# Algorithm for detection of low-flying small objects from background radar

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Abstract. The proposed material explores the possibilities for creating and practical implementation of an optimal algorithm for detection of low-flying small-sized objects from background radar, with linear frequency modulation of the emitted signal and a relatively uniform reflecting ground surface. A method and a block diagram implementing a close-to-optimal algorithm for a series of time periods are proposed, enabling, in the presence of sufficient computing resources, the optimal algorithm to be realized for the entire area of operation of the background radar.

# **INTRODUCTION**

The development of technologies for countering small-sized low-flying objects (SLFOs) leads to a greater use of the so-called background radar. Most of them have active radars with continuous operation and have linear frequency modulation, and for optimal or quasi-optimal detection of SLFOs it is necessary to develop algorithms that take into account the features of the reflected signals, mainly reduced to the bistatic nature of their effective reflecting surface (ERS) and the interference of radio waves reflected by the SLFOs and the earth's surface. The proposed material explores the possibilities for creating and practical implementation of an optimal algorithm for detection of low-flying small-sized objects from background radar, with linear frequency modulation (LFM) of the emitted signal and a relatively uniform reflecting ground surface.

# DESCRIPTION

# **Description of the Problem**

The detection of flying and other objects is most often performed by algorithms based on the likelihood ratio, which requires knowledge of the distribution law of the useful signal and the disturbance. A characteristic feature of background radars is [1-5] that the useful signal used to detect the objects is not the signal directly reflected by them, but the signal reflected from the nearby earth's surface (Figure 1). This is due to the fact that for low-flying objects the bistatic angle is relatively large, usually close to 180°, whereby the signals that are reflected from the object and then re-reflected from the earth's surface are significantly larger than the signals that are reflected directly from the object to the radar [6-9]. In the existing literature [10-13], it is usually assumed that the geometric dimensions of the SLFOs and of the surface irregularities are much smaller than the resolving power of the radar. In this case, the object and the Earth's surface can be assumed to be isotropic emitters, with the background radar input signal resulting from the interference between the signal reflected from the nearby Earth's surface (signal 1 in **FIGURE 1** and the signal reflected only from the Earth's surface (signal 2 from **FIGURE 1**).



trajectory of the signal 1 reflected from the SLFOs and re-reflected from the Earth's surface trajectory of the signal 2 reflected from the earth's surfacer

FIGURE 1. Background radar signal trajectory

Under these assumptions, it is assumed that the model of the input signal in the presence of an SLFO is described by the expression [7-9]:

$$Y(t) = \hat{A}_{0} \cdot \sqrt{\sigma_{\phi}(\sigma_{62}, \varepsilon_{62})} \cdot \sqrt{1 + K_{A}(t) + 2 \cdot K_{A}(t) \cdot \cos[\Delta_{\varphi}(t)]} = \hat{A}_{0} \cdot \sqrt{\sigma_{\phi}(\sigma_{62}, \varepsilon_{62})} \cdot f(t)$$
(1)

where:  $\hat{A}_0 = A_0 \sqrt{\sigma_{\phi}(\sigma_{62}, \varepsilon_{62})}$ 

 $\rm A_o~$  - amplitude of the signal at the radar input in the presence of an SLFO;

 $\sigma_{\phi}(\sigma_{62}, \varepsilon_{62})$  - bistatic ERS of the SLFO, which is ERS relative to the receiving station at specified bistatic angles in azimuth ( $\sigma_{62}$ ) and elevation ( $\varepsilon_{62}$ );

 $K_A(t)$  – ratio of the radar input between the amplitude of the signal reflected from the earth's surface and the signal in the presence of an SLFO;

 $\Delta_{\varphi}(t) = 2\pi f_{\circ}(\tau - \tau_{\circ}(t)) - \varphi_{\circ} \text{ - phase difference between the interfering ground-reflected signals of the SLFO and the ground-reflected signal in radians;}$ 

 $f_{\circ}$  - basic operating frequency of the radar;

au - travel time of the signal from the radar to the ground surface;

 $\tau_{o}(t)$  - travel time of the signal from the radar to the SLFO and from there to the earth's surface;

 $\varphi_{\circ}$  - dephasing of the signal when it is reflected by the SLFO.

$$f(t) = \sqrt{1 + \mathcal{K}_{A}(t) + 2.\mathcal{K}_{A}(t).\cos[\Delta_{\varphi}(t)]}$$
<sup>(2)</sup>

In the existing literature [10-13], solutions are proposed for finding a realizable quasi-optimal algorithm that avoids considering the complex changing function f(t), using the simplified expression represented by Formula 3.

$$Y(t) = \dot{A}_{\rm o} \cdot \sqrt{\sigma_{\phi}(\sigma_{62}, \varepsilon_{62})} + N \tag{3}$$

where N is additive amplitude composite with random time uncorrelated distribution.

