Difference Kalman Filter Robust to Systematic Measurement Errors Related to Inertial Navigation

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***Abstract* — Systematic components of measurement errors that are non-random in nature can lead to a significant decrease in the efficiency of the estimation process. This case occurs, in particular, in inertial navigation systems. Due to the bias in measurements, the Kalman filter estimates will be biased. This study proposes a new discrete Kalman filter that takes into account unknown systematic measurement errors. In this case, the filtering algorithm estimates the differences between two successive states. The differences between two successive measurements are used as measurements, as a result of which the systematic measurement errors mutually exclude each other. Corresponding expressions for estimating the state, the variance of the estimation errors, and the gain coefficient of the difference filter are derived. The proposed filter significantly improves the quality of estimation in the presence of constant or slowly changing offsets in measurements. Theoretical studies and computer simulations are carried out for a one-dimensional dynamic system.**

***Keywords*—Inertial Navigation, Estimation, Kalman Filter, Measurement Errors, Calibration.**

I. INTRODUCTION

When dynamic systems operate in real conditions, the task of assessing their state and parameters is usually solved under conditions of systematic measurement errors, the average value of which cannot be zero. The emergence of a systematic error is associated with the presence of unaccounted constant or slowly changing factors, such as the conditions of radio wave propagation, variations in the reference frequencies of generators, discrepancies between the readings of consumer clocks and satellite time scales when using navigation satellites to determine location, errors in the geodetic referencing of measuring instruments, etc.

Systematic components of measurement errors that are non-random in nature can lead to a significant decrease in the efficiency of the evaluation process. Due to the bias in measurements, the Kalman filter estimates will be biased. Therefore, taking into account systematic errors in the development of filtering and identification algorithms is an urgent task of great practical importance.

Below we propose a filtering algorithm that is robust to systematic measurement errors in the case where the measurement errors are additive. This case occurs, in particular, in inertial navigation systems (INS) [1]. In INS, information about the altitude, speed, and location is obtained by integrating the acceleration vector of the aircraft, which is measured by a system of sensitive elements-accelerometers installed either on a gyrostabilized platform or rigidly fixed in the body [2]. Due to the nature of the INS components (accelerometers and gyroscopes), the accuracy of the position and velocity data degrades over time. INS errors accumulate over time and they are limitless.

INS errors are classified as methodological or instrumental, and they can include both systematic and random components. In most applications, it is assumed that the INS operates in a linear mode and the measurement errors are additive. In the current literature [1-4], assuming that the INS operates in a linear mode, the measurement signal at the system output is stated as,

$$x(t) = x_0(t) + \Delta x_{INS}(t) \tag{1}$$

where $x_0(t)$ is the actual value of the navigation parameter measured by the INS, $\Delta x_{INS}(t)$ is the INS error.

As can be seen from equation (1), measurement errors are additive. INS errors $\Delta x_{INS}(t)$ are divided into methodological and instrumental and can have both systematic and random components, therefore, taking into account (1), the INS error can be written as a linear superposition of individual error components [1]:

$$\Delta x_{INS}(t) = L_{(i)}\left[f_i(t) + \phi_i(t)\right], i = 1, 2, ..., n, \tag{2}$$

where $L_{(i)}$ is the linear operator; $f_i(t), \phi_i(t)$ are the systematic and random components of the INS error, respectively.

As is known [5], in the case of a change in the variance of the measurement error, the Kalman filter estimates remain unbiased, but they will no longer be linear estimates with minimal variance. This case is studied in detail in [5], where an effective algorithm for eliminating this drawback is proposed. When the mathematical expectation of the measurement error is biased, the Kalman filter estimates will be biased. In this case, two cases are possible:

1) The mathematical expectation of the measurement error is known. As applied to this problem, the filtering equation is given in [6]. In this case, the known systematic errors at each filtering iteration are subtracted from the innovation sequence. The filter of this type is a special case of the generalized discrete Kalman filter. Note that in most cases encountered in practice, the mathematical expectation of the measurement error is not known and therefore this case is not widely used.

2) The mathematical expectation is not known. In this

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case, another method for eliminating the influence of the systematic error [7, 8] is envisaged, which consists in the fact that the parameters characterizing the systematic error are included in the number of unknowns and are estimated based on the measurement results. For this purpose, the systematic error is approximated by a polynomial and the estimates of the unknown coefficients are determined along with the estimates of the object parameters by some statistical method. Obviously, since part of the information contained in the measurements is spent on determining the systematic measurement errors, the accuracy of determining the desired vector decreases in this case.

