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INVESTIGATION OF THE TRANSVERSE VELOCITY AND TRANSVERSE DISPLACEMENT OF A SOLID PARTICLE IN A HORIZONTAL BOUNDARY LAYER WITH LONGITUDINAL VELOCITY PULSATIONS

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Summary: *The article examines the movement of a single solid particle in the boundary layer of a flat horizontal flow with a fixed velocity, with longitudinal velocity pulsations, under the action of forces implementing the so-called inertial transport.*

Keywords: *solid particle; boundary layer; horizontal flat current with longitudinal velocity pulsations; mass and surface forces.*

Introduction

The movement of a solid particle in a fluid flow is determined by the action of forces of different magnitude and nature, as well as by the nature of the movement of the carrier phase. Under certain conditions, some of them can significantly change the direction and magnitude of the velocity, as well as the position of the particle in the volume of the flow of the carrier phase [Crowe et al. 1998], [Maxey et al. 1983], [Карапетков и др. 2014], [Карапетков и др. 2018], [Карапетков и др. 2015], [Л.Д. Ландау и др. 1988], [Лойцянский и др. 1973], [Петров 2018], [Тихонов и др. 1972], [Славчева и др. 2022].

The purpose of the publication is to investigate the movement of solid impurities with low concentration in the boundary layer of a plane horizontal, quasi-steady flow with a low degree of turbulence and to establish the influence of certain hydrodynamic forces, the force of gravity, as well as inertial forces generated by longitudinal pulsations of the velocity field of the carrier phase on the velocity, trajectory, and redistribution of inertia-dominated solid particles.

Exposure

Longitudinal harmonic velocity pulsations are created in the boundary layer of a horizontal plane flow with a fixed velocity. At a point of the initial section (O-O) (Fig. 1), solid spherical particles of the same diameter (d_p) are periodically released.

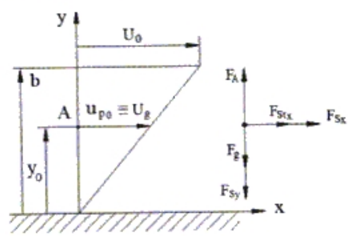

Figure 1

Statement of the task: To investigate the influence of longitudinal velocity pulsations in the current on the motion of the solid particle.

The velocity field of the current in the boundary layer consists of one established longitudinal velocity $(U_g)$and one longitudinal pulsation component (u'_g):

The components of the current velocity along the coordinate axes are: along axis x $U_g + u'_g$; along axis y $V_g = 0$.

The ripple component is a harmonic function of time (t):

$$u'_g = u_0.\cos(\omega.t), \qquad (1)$$

for which the amplitude (u_0) and the frequency (ω) are known.

The range of frequency variation (ω) is given by the expression

$$\omega = \frac{U_g}{d_p}.\varepsilon,$$

where (ε) is degree of turbulence. Moderate turbulence is assumed to be $(\varepsilon = 0{,}08)$.

Thus, by observing the above dependence, the low-frequency large-scale vortex range is covered, in which the so-called inertial transport of impurities falls.

The forces that determine the inertial transfer of particles are: Aerodynamic resistance force, Inertia force from the added mass, Gravity force, Safman force and Archimedes force.

The equations describing the motion of the solid particle under the described hydrodynamic conditions are compiled using the Lagrange method. They form the following system:

$$\frac{du_p}{dt} = A\left(U_g + u'_g - u_p\right) + B.v_p; \qquad (2)$$

$$\frac{dv_p}{dt} = \begin{Bmatrix} -A.v_p + B\left(U_g + u'_g - u_p\right) - \\ -g.\left(1 - {}^{1}/_{\varepsilon_p}\right) \end{Bmatrix}. \qquad (3)$$

These equations describe the motion of single solid impurities in the boundary layer of a horizontal plane quasi-steady flow with longitudinal harmonic velocity pulsations.

The right-hand side of the equations includes the projections of: The resistive hydrodynamic force; Safman's power; The force of gravity corrected by Archimedes' force.