In this case, the quasi-optimal algorithms are reduced to the classic correlation filters, which can be implemented by both analog and digital methods and means [11-13]. However, to improve the effectiveness of background radars, it is expedient to synthesize an optimal algorithm for detecting a signal of the type of readings using Formula 1 and to analyze the possibilities for its practical implementation.

#### Suggestion to solve the problem

Let us reduce the task of binary detection of SLFO to determining the fact of the presence of a useful signal in the realizations of the fluctuating amplitude of the radio signal reflected from the background. Hypothesis H<sub>1</sub> corresponds to the case of a SLFO moving in front of a background surface, when the radio signal reflected from the background is modulated by the radio signal reflected from it. Hypothesis H<sub>0</sub> corresponds to the case of reception of a radio signal reflected from the background. The optimization task in this case is reduced to processing the data for time t, corresponding to the observation period T of a given section of the space t  $\epsilon$  [0, T]. When optimizing according to the minimum average risk criterion [11-13], in this case, the likelihood ratio L(Y) is decided and it is compared with a likelihood threshold  $\gamma_0$ , (Formula 4), upon exceeding which it is assumed that a SLFO exists.

$$L(Y) = \frac{W(Y/H_{1},t)}{W(Y/H_{0},t)} > \gamma_{0}$$
(4)

where:  $W(Y/H_1, t)$  - density distribution of the input signal in the presence of SLFO;  $W(Y/H_0, t)$  - density distribution of the input signal in the absence of SLFO.

The randomness of the process Y(t) is determined by the fluctuations of the ERS  $\sigma_{\Phi}(\sigma_{62}, \varepsilon_{62})$  and the law of modulations Y(t) is determined by the function f(t), characterizes the non-stationary nature of the density distribution of Y in the conditions of movement of SLFO. Considering that the correlation interval of ERS fluctuations is much shorter than the duration of the useful signal, we record the density of the distribution of the discrete values of  $Y_i$  under the condition of the presence of a moving SLFO with the following expression [10-13]:

$$W(Y/H_{1},t) = \frac{2}{|A_{0}|.\sqrt{2\pi}.\sigma_{\sigma_{\phi}}} \cdot \prod_{i=1}^{n} \frac{Y_{i}}{|f(t_{i})|} \cdot e^{-\frac{1}{2.A_{0}^{2}.\sigma_{\sigma_{\phi}}^{2}} \sum_{i=1}^{n} \left(\frac{Y_{i}^{2} - m_{\sigma_{\phi}}}{f(t_{i})}\right)^{2}}$$
(5)

where:

 $\begin{array}{l} Y_i = \mathrm{Y}(t_i) - \mathrm{discrete\ sample\ of\ the\ input\ signal\ at\ time\ t_i\ ;} \\ m_{\sigma_{\phi}} & -\mathrm{mathematical\ expectation\ of\ the\ bistatic\ ERS\ \sigma_{\phi}(\sigma_{62},\varepsilon_{62}); \\ \sigma_{\sigma_{\phi}} & -\mathrm{rms\ deviation\ of\ the\ bistatic\ ERS\ \sigma_{\phi}(\sigma_{62},\varepsilon_{62}); \\ n & -\mathrm{number\ of\ discrete\ samples\ of\ the\ input\ signal\ for\ the\ period\ T.} \end{array}$ 

The distribution density of the sample Y under the hypothesis H<sub>0</sub>, in turn, is:

$$W(Y/H_0) = \frac{2}{|A_0| \sqrt{2\pi} \sigma_{\sigma_{\phi}}} \cdot \prod_{i=1}^n Y_i \cdot e^{-\frac{1}{2A_0^2 \sigma_{\sigma_{\phi}}^2} \sum_{i=1}^n \left(Y_i^2 - m_{\sigma_{\phi}}\right)^2}$$
(6)