Numerous studies [5–8] have shown that by using integrated navigation systems that combine inertial navigation systems with other navigation aids, INS errors are eliminated. As a result, independent measurements of comparable values from an external sensor are compared with the INS output data. Based on the measured discrepancies between the readings of various navigation systems, the inertial navigation system is adjusted.

In this paper, a new difference Kalman filter is proposed, which takes into account constant and slowly changing systematic measurement errors and significantly increases the quality of estimation in such cases. This approach can provide autonomous navigation without using external navigation sources.

II. Some Preliminary Remarks

Let us consider a linear one-dimensional dynamic system represented by state and measurement equations:

$$x(k) = ax(k-1) + w(k) \tag{1}$$

$$z(k) = x(k) + v(k)\,, \tag{2}$$

where a is a known system parameter; $\{w(k)\}$ is a sequence of independent, identically distributed random variables (system noise) with distribution $N\left(0,\sigma_w\right)$; $\{v(k)\}$ is a sequence of independent identically distributed random variables (measurement noise) with distribution $N\left(0,\sigma_v\right)$, independent of $\{w(k)\}$.

The state $x(k)$ is estimated by the linear optimal Kalman filter [9]:

$$\hat{x}(k) = a\hat{x}(k-1) + \frac{a^2P(k-1)+\sigma_w^2}{a^2P(k-1)+\sigma_w^2+\sigma_v^2}\left[z(k)-a\hat{x}(k-1)\right], \tag{3}$$

$$P(k) = a^2P(k-1)+\sigma_w^2 - \frac{\left[a^2P(k-1)+\sigma_w^2\right]^2}{a^2P(k-1)+\sigma_w^2+\sigma_v^2} = \frac{\sigma_v^2\left[a^2P(k-1)+\sigma_w^2\right]}{a^2P(k-1)+\sigma_w^2+\sigma_v^2} \tag{4}$$

Here $\hat{x}(k)$ is the estimation value, $P(k)$ is the variance of estimation error.

Filter (3)-(4) will be presented in a form more convenient for research:

$$\hat{x}(k) = a\hat{x}(k-1) + K(k)\left[z(k)-a\hat{x}(k-1)\right], \tag{5}$$

$$K(k) = \frac{a^2P(k-1)+\sigma_w^2}{a^2P(k-1)+\sigma_w^2+\sigma_v^2}, \tag{6}$$

$$P(k) = \sigma_v^2 K(k), \tag{7}$$

where $K(k)$ is the gain coefficient of the filter.

Systematic errors may exist from the start of the system's operation or may appear during its functioning. The first case occurs, in particular, in inertial navigation systems. In INS, information on the parameters of motion is obtained by integrating the aircraft acceleration vector. Due to the characteristics of the accelerometers and gyroscopes included in the INS, errors in determining location and speed slowly increase over time.

The second case is related to sudden shifts appearing in the measurement channel, changes in the reference frequency of the generators, etc. Generalizing these two cases, let us assume that at the time τ a shift occurred in the measurement channel. Obviously, in the first case (for example, as applied to the INS) $\tau = 0$. Due to the shift in measurements at, the estimates of the optimal Kalman filter (5)-(7) will also be shifted. Below, a difference Kalman filter is proposed that significantly reduces the shifts in the estimates of the output coordinate of the system in the presence of constant or slowly changing systematic errors in the measurements.

III. Difference Kalman Filter

In order to eliminate the bias of estimates due to systematic measurement error, it is proposed to use the Kalman filter to estimate not the state $x(k)$ but the increment

$$\Delta x(k) = x(k) - x(k-1) \tag{8}$$

In this case, the difference between two consecutive measurements will be used as a measurement

$$\Delta z(k) = z(k) - z(k-1) \tag{9}$$

As a result, systematic measurement errors are mutually exclusive and, most importantly, no information about the magnitude and sign of these errors is required. Note that this approach is applicable when systematic errors are additive. In addition, effective results are obtained when they are constant or slowly changing.

Definition. The differences between two consecutive measurements $\Delta z(k) = z(k) - z(k-1)$ are called differential measurements.