The initial conditions used for the solution are:

$$t = 0 \quad u_{p0} = U_g; \ \frac{du_p}{dt} = 0; \qquad (4)$$

$$v_p = 0; \ \frac{dv_p}{dt} = 0.$$

With the initial conditions thus formulated, the mathematical model of the considered process is complete.

Solution of the task: By eliminating the velocity (u_p) and its derivatives, from the system (2) and (3), the differential equation for the velocity (v_p) is obtained:

$$\begin{Bmatrix} \frac{d^2v_p}{dt^2} + \\ +2.A.\frac{dv_p}{dt} + (A^2 + B^2).v_p + \\ +A.g.\left(1 - {}^{1}/_{\varepsilon_p}\right) \end{Bmatrix} = -B.u_0.\omega.\sin(\omega.t) \qquad (5)$$

This equation describes the motion of a single solid particle in a resistive medium and the presence of forced velocity pulsations in the surrounding fluid.

The solution of (5) is a sum of the solution of the homogeneous equation describing the self-damping oscillatory motions (left-hand side) and a partial solution describing the influence of the forced external velocity pulsations (right-hand side).

The characteristic equation of the homogeneous equation on the left side

$$\alpha^2 + 2.A.\alpha + (A^2 + B^2) = 0$$

It has complex roots $\alpha_{1,2} = -A \pm i.B$.

As known from mathematics, the general solution of the left side of (5) has the form:

$$v_p = \begin{Bmatrix} \begin{pmatrix} C_1.\cos(B.t) + \\ +C_2.\sin(B.t) \end{pmatrix} e^{-At} - \\ -\frac{A.g}{A^2 + B^2}\left(1 - {}^{1}/_{\varepsilon_p}\right) \end{Bmatrix}. \qquad (6)$$

The partial integral, in accordance with (1), has the form

$$\eta = H_1.\cos(\omega.t + \delta_1) = = \begin{Bmatrix} H_1.\cos(\omega.t).\cos(\delta_1) - \\ -H_1.\sin(\omega.t)\sin(\delta_1) \end{Bmatrix}. \qquad (7)$$

Here (H_1) and (δ_1) are constants. Their determination takes place after (7) is differentiated twice and substituted together with the derivatives in (5):

$$\eta' = \left\{-H_1.\omega.\begin{pmatrix}\sin(\omega.t).\cos(\delta_1) + \\ +\cos(\omega.t).\sin(\delta_1)\end{pmatrix}\right\}; \qquad (8)$$

$$\eta'' = \left\{-H_1.\omega^2.\begin{pmatrix}\cos(\omega.t).\cos(\delta_1) - \\ -\sin(\omega.t).\sin(\delta_1)\end{pmatrix}\right\}. \qquad (9)$$

By summing the coefficients in front of $(\cos(\omega.t))$ from (5), (7), (8) and (9), we get:

$$H_1 \cdot \begin{pmatrix}(A^2 + B^2 - \omega^2).\cos(\delta_1) - \\ -2.A.H_1.\omega.\sin(\delta_1)\end{pmatrix} = 0 \qquad (10)$$

From the last for(δ_1) we get:

$$\delta_1 = \operatorname{atan}\left(\frac{A^2 + B^2 - \omega^2}{2.A.\omega}\right). \qquad (11)$$

By summing the coefficients in front of $(\sin(\omega.t))$ from the same dependences, for (H_1) we get:

$$H_1 = \frac{B.u_0.\omega}{\begin{Bmatrix}(A^2 + B^2 - \omega^2).\sin\delta_1 + \\ +2.A.\omega.\cos\delta_1\end{Bmatrix}}. \qquad (12)$$

The integration constants (C_1) and (C_2) are determined according to the initial conditions (4):

$$\text{At } t = 0 \quad v_p = v_{p0}; \quad \frac{dv_p}{dt} = 0 .$$

These conditions are replaced in the general solution formed by the sum (6) and (7):

$$v_p = \begin{Bmatrix}\begin{pmatrix}C_1\cos(B.t) + \\ +C_2\sin(B.t)\end{pmatrix}.e^{-At} - \\ -\frac{A.g}{A^2 + B^2}\left(-1/\varepsilon_p\right) + \\ +H_1.\cos(\omega.t + \delta_1)\end{Bmatrix}. \qquad (13)$$

