

A VARIANT OF MODIFIED ALGORITHM FOR PROCESSING A PACKET OF REFLECTED ULTRAWIDEBAND PULSES

ВАРИАНТ НА МОДИФИЦИРАН АЛГОРИТЪМ ЗА ОБРАБОТКА НА ПАКЕТ ОТРАЗЕНИ СВРЪХКРАТКИ ИМПУЛСИ

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Abstract: This study synthesizes a variant of a modified algorithm for measuring the time delay of a signal reflected from a small-sized (point) target, using an ultrawideband (UWB) sequence of ultra-short (US) probing pulses. The research is a continuation of a series of publications dedicated to the possibilities of reducing the number of operations during analog-to-digital converter (ADC) discretization of the reflected signal.

Keywords: modified algorithm; ultrawideband (UWB) sequence; reflected signal discretization.

1. Introduction.

In recent years, the use of ultra-short transmitted pulses (USP), as a typical representative of ultrawideband signals (UWBS), has become relevant in various fields: radio reconnaissance, electronic warfare (EW), for high-precision distance detection to ultra-small objects (UAVs, drones, SLAs, etc., commonly referred to as remotely piloted aircraft systems - RPAS)². In radiolocation, UWBS are used for:

detection, tracking, and control of small-sized and point target; for obtaining high-resolution radio images; radio mapping of specific terrains, and more. Finding and developing new and modified algorithms for digital processing of US pulses (USP) is one of the constant tasks, with the aim of finding a relatively universal processing method.

2. Prerequisites and ways to solve the problem.

This study synthesizes a variant of a modified algorithm for measuring

the time delay of a signal reflected from a small-sized (point) target, using an ultrawideband (UWB) sequence of ultra-short (US) probing pulses. The research is a continuation of a series of publications dedicated to the possibilities of reducing the number of operations during analog-to-digital converter (ADC) discretization of the reflected signal and increasing the radar's resolution.

When synthesizing the algorithm, a UWB reflected signal with a known "bell-shaped" envelope is used. In one of the subsequent publications on the topic, the operation of the proposed variant of the modified algorithm will be simulated and the algorithm's performance will be checked even with a priori unknown envelope shape of the reflected US signal, and under conditions of Gaussian noise in the received sequence of reflected pulses.

One of the challenges in processing received US signals is the need for high-speed and expensive analog-to-digital converters (ADCs) and a large volume of resources, which makes real-time processing of USPs reflected from the target, often lasting several picoseconds, impossible. Most often, the processing of signals reflected from point targets (RPAS) is carried out in three stages:

1. Discretization of the received sequence of USPs, reflected from the illuminated small-sized target;
2. Determination of the amplitudes of the pulses in the packet, usually in complex form;
3. Determination of the moment of appearance of the reflected signal by applying known and well-developed spectral methods, after which the distance to the illuminated object is determined by known procedures.

3. Solution to the investigated problem.

One approach to solving the problem described above is to apply a low-pass filter (LPF), briefly called a "preliminary filter", at the receiver input.

The study was conducted in three stages:

1. In the first stage, an algorithm for processing USPs reflected from small-sized targets was synthesized;
2. During the second stage, simulation procedures were conducted in a MATLAB environment to determine a suitable resonant frequency for tuning the "preliminary filter" with the aim of the best matching with the frequency of the input packet of received USPs;
3. In the third stage, the algorithm's performance under noisy conditions was tested.

To simplify the study, a "bell-shaped" envelope form and a "Nyquist filter" will be used, which provides some freedom in selecting the initial conditions for algorithm synthesis.

Initial conditions for conducting the first stage of the study:

1. At the receiver input, a signal reflected from the observed object is observed, which can be represented as: $s(t - \tau)$, where τ is the unknown moment of appearance of the reflected signal taking values in the interval $[0 - T]$, and $T \gg t$.
2. Envelope shape of the pulse – "bell-shaped", with a mathematical model of the form:

$$s(t) = A \exp(-a \tau t^2)$$

Such an envelope shape was chosen for the simplicity of analytical analysis and the relatively easy physical realization.

Duration of USP - 20 ps, then f_m will take a value equal to $\alpha > 2 * 10^{-11}$ s.

3. Discretization interval: According to the Sampling Theorem, the condition $\Delta t \leq 5 * 10^{-12}$ s must be met.

4. Observation interval: $T = 1 * 10^{-3}$ s (1ms), and for the entire observation period, $T/\Delta t$ samples will be required.

With these accepted time intervals, the number of samples for subsequent processing will exceed $2 * 10^8$. Thus, for discretization, even if it is considered principally possible to perform, excessively large resources and corresponding high ADC speed will be required.

Modern publications describe various methods for processing USPs, including under noisy conditions. The goal is to measure the moment of reception of the reflected signal and determine the time delay relative to the emitted signal, and from there, the distance to the target.