In this case, the state estimate $\hat{x}(k)$ is calculated by adding the found increment estimate $\Delta\hat{x}(k)$ to the previous estimate $\hat{x}(k-1)$, i.e.

$$\hat{x}(k) = \hat{x}(k-1) + \Delta\hat{x}(k) \tag{10}$$

Let us define the necessary equation for the Kalman filter, which allows us to estimate the increment $\Delta x(k)$. The equation of the estimated state is represented as

$$\Delta x(k) = a\Delta x(k-1) + \Delta w(k) \tag{11}$$

where

$$\Delta x(k-1) = x(k-1) - x(k-2);$$
$$\Delta w(k) = w(k) - w(k-1).$$

Measurement equation

$$\Delta z(k) = z(k) - z(k-1) = \Delta x(k) + \Delta v(k), \quad (12)$$

Here $\Delta v(k) = v(k) - v(k-1)$.

As is known, random variables obtained as a result of any linear transformations of normally distributed random variables are normally distributed. Based on this, we can state that the variables $\Delta w(k)$ and $\Delta v(k)$ are distributed according to the normal (Gaussian) law. It is easy to show that their average values are equal to zero:

$$E\{\Delta w(k)\} = 0; E\{\Delta v(k)\} = 0.$$

Taking into account the independence of system noise from measurement noise, the dispersion of variables $\Delta w(k)$ and $\Delta v(k)$ can generally be represented as follows:

$$D_{\Delta w}(k) = 2\sigma_w^2; D_{\Delta v} = 2\sigma_v^2.$$

Thus, based on the above, we can write:

$$\Delta w(k) \sim N\left(0, \sigma_w\sqrt{2}\right); \quad \Delta v(k) \sim N\left(0, \sigma_v\sqrt{2}\right).$$

Under the above conditions, the state estimation equation can be written similarly to equation (5) of the linear optimal Kalman filter

$$\Delta\hat{x}(k) = a\Delta\hat{x}(k-1) + G(k)\left[\Delta z(k) - a\Delta\hat{x}(k-1)\right] \quad (13)$$

where $G(k)$ is the gain coefficient.

Obtaining the variance of the value $\Delta\hat{x}(k) = \hat{x}(k) - \hat{x}(k-1)$ is a complex task, since the errors in two successive state estimates are correlated due to the use of the same system model and the same initial conditions in their calculation. In addition, the system and measurement noises are also the same. As a result, correlation moments appear between the indicated errors.

We determine the estimation errors $e(k-1)$ and $e(k)$ as follows:

$$e(k-1) = \hat{x}(k-1) - x(k-1);$$
$$e(k) = \hat{x}(k) - x(k) \quad (14)$$

As is known [6], errors (14) are distributed according to the normal law. The difference between the errors of two consecutive estimates will be denoted by

$$\delta(k) = e(k) - e(k-1) \quad (15)$$

Taking into account the linearity of equation (13), it is easy to show that when $k < \tau$

$$E\{e(k-1)\} = 0; E\{e(k)\} = 0;$$
$$E\{\delta(k)\} = E\{e(k)\} - E\{e(k-1)\} = 0$$

Let us apply the results obtained in [9, 10] to find the dispersion of the quantity $\delta(k)$

$$\begin{aligned} &E\left\{\delta^2(k)\right\} = \\ &E\left\{e^2(k) - e(k)e(k-1) - e(k-1)e(k) + e^2(k-1)\right\} \quad (16) \\ &= P(k) + P(k-1) - P_{\hat{x}(k)\hat{x}(k-1)} - P_{\hat{x}(k-1)\hat{x}(k)} \end{aligned}$$

where $P_{\hat{x}(k)\hat{x}(k-1)}$ and $P_{\hat{x}(k-1)\hat{x}(k)}$ are the correlation moments between the errors of two consecutive estimates, and

$$P_{\hat{x}(k)\hat{x}(k-1)} = P_{\hat{x}(k-1)\hat{x}(k)} \quad (17)$$

Thus,

$$E\left\{\delta^2(k)\right\} = P(k) + P(k-1) - 2P_{\hat{x}(k)\hat{x}(k-1)} \quad (18)$$