Substituting the conditional distribution densities (5) and (6) into (4), we get:

$$L(Y) = \frac{\left(\frac{2}{|A_0|\sqrt{2\pi}.\sigma_{\sigma_{\phi}}}\right)^n \cdot \prod_{i=1}^n \frac{Y_i}{|f_{t1}|} e^{-\frac{1}{2\dot{A}_0^2 \sigma_{\sigma_{\phi}}^2} \sum_{i=1}^n \left(\frac{Y_i^2 - m_{\sigma_{\phi}}}{f(t_i)}\right)^2}}{\left(\frac{2}{|A_0|\sqrt{2\pi}.\sigma_{\phi}}\right)^n \cdot \prod_{i=1}^n Y_i \cdot e^{-\frac{1}{2\dot{A}_0^2 \sigma_{\phi_{\phi}}^2} \sum_{i=1}^n \left(Y_i^2 - m_{\sigma_{\phi}}\right)^2}}$$
(7)

$$L(Y) = \frac{\prod_{i=1}^{n} \frac{Y_{i}}{|f_{t1}|} e^{-\frac{1}{2\dot{A}_{0}^{2}\sigma_{\phi}^{2}} \sum_{i=1}^{n} \left(\frac{Y_{i}^{2} - m_{\sigma_{\phi}}}{f(t_{i})}\right)^{2}}}{\prod_{i=1}^{n} Y_{i} \cdot e^{-\frac{1}{2\dot{A}_{0}^{2}\sigma_{\phi}^{2}} \sum_{i=1}^{n} \left(Y_{i}^{2} - m_{\sigma_{\phi}}\right)^{2}}}$$
(8)

Or simplifying

$$L(Y) = \prod_{i=1}^{n} \frac{1}{|f(t_i)|} \cdot e^{-\frac{1}{2\hat{h}_0^2 \sigma_{\sigma_{\phi}}^2} \left\{ \sum_{i=1}^{n} \left[ \frac{Y_i^2}{f(t)} \right]^2 + \sum_{i=1}^{n} \left[ \frac{m\sigma_{\phi}}{f(t)} \right]^2 - \left[ \sum_{i=1}^{n} (Y_i^2)^2 + \sum_{i=1}^{n} m_{\sigma_{\phi}}^2 \right] + 2m_{\sigma_{\phi}} \sum_{i=1}^{n} Y_i^2 \left[ 1 - \frac{1}{f(t_i)} \right] \right\}}$$
(9)

The resulting expression describes the likelihood ratio for the task of detecting SLFOs under background radar, where the algorithm for detecting SLFOs consists of comparing an expression L(Y) with a certain threshold  $\gamma 0$ . If we simplify the optimal detection algorithm described by formula (8), by logarithmizing both sides of the equation we get:

$$ln[L(Y)] = \sum_{i=1}^{n} ln[|f(t_i)|^{-1}] - \frac{\sum_{i=1}^{n} \left[\frac{Y_i^2}{f(t_i)}\right]^2 + \sum_{i=1}^{n} \left[\frac{m_{\sigma_{\phi}}}{f(t_i)}\right]^2 - \left[\sum_{i=1}^{n} (Y_i^2)^2 + n.m_{\sigma_{\phi}}^2\right] + 2.m_{\sigma_{\phi}}^2 \sum_{i=1}^{n} Y_i^2 \cdot \left[1 - \frac{1}{f(t_i)}\right]}{2.A_0^2 \cdot \sigma_{\sigma_{\phi}}^2}$$
(10)

Thus, the optimal SLFOs detection algorithm, taking into account the multiplicative interaction of a known useful signal with the reflections from the earth's surface, according to the Bayesian criterion, takes the following form:

$$Z = \frac{\sum_{i=1}^{n} Y_{i}^{4} - \sum_{i=1}^{n} \frac{Y_{i}^{4}}{f^{2}(t_{i})} - 2.m_{\sigma_{\Phi}} \sum_{i=1}^{n} Y_{i}^{2} \left[ 1 - \frac{1}{f(t_{i})} \right]}{2.\hat{A}_{0}^{2} \sigma_{\phi_{\Phi}}^{2}} > Z_{\Pi}$$
(11)

where:

$$Z_{\pi} = \frac{A_1 - B_1}{2.\dot{A_0^2} \cdot \sigma_{\sigma_{\phi}}^2} - \sum_{i=1}^n \ln|f(t_i)|^{-1} + \ln\gamma_0 \text{ is modified detection threshold;}$$
$$A_1 = \sum_{i=1}^n \left[\frac{m_{\sigma_{\phi}}}{f(t_i)}\right]^2$$
$$B_1 = n. m_{\sigma_{\phi}}^2$$