Such examples are:

- Using classical correlation methods - these methods would require ADCs with very high sampling rates for direct digital processing to measure the time delay. For correlation reception with accumulation (Kalman filtering), a delay line (DL) with a very large number of outputs would be needed. Given requirements for high probability of correct detection, measurement accuracy, and high distance resolution the practical impossibility of such classical measurement of the arrival time of the reflected signal is observed.

- Applying the "Compressive Sensing" approach - this method presupposes the availability of complex and expensive equipment for signal compression and ADCs with very high sampling rates.

An approach is proposed where the packet of useful periodic input signals is fed to the input of a so-called "preliminary filter" with selected parameters:

1. The mathematical model of the useful reflected signals can be represented as a12

periodic sequence (packet) of ultra-short pulses with the expression:

$$(1) S(t - \tau) = A \sum_{n=-\infty}^{\infty} F(t - \tau - nT),$$

where: T – observation period; τ – the unknown moment of arrival of the useful signal, which needs to be determined; $F(t)$ – a function describing the chosen envelope – "bell-shaped" form of the reflected pulse within the packet.

2. Since, according to the chosen model - (1), the signal is a periodic function, it can be expanded into a Fourier series, i.e.:

$$(2) F(t - nT) = \sum_{m=-\infty}^{\infty} a_m e^{(j2\pi mt / T)}$$

where: $a_m = 1/T \int_0^T F(t) e^{(-j2\pi mt / T)} dt$ – Fourier series coefficients (these coefficients can also be in complex form).

3. Then expression (1) can be transformed into the following form:

$$(3) s(t, \tau) = A \sum_{m=-\infty}^{\infty} \dot{a}_m e^{(j2\pi m(t-\tau) / T)} = \sum_{m=-\infty}^{\infty} \dot{a}_m \dot{b}_m e^{(j2\pi mt / T)},$$

where: $\dot{b}_m = A e^{(-j2\pi m\tau / T)}$ – is the exponential component in complex form, which depends on the sought unknown moment of arrival of the reflected signal - τ .

Let the input signal described by expression (1) now be fed to the input of a low-pass filter (LPF) with the following main characteristics – impulse response, most often denoted as $h_i(t)$, which, for a chosen "ideal" LPF, has the form :

$h(t) = \frac{w_c}{\pi} \text{sinc}\left(\frac{w_c t}{\pi}\right)$, where $\text{sinc}(x) = \sin(\pi x)/\pi x$. It should immediately be noted that with such an impulse response, such an LPF cannot be technically realized due to the fact that the described function has infinite duration. As a first approximation and for the simulation study later, the well-known "Nyquist filter" will be applied, which is standard in parameters. For clarification, the main mathematical models and characteristics of the "Nyquist filter" are as follows:

- Impulse response – usually represented by the following mathematical model:

$$(4) h_i(t) = \frac{w_c}{\pi} \text{sinc}\left(\frac{w_c t}{\pi}\right) \frac{\cos(\beta w_c t)}{1 - \left(\frac{2\beta w_c t}{\pi}\right)^2},$$

- Amplitude-frequency characteristic (AFC), which is described by the following expression:

$$(5) K(w) = \begin{cases} 1, & \text{при } |w| < \frac{1-\beta}{1+\beta} w_c \\ \cos^2 \left[\frac{\pi(1+\beta)}{4\beta w_c} \left(|w| - \frac{1-\beta}{1+\beta} w_c \right) \right], & \text{при } \frac{1-\beta}{1+\beta} w_c \leq |w| \leq w_c \\ 0, & \text{при } |w| > w_c \end{cases}$$

- Roll-off factor (or smoothing coefficient) – in this case denoted by β , and takes values: $0 < \beta < 118$. When $\beta = 0$, the filter transforms into an "ideal LPF with a rectangular AFC envelope". At the maximum value, i.e., when $\beta = 1$, the filter has the properties of a so-called "raised cosine". The following feature of the filter should also be taken into account: as the values of β (the roll-off factor) increase, the decaying processes, which are typically called "tails," decrease much faster in time. These characteristics justify the application of the "Nyquist filter" in solving problems of compensation of passive interference [5,9] penetrating the receiver through the antenna. The presence of parasitic re-emissions with frequencies close to the filter's passband, such as passive refractions (re-reflections) from stationary objects, are characteristic of densely populated regions. At the filter's output, a signal will be realized that can be described by the expression, where $\dot{a}_m, \dot{b}_m$ are respectively:

$$\dot{a}_m = f_m, \quad \dot{b}_m = C_m$$

$$(6) \quad \tilde{s}_{out}(t) = \int_{-\infty}^{+\infty} s(\mu, \tau) h(t - \mu) d\mu = \sum_{m=-\infty}^{+\infty} \dot{a}_m \dot{b}_m \int_{-\infty}^{+\infty} h(t - \mu) e^{(j2\pi m\mu / T)} d\mu,$$

which can be written in the form:

$$(7) \quad \tilde{s}_{out}(t) = \sum_{m=-\infty}^{\infty} \dot{a}_m \dot{b}_m K\left(\frac{2\pi m}{T}\right) e^{\left(\frac{j2\pi m t}{T}\right)}$$