The Kalman filter estimates $\hat{x}(k)$ and $\hat{x}(k-1)$ under consideration are computed using the same system state model, have similar initial circumstances, and have equal initial variances for the errors of the designated estimates. In [9] it is shown that in this case, with the equality of the initial variance and the initial correlation moment, the following relationship is valid $P_{\Delta x(k)\Delta x(k-1)} = P(k)$. Then the variance of the quantity $\delta(k)$ can be represented in the following form

$$E\left\{\delta^2(k)\right\} = P_{\Delta\hat{x}}(k) = P(k-1) - P(k) \quad (19)$$

The gain of the difference Kalman filter is written as follows:

$$G(k) = \frac{P_{\Delta\hat{x}}(k)}{2\sigma_v^2} \quad (20)$$

By grouping the necessary equations for the operation of the difference Kalman filter, we obtain the following system of equations

$$\begin{aligned} &\Delta\hat{x}(k) = a\Delta\hat{x}(k-1) + G(k)\left[\Delta z(k) - a\Delta\hat{x}(k-1)\right] \\ &\Delta z(k) = z(k) - z(k-1) \\ &G(k) = \frac{P_{\Delta\hat{x}}(k)}{2\sigma_v^2} \quad (21) \\ &P_{\Delta\hat{x}}(k) = P(k-1) - P(k) \\ &P(k) = \frac{\sigma_v^2\left[a^2P(k-1) + \sigma_w^2\right]}{a^2P(k-1) + \sigma_w^2 + \sigma_v^2} \end{aligned}$$

The Kalman filter represented using equations (21) is called the difference Kalman filter. Taking into account expression (10) and denoting the estimates of the state of the difference filter as $\hat{x}_d(k)$, we can write the estimation value and variance of the estimation errors of the system state estimates in the following form

$$\hat{x}_d(k) = \hat{x}_d(k-1) + \Delta\hat{x}(k) \quad (22)$$

$$P_{\hat{x}}(k) = P_{\hat{x}}(k-1) + P_{\Delta\hat{x}}(k) \quad (23)$$

From the point of view of the accuracy of the estimation of the proposed difference Kalman filter, the variance of the estimate $\hat{x}(k-1)$ at the moment of the beginning of the difference filter operation is of great importance. From equation (23) it follows that it is necessary to strive to ensure that at the moment of the beginning of the difference filter operation the variance of the specified estimate would be as small as possible. For this purpose, it makes sense to use a combination of measuring instruments or different data processing algorithms at the specified point.

IV. Performance Analysis of Estimates of the Difference Kalman Filter

In order to study the optimality of estimates of the difference Kalman filter, we will consider the main properties of the innovation sequence. The innovation sequence of the difference Kalman filter has the following form

$$\Delta_d(k) = \Delta z(k) - a\Delta\hat{x}(k-1) \tag{24}$$

Let us prove the following theorems about the properties of the innovation sequence.

Theorem 1. In the case of normal measurements (in the absence of failures in the measurement channel), the innovation sequence of the difference Kalman filter (21) is distributed according to the Gaussian law with zero mathematical expectation:

$$\Delta_d(k) \sim N; \quad E\left[\Delta_d(k)\right] = 0\,. \tag{25}$$

Theorem 2. In the case of normal measurements (in the absence of failures in the measurement channel), the variance of the innovation sequence of the difference Kalman filter (21) is determined by the expression

$$P_{\Delta_d}(k) = a^2 P_{\Delta\hat{x}}(k-1) + 2\sigma_v^2 \tag{26}$$

where $P_{\Delta\hat{x}}(k) = P(k-1) - P(k)$.

Thus, during normal operation of the difference Kalman filter, the innovation sequence (24) is a white Gaussian noise with zero mean and variance

$$P_{\Delta_d}(k) = a^2 P_{\Delta\hat{x}}(k-1) + 2\sigma_v^2\ :$$

$$\Delta_d(k) \sim N\left[0, a^2 P_{\Delta\hat{x}}(k-1) + 2\sigma_v^2\right]. \tag{27}$$

In order to check the quality of the functioning of the difference Kalman filter, it is more convenient to use a normalized innovation sequence

$$\tilde{\Delta}_d(k) = \frac{\Delta_d(k)}{\sqrt{a^2 P_{\Delta\hat{x}}(k-1) + 2\sigma_v^2}} \tag{28}$$

because in this case $P_{\tilde{\Delta}_d} = 1$ and $\tilde{\Delta}_d(k) \sim N(0,1)$.