It follows from Formula 11 that in order to decide on the presence of a signal caused by a SLFO from a background radar, a number of summation operations must be performed on non-linearly transformed samples of the input signal Y(t), for the considered time period as obtained results are compared with the modified likelihood threshold  $Z_{\pi}$ . It can be seen that in order to calculate the likelihood ratio and the likelihood threshold, it is necessary to have data on the values of two types of parameters.

The first type are quasi-stationary, i.e. those that for the given situation do not change over time  $(A_0, \sigma_{\sigma_{\phi}}, m_{\sigma_{\phi}})$  and on the basis of the data on the parameters of the radar, the SLFO and the type of the earth's surface in the zone of reflection of the useful signal can be calculated and/or measured. By dividing the area of action of the background radar into separate elementary zones corresponding to the resolution of the radar and the given type of SLFO, it is possible to fix and record the data on the quasi-stationary parameters in advance in a given information database.

The second type of parameters are dynamic and change over time. These are the arguments of the function f(t) from Formula 2 and in particular  $K_A(t)$  and  $\Delta_{\varphi}(t)$ .

The ratio between the amplitude of the signal reflected from the earth's surface and the signal in the presence of a SLFO  $K_A(t)$  can be calculated and/or measured for each type of SLFO and for each individual elemental zone, then recorded in an information database analogously to the case with the quasi-static parameters.

Phase difference between the interfering ground-reflected signals reflected by the SLFO and reflected by the ground in radians  $\Delta_{\varphi}(t)$  depends on:

- $\tau$  the travel time of the signal from the radar to the earth's surface, which can be calculated for each elementary zone and recorded in an information database;
- he time for the signal to travel from the radar to the SLFO and from there to the earth's surface, which, at a given height and distance to the SLFO, can also be calculated for each elementary zone and recorded in an information database. The distance to a given elementary zone is known, i.e. with sufficient accuracy it can be assumed that the value of τ<sub>a</sub>(t) depends mainly on the height of the SLFO (H). Therefore, despite the

relatively small altitude range of the SLFO flight, it is possible to divide it into several sections and calculate the value of  $\tau_{o}(t)$  for each of them. Therefore, for each elemental zone in the information database, there will be several values of  $\tau_{o}(t)$  recorded, corresponding to several height ranges;

- the time for the signal to travel from the radar to the SLFO and from there to the earth's surface, which, at a given height and distance to the SLFO, can also be calculated for each elementary zone and recorded in an information database. The distance to a given elementary zone is set, it can be assumed with sufficient accuracy that the value of  $\tau_{o}(t)$  depends mainly on the height of the SLFO (H). Therefore, despite the relatively small altitude range of the SLFO flight, it is possible to divide it into several sections and calculate the value of  $\tau_{o}(t)$  for each of them. Therefore, for each elemental zone in the information array, there will be several values of  $\tau_{o}(t)$  recorded, corresponding to several height ranges;
- φ<sub>o</sub> the dephasing of the signal when it is reflected by the SLFO is a random quantity, but to reduce the influence of its uncertainty, it is possible to calculate radians Δ\_φ (t) for several possible values for example, for 0 and 0.5 π radians.

In this way, for each elementary zone in the information database, it is possible to calculate and record the quasi-stationary parameters and  $K_A(t)$  for each potential type of SLFO, as well as several data for  $\Delta_{\varphi}(t)$ , corresponding to the corresponding height range and value of  $\varphi_o$ . The sample form of entry in the information array for a given SLFO and for a conditional elemental area is shown in TABLE 1.

