$$J_d\ddot{q} + mgl\sin(q) + f_d\dot{q} + k_s q = k_s q_m, \quad (17a)$$

$$J_m\ddot{q}_m + f_m\dot{q}_m + k_s(q_m - q) = k_t u, \quad (17b)$$

where q and q_m are the angular positions of the arm and the motor shaft, $u = i_d$ is the torque current of the motor. The system parameters are m, l – mass and length of the rotating link, k_s – spring constant, J_d and J_m – inertias of the link and the motor shaft, f_d and f_m – viscous friction coefficients, k_t – torque constant. The measured system variables are the angular positions of the arm q and the motor shaft q_m. By choosing the state space vector

$$\mathbf{x} = [x_1, x_2, x_3, x_4]^T = [q, \frac{f_d}{J_d}q + \dot{q}, q_m, \frac{f_m}{J_m}q_m + \dot{q}_m]^T$$

the model (17) is transformed in RRGOCF (2) reading

$$\begin{aligned} \dot{x}_1 &= x_2 - \theta_1 x_1 \\ \dot{x}_2 &= -\theta_2\sin(x_1) - \theta_3(x_1 - x_3) \end{aligned} \quad (18a)$$

$$\begin{aligned} \dot{x}_3 &= x_4 - \theta_4 x_3 \\ \dot{x}_4 &= \theta_5(x_1 - x_3) + \theta_6 u \end{aligned} \quad (18b)$$

$$y_1 = x_1, \ y_2 = x_3. \quad (18c)$$

where the model parameters are

$$\theta_1 = \frac{f_d}{J_d}, \ \theta_2 = \frac{mgl}{J_d}, \ \theta_3 = \frac{k_s}{J_d}, \ \theta_4 = \frac{f_m}{J_m}, \ \theta_5 = \frac{k_s}{J_m}, \ \theta_6 = \frac{k_t}{J_m}.$$

The system is single-input multi-output with one input u, two outputs y_1, y_2 and observability indices $n_1 = 2$, $n_2 = 2$. The matrices for the filtered transformation dynamics (3) are

$$\mathbf{M}_1 = [m_{11}, m_{12}, m_{13}, m_{14}, m_{15}, m_{16}]$$

$$\mathbf{M}_2 = [m_{21}, m_{22}, m_{23}, m_{24}, m_{25}, m_{26}]$$

$$\mathbf{A}_{b_1} = -b_{11}, \ \mathbf{B}_{b_1} = [-b_{11}, 1], \ \mathbf{A}_{b_2} = -b_{21}, \ \mathbf{B}_{b_2} = [-b_{21}, 1]$$

$$G_1 = \begin{bmatrix} -y_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\sin(y_1) & y_2 - y_1 & 0 & 0 & 0 \end{bmatrix},$$

$$G_2 = \begin{bmatrix} 0 & 0 & 0 & -y_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & y_1 - y_2 & u \end{bmatrix}.$$

The AOCF (4) is defined by the vectors and matrices $\mathbf{b}_1 = [1, b_{11}]^T$, $\mathbf{b}_2 = [1, b_{21}]^T$, $\mathbf{B} = \mathrm{diag}[\mathbf{b}_1, \mathbf{b}_2]$, $\mathbf{g}(\mathbf{y}, \bar{\mathbf{u}}) = \mathbf{0}_{4\times1}$,

$$\boldsymbol{\omega}^T = \begin{bmatrix} m_{11} - y_1 & m_{12} & m_{13} & m_{14} & m_{15} & m_{16} \\ m_{21} & m_{22} & m_{23} & m_{24} - y_2 & m_{25} & m_{26} \end{bmatrix},$$

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \ \mathbf{A}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \ \mathbf{A} = \mathrm{diag}[\mathbf{A}_1, \mathbf{A}_2],$$

with unknown parameter vector $\boldsymbol{\theta} = [\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6]^T$. The observer gain matrix $\mathbf{N}$ and the filters (6) matrix $\mathbf{K}$ are

$\mathbf{n}_1 = [n_{11}, n_{12}]^T$ $\mathbf{n}_2 = [n_{21}, n_{22}]^T$, $\mathbf{N} = \mathrm{diag}[\mathbf{n}_1, \mathbf{n}_2]$,

$\mathbf{k}_1 = [k_{11}, k_{12}]^T$ $\mathbf{k}_2 = [k_{21}, k_{22}]^T$, $\mathbf{K} = \mathrm{diag}[\mathbf{k}_1, \mathbf{k}_2]$.