Let the condition $K(w_c) = 0$ be satisfied for the filter, and by setting $w_c = 2\pi M / T$, where $M \geq 1$. After such a setting, the expression $K\left(\frac{2\pi m}{T}\right) = 0$ for values of $|m| \geq M$. After performing the mathematical operations, expression (1.7) can be written as:

$$(8) \quad \tilde{s}_{out}(t) = \sum_{m=-M}^M \dot{a}_m \dot{b}_m K(mw_c) e^{\left(\frac{j2\pi m t}{T}\right)}$$

From expression (8), it can be concluded that for a value $M = 1$, the input information regarding the moment of arrival of the incoming US reflected pulse – τ – is completely compensated. Since when solving (8) for $M=1$, the signal function becomes $\tilde{s}_{out}(t) = a_o = A$. For further research and simulation procedures, the condition $M \geq 2$ will be applied.

With this analysis, mathematical operations, assumptions, and initial conditions, one can proceed to calculating the necessary minimum number of samples for discretizing the signal output from the LPF – expression (8). In doing so, it is necessary to consider that the maximum frequency in the output signal's spectrum – w_{max} – is equal to the resonant frequency – w_p – to which the LPF is tuned. The theorem for calculating the necessary minimum number of samples for discretization states: $\Delta t \leq \pi / w_{max} = \pi / w_c = T / 2M$. In fulfilling this condition, in this specific case, the minimum required number of samples must satisfy the condition: $N \geq T / \Delta t + 1 = 2M + 1$. With the understanding that $M \geq 2$, it turns out that after applying "preliminary filtration," with an LPF tuned to $w_p = w_{max}$, the necessary minimum number of samples drastically decreases. The obtained result confirms the expected reduction in the number of required samples from the input packet of ultra-short pulses reflected from the target. The number of samples for unambiguously determining the arrival time of the pulse is about 5-10, which will allow determining the distance to the observed target in real time and with minimal ADC resource consumption. For further analysis, the equation can be used: $C_m = A e^{\frac{-j2\pi m \tau}{T}}$, which represents the exponential component in complex form. For determining the amplitude and initial phase of the sum of complex components (K) [10,11], $N > 2K$ samples are required. In the analyzed specific case, i.e., $K=1$, the minimum required number of samples can be reduced to 3-5. For further analysis, let's set: $\dot{X}_m = \dot{f}_m \dot{C}_m K(mw_c)$. When this assumption is applied to (1.8), it can be written as:

$$(9) \quad \tilde{s}_{out}(t) = \tilde{s}(n\Delta t) \sum_{m=-(M-1)}^{M-1} \dot{X}_m e^{\left(\frac{j2\pi m \Delta t}{T}\right)},$$

for $n = 0, 1, 2, \dots, N-1$

And in this case, N can take very small values, for example, $N \geq 3$.

If we now consider the last equation - (9), we can conclude that it represents a system of N equations with $2M+1$ unknown values of $\dot{X}_m$. If $T/\Delta t = N = 2M + 1$ is chosen, then equation (9) transforms into an "inverse Fourier transform," i.e., the unknown complex coefficients can be found by applying the "forward Fourier transform" directly from the acquired samples for.

However, if $N > 2M + 1$, the solution to the system can be obtained after pseudo-inversion of the corresponding signal matrix [12].

4. Results and discussion.

Thus: The general formulation of the proposed modified algorithm for measuring the exact arrival time of the reflected signal can be presented as follows:

1. The input sequence of ultra-short pulses (signals reflected from the target) must be fed to the input of a low-pass filter with a resonant frequency $w_c = 2\pi M / T$, where $M > 1$;
2. The output signal from the "preliminary filter" is discretized in the interval $[0 - T]$, resulting in $N \geq 2M + 1$ samples of $\tilde{s}_{out}$;
3. After matrix inversion (if $N > 2M + 1$) or by applying the "forward Fourier transform" (if $N = 2M + 1$), the coefficients - X_m - are found.
4. As a result of these operations, the complex amplitude –

$$\dot{C}_m = \frac{\dot{X}_m}{\dot{f}_m K(mw_p)}$$

5. Furthermore, the unknown values of the complex exponential dependence - (A and τ) – will be determined by standard spectral methods, such as Prony's Method; Annihilating Filter; MUSIC, etc. [13,15].

5. Conclusion.

The algorithm proposed for calculating the arrival time of the ultra-short reflected pulse is synthesized under the condition that only reflected signals are considered and noise samples, which are necessarily present in the input signal under real conditions, are not accounted for. Under "noise component", it will further be assumed that the input sequence contains "white" (Gaussian noise) with a one-sided noise power spectral density - N_0 . In this case, the estimation of the time moment – τ – will be obtained with a certain error, which will depend on the signal-to-noise ratio.

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