Number	Áo	$\sigma_{\sigma_{\Phi}}$	$m_{\sigma_{\Phi}}$	$K_A(t)$			
zone					(0)	$\tau(t)$	
					$\psi_{\circ}$	$\iota_{\circ}(\iota)$	
					0	H20 m	$\Delta_{11\varphi}(t)$
					0	<i>H</i> 40 m	$\Delta_{12\varphi}(t)$
					0	<i>H</i> 60 m	$\Delta_{13\varphi}(t)$
1	Á <sub>10</sub>	$\sigma_{1\sigma_{igoplus}}$	$m_{1\sigma_{igoplus}}$	$K_{1A}(t)$	0	H80 m	$\Delta_{14\varphi}(t)$
					0	<i>H</i> 100 m	$\Delta_{15\varphi}(t)$
					$0.5 \pi$	<i>H</i> 20 m	$\Delta_{16\varphi}(t)$
					$0.5 \pi$	<i>H</i> 40 m	$\Delta_{17\varphi}(t)$
					$0.5 \pi$	<i>H</i> 60 m	$\Delta_{18\varphi}(t)$
					$0.5 \pi$	H80 m	$\Delta_{19\varphi}(t)$
					$0.5 \pi$	<i>H</i> 100 m	$\Delta_{10\varphi}(t)$

TABLE 1. Record for a given elemental zone for a given type of SLFO

As a result of the above, it is possible to propose a near-optimal algorithm for detecting SLFOs from background radar (Figure 2), designed to detect several types of SLFOs, it is possible to calculate this type of parameters for each individual SLFO type and subsequently perform parallel processing of received signals. The block diagram digitally implementing the optimal algorithm for background detection of SLFOs for a given period of time is shown in Figure 2.

Its implementation requires preliminary preparation, related to the deployment of the background radar in the area, determination of its expected area of operation and the types of SLFOs it is intended to detect, and gathering of data of the type shown in **TABLE 1** in the Information block. These data can be calculated based on the parameters of the background radar, the SLFOs and the Earth's surface and can be refined by conducting flybys with real SLFOs. Subsequently, in the process of radar reconnaissance, the quasi-optimal algorithm performs the processing of the input information in a parallel way, as for each direction of the radar diagram for each frequency interval (corresponding to a time interval, i.e., to an elementary area), for each type SLFOs and for at least two orthogonal values of  $\varphi_{e}$ .

The implementation of the algorithm is related to the pre-calculation of the necessary data and their recording in an information array in a manner similar to that shown in **TABLE** 1.

Subsequently, for each direction of the background radar antenna and for each elemental section, digital conversion of the received signals and their parallel processing is performed for each possible type of SLFOs, for each of the possible height ranges and for values of  $\varphi_{0}$  equal to 0.5  $\pi$  and to 0 radians.



FIGURE 2. Block diagram of a quasi-optimal algorithm for detecting SLFOs from background radar

If the calculated likelihood ratio for at least one type of SLFOs and for one altitude range exceeds the probability threshold, a decision is made to detect and determine the type and altitude of the given SLFO. If the likelihood ratio exceeds the probability threshold for several types of SLFOs and/or for several altitude ranges, a decision is made only to detect SLFOs in the corresponding elementary section.

# CONCLUSION

The material explores the possibilities of creating and practical implementation of an optimal algorithm for detecting low-flying small-sized objects from a background radar, with linear frequency modulation of the emitted signal and a relatively uniform reflecting ground surface. An optimal algorithm and a block diagram of its digital implementation for a given time period T are proposed.

A method and a block diagram implementing a close-to-optimal algorithm for a series of time periods are proposed, enabling, in the presence of sufficient computing resources, the optimal algorithm to be realized for the entire area of operation of the background radar. The proposed algorithm give the possibility for even further digitalization of the whole SLFO's detection process and thus reducing the required reaction time. A feature of the proposed algorithm is the need for higher computing power, but within the modern capabilities of computing technology, this feature does not appear to be problematic.

# ACKNOWLEDGMENTS

The results of the study will be used in the implementation of the second stage of a Project BG05M2OP001-1.002-0006 - Creation and Development of a Center of Competence "Quantum Communication, Intelligent Security Systems and Risk Management" (Quasar), founded by the European Regional Development Fund through the Operational programme "Science and education for smart growth".

The authors would like to thank the Research and Development Sector at the Technical University of Sofia for the financial support.

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