SIMULATION AND SYSTEM TIME RESPONSES

In this section the nonlinear adaptive system, including the nonlinear adaptive observer and the parameter estimator is dynamically simulated. The control input $u(t)$ implements an output feedback tracking closed-loop system and it follows the example in [7]. The PMSM servomotor used is Lenze MDSKS071-03 with shaft inertia $J = 6\times10^{-4}$ kg.m^2, viscous friction coefficient $f_m = 0.002$ Nm.s rated current $I_N = 4.2$ A, rated angular speed $\omega_N = 356$ rad/s, and torque constant $k_t = 1.37$ Nm/A. The link with length $l = 0.5$ m and mass $m = 0.5$ kg is attached to the motor shaft via a linear torsional spring characterized by the constant $k_s = 1.6$. The attached link viscous friction coefficient is $f_d = 0.001$ Nm.s and the gravity constant is $g = 9.81$ m s^{-2}. The initial conditions that are not defined yet are $\mathbf{x}_0 = [-\pi, 0, -\pi, 0]$, $\mathbf{z}_0 = \mathbf{x}_0$, $\hat{\mathbf{z}}_0 = \mathbf{0}_{1\times4}$, $\hat{\mathbf{a}}_0 = \mathbf{0}_{1\times10}$, and $\mathbf{\Gamma} = \mathbf{I}_{10\times10}$. The data accumulation process performance specifying matrices and the full rank reference matrix are $\mathbf{\Lambda}_q = 10\,\mathbf{I}_{10\times10}$, $\mathbf{R}_w = \mathrm{diag}\,(80, 80)$, $\mathbf{R}_q = 80\,\mathbf{I}_{10\times10}$ $\mathbf{\Gamma}_\psi = 4\,\mathbf{I}_{10\times10}$, $\mathbf{Q}_r = 2\,\mathbf{I}_{10\times10}$. The vectors $\mathbf{b}_1$, $\mathbf{b}_2$ coefficients are $b_{11} = 5$, $b_{21} = 5$. The elements of the matrices $\mathbf{N}$, $\mathbf{K}$ are $n_{11} = 40$, $n_{12} = 400$, $n_{21} = 40$, $n_{22} = 400$, $k_{11} = 10$, $k_{12} = 25$, $k_{21} = 10$, $k_{22} = 25$ providing desired poles double observer pole $p_{obsv} = -20$ and double filters pole $p_f = -5$ for the two subsystems. The dynamic equations (4), (6), (8), (10), (12) with the vector and the matrices defined in the previous section are dynamically simulated. The nonlinear adaptive state observer time evolution in adaptive observer canonical form is displayed on figure 1. The parameter estimator responses are depicted on figure 2. The box on figure 2 shows the time evolution of the $\mathbf{\Phi}$ matrix rank that determines the