Whence for C_1 we get:

$$C_1 = \frac{A.g}{A^2+B^2}\left(1 - 1/\varepsilon_p\right) - H_1\cos(\delta_1). \qquad (14)$$

And for the second integration constant it follows:

$$C_2 = C_1.\frac{A}{B} + \frac{H_1\omega}{B}\sin(\delta_1). \qquad (15)$$

Dependencies (11), (12), (14), (15) and (13) form the mathematical model for the transverse velocity (v_p) of single solid impurities in the boundary layer of a horizontal flow with longitudinal velocity pulsations in the surrounding fluid. This model allows the transverse velocity to be studied as a function of time, i.e., $v_p = v_p(t)$ and is used to derive the particle's transverse displacement function of time, i.e., a function of the form $y = y(t)$.

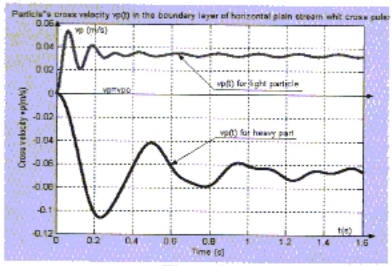

Figure 2

In **fig. 2** shows the graphical representation of the variation of the particle's transverse velocity as a function of time for a "light" and a "heavy" particle. The two particles start from the same point of the initial cross-section with no initial transverse velocity $(v_{p,0} = 0)$.The upper line shows the variation of the transverse velocity of the "light" particle. After a very short transition period, the particle begins to move with an oscillating change in speed around the constant value $(v_p \approx \frac{0.035m}{s})$. The direction is vertical, up towards the higher speed zone. The "heavy" particle has a significantly longer transition period. It is directed downwards, towards the area with lower current velocities, and oscillates around a fixed vertical velocity $(v_p) = -0.065\frac{m}{s}$. Unlike the "light" particle, here the fluctuations are significantly larger and the decay is much slower. The (-) sign indicates that the particle is pointing downwards, towards the wall.

The displacement of the particle across the streamlines is determined by the solution of the integral $y = \int v_p.dt$ in which the velocity is replaced by (13). It is obtained:

$$y = \left\{ \begin{array}{c} \left(\begin{pmatrix} B.C_1 - \\ -A.C_2 \end{pmatrix}.sin(B.t) - \\ -\begin{pmatrix} A.C_1 + \\ +B.C_2 \end{pmatrix}.cos(B.t) \right) \cdot \dfrac{e^{-At}}{A^2 + B^2} + \\ + \dfrac{H_1}{\omega} \sin(\omega.t + \delta_1) - \\ - \dfrac{A.g}{A^2 + B^2} \cdot \left(1 - {}^{1}/_{\varepsilon_p}\right).t + \\ + \dfrac{A.C_1 + B.C_2}{A^2 + B^2} - \dfrac{H_1}{\omega} sin(\delta_1). \end{array} \right\} \qquad (16)$$

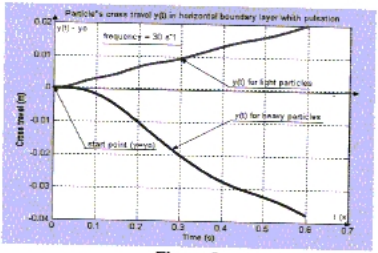

Figure 3

In **fig.3** the graphical form of the function $y = y(t)$ for a "light" and a "heavy" particle is shown. The upper line shows a monotonous rise of the "light" particle towards the region of higher carrier phase velocities. Irregularities in linearity are due to the influence of forced velocity ripples. Irregularities in linearity are due to the influence of forced velocity ripples. The displacement of the "heavy" particle (bottom line) in this case indicates the decrease in height relative to the initial position. In this way, it passes into the zone of lower velocities of the carrier phase. This contributes to the reduction of the longitudinal velocity of the particle and its further settling on the boundary wall.

Conclusion

According to the obtained results, the longitudinal velocity pulsations create a transverse inertial force that acts on the particle and thus overcomes the action of the Safman force. This was to be expected given the constraint on the pulsation parameters: amplitude and frequency as a function of particle size, carrier phase velocity and degree of turbulence.

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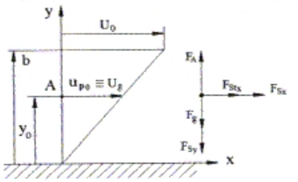
ИЗСЛЕДВАНЕ НА НАПРЕЧНАТА СКОРОСТ И НАПРЕЧНОТО ПРЕМЕСТВАНЕ НА ТВЪРДА ЧАСТИЦА В ХОРИЗОНТАЛЕН ГРАНИЧЕН СЛОЙ С НАДЛЪЖНИ СКОРОСТНИ ПУЛСАЦИИ

Румен ЯНКОВ   Марияна ИВАНОВА   Мария ГРАМЕНОВА-АНГЕЛОВА

В работа се изследва движението на единична твърда частица в граничния слой на равнинно хоризонтално, квазистационарно течение с ниска степен на турбулентност, с цел установяване на влиянието на определени хидродинамични сили, силата на тежестта, както и инерционните сили породени от надлъжните пулсациите на скоростното поле на носещата фаза върху скоростта, траекторията и преразпределението на твърди частици с преобладаващо инерционно движение.

В граничния слой на хоризонтално равнинно течение с установена скорост се създават надлъжни хармонични скоростни пулсации, фиг.1. Изследва се влиянието им върху движението на единични твърди примеси, представено със зависимостите:



Фигура 1

$$\frac{du_p}{dt} = A\left(U_g + u'_g - u_p\right) + B.v_p; \tag{2}$$

$$\frac{dv_p}{dt} = -A.v_p + B\left(U_g + u'_g - u_p\right) - g.\left(1 - {}^{1}\!/_{\varepsilon_p}\right). \tag{3}$$

Началните условия, използвани за решението са:

$$t = 0;\ u_{p0} = U_g;\ \frac{du_p}{dt} = 0;\ v_p = 0;\ \frac{dv_p}{dt} = 0. \tag{4}$$

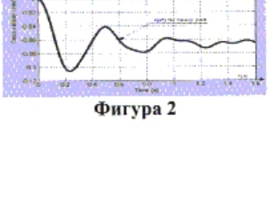
Решава се диференциалното уравнение:

$$\frac{d^2v_p}{dt^2} + 2.A.\frac{dv_p}{dt} + (A^2 + B^2).v_p + A.g.\left(1 - {}^{1}\!/_{\varepsilon_p}\right) = -B.u_0.\omega.\sin(\omega.t) \tag{5}$$

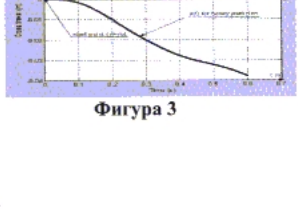
От (5) е получен математическия модел за напречната скорост на единични твърди примеси в граничния слой на хоризонтално течение с надлъжни скоростни пулсации в заобикалящия флуид (16).

$$y = \left\{ \begin{array}{c} \left((B.C_1 - A.C_2).sin(B.t) - (A.C_1 + B.C_2).cos(B.t)\right).\dfrac{e^{-At}}{A^2 + B^2} + \dfrac{H_1}{\omega}\sin(\omega.t + \delta_1) - \\ -\dfrac{A.g}{A^2 + B^2}.\left(1 - {}^{1}\!/_{\varepsilon_p}\right).t + \dfrac{A.C_1 + B.C_2}{A^2 + B^2} - \dfrac{H_1}{\omega}sin(\delta_1) \end{array} \right\}. \tag{16}$$

На фиг. 2 е представено графично изменението на напречната скорост на„лека" и „тежка" частица във функция от времето, а на фиг. 3 е представен графичен вид на функцията $y = y(t)$ по зависимост (16).



Фигура 2



Фигура 3

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