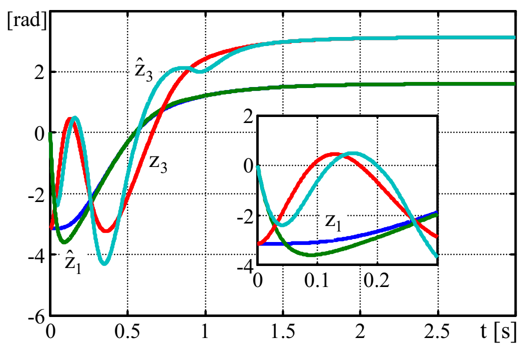

Figure 1: Nonlinear adaptive state observer responses

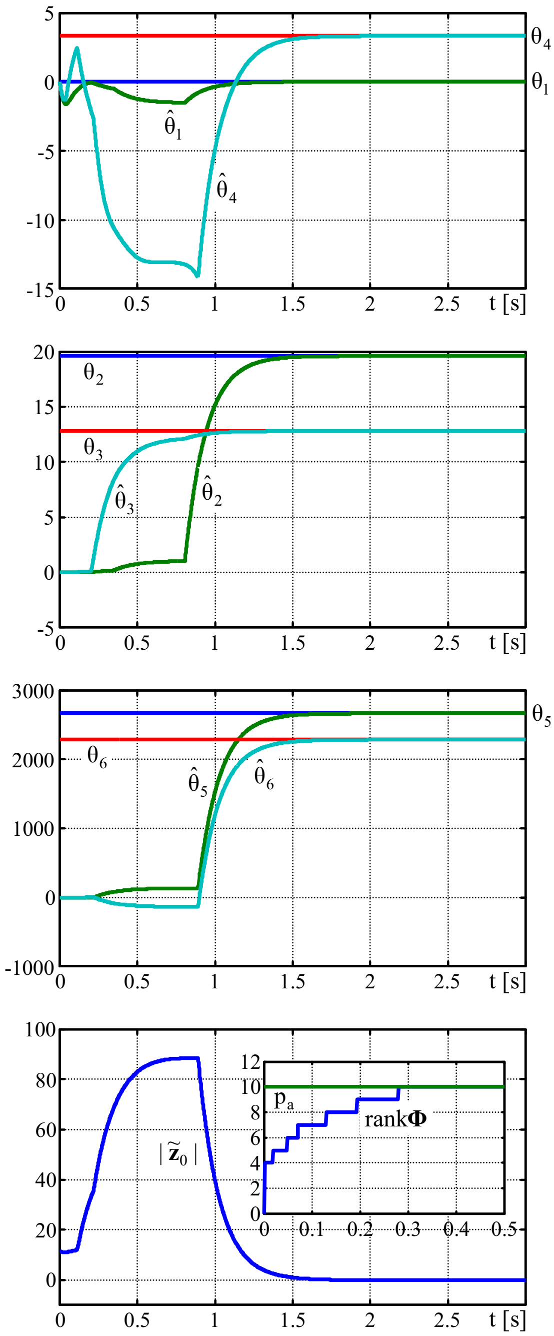

Figure 2: Parameter estimator dynamics

parametric identifiability property of the objective nonlinear system. The necessary condition for exponential estimation of the state is the exponential parameter estimation which is confirmed by the simulation. The overall adaptive system time evolution has settling time 1.5s conditioned by the matrices $\mathbf{\Lambda}_q$, $\mathbf{Q}_r$, $\mathbf{R}_w$, $\mathbf{R}_q$, $\mathbf{\Gamma}_\psi$ $\mathbf{\Gamma}_a$, and $\mathbf{N}$ where bigger values lead to faster responses.

CONCLUSIONS

The paper has presented a new approach for globally exponentially stable nonlinear adaptive state observer design in AOCF. The approach is based on stable data accumulation process and the parametric identifiability property of the objective nonlinear system. The performance specifications of the nonlinear adaptive observer, parameter estimator and data accumulation process is controllable via the design matrices $\mathbf{N}$, $\mathbf{K}$, $\mathbf{\Gamma}$, $\mathbf{\Gamma}_\psi$, $\mathbf{\Lambda}_q$, $\mathbf{Q}_r$, $\mathbf{R}_w$, and $\mathbf{R}_q$ where the bigger the values the faster the responses. The exact parameter estimation process does not require the unacceptable condition for persistent excitation of the objective nonlinear system.

The method proposed for nonlinear adaptive observer design can be applied to nonlinear systems which are transformable in AOCF. The approach provides exponential state and parameter estimation with controllable performance without the inconsistent with the control goals persistency of excitation condition and can be used in advanced high performance nonlinear adaptive closed-loop systems design.

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