Statement 1. The estimates of the difference between two successive states $\Delta\hat{x}(k)$ of the difference Kalman filter are unbiased.

Statement 2. With a constant bias of measurements, the innovation sequence, as well as the estimates of the difference between two successive states $\Delta\hat{x}(k)$ of the difference Kalman filter, will be unbiased.

Statement 3. With a time-varying measurement bias, the innovation sequence, as well as the estimates of the difference between two successive states $\Delta\hat{x}(k)$ of the difference Kalman filter, will be biased.

V. Advantages of Using Difference Measurements in Inertial Navigation

In the presence of accelerometer bias, the systematic error of positioning in the INS is $0.5\,bt^2$ [11], where b is the accelerometer bias. The measurements $z(k)$ and $z(k-1)$ (instead of time t the iteration number k was used) include the systematic errors $0.5bk^2$ and $0.5b(k-1)^2$, respectively. Consequently, the systematic error present in the difference measurement $\Delta z(k) = z(k) - z(k-1)$ is equal to: $0.5b\left[k^2 - (k-1)^2\right] = 0.5b(2k-1)$. Thus, the systematic error of the difference measurement $\Delta z(k)$ is $\frac{k^2}{2k-1}$ times less than the systematic error of the conventional measurement $z(k)$. This is the main advantage of the difference measurement method. Thus, 1 hour after the INS has been operating, the systematic error of the difference measurement $\Delta z(k)$ due to the accelerometer bias, at $k = 3600$ sec. is approximately 1800 times less than the systematic error of the conventional measurement $z(k)$. This is a significant gain and is of great importance in navigation.

VI. Numerical Example

We examine a one-dimensional linear dynamic system as an example

$$\begin{aligned} x(k) &= ax(k-1) + w(k), \\ z(k) &= x(k) + v(k) \end{aligned} \tag{29}$$

with parameters a = 0,95; σ_w^2 =0,36; σ_v^2 = 1; $\hat{x}(0) = 2$; $x(0) = 1$; $P(0) = 10$.

Let the measurement processing be performed using two Kalman filters:

- the optimal Kalman filter (5) - (7), which determines the state estimate $\hat{x}(k)$ and variance of estimation errors $P(k)$ at each k-th step;
- the difference Kalman filter (21) proposed above, which finds the state estimate $\hat{x}(k)$ based on the estimate of the increment between two consecutive states $\Delta x(k)$.

We will show that the difference Kalman filter significantly improves the accuracy of the estimate in the presence of constant and slowly changing additive systematic errors in the measurements.

Scenario 1. During the simulation, the difference filter functioned starting from the 2nd estimation step. Starting from the step k = 50, the average value of the measurement error changed due to the introduced constant bias $\mu = 3$. The simulation results are shown in Figs. 1-4.

The behavior graph of the estimates of the difference Kalman filter is shown in Fig. 1. In the figure, the true values of the difference between two successive states $\Delta x(k)$ are shown by a solid line, and the estimates $\Delta\hat{x}(k)$ by a dashed line. As can be seen from the graphs, the estimates of the difference between two successive states of the difference filter with a constant measurement bias are unbiased.

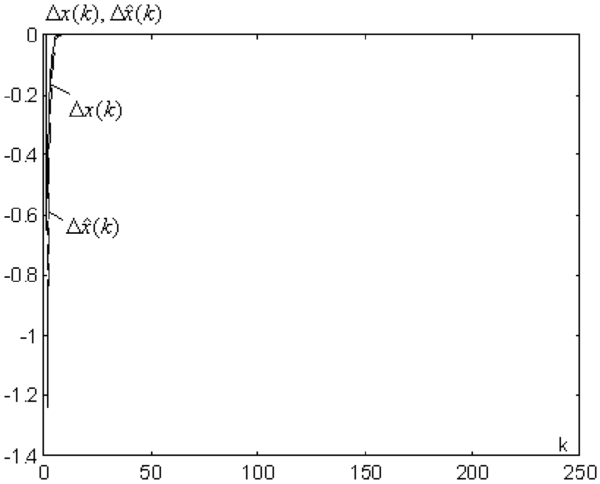

Fig.1. Behavior of the estimates of the difference Kalman filter

The normalized innovation sequences of the optimal and difference Kalman filters are shown in Fig. 2 and Fig. 3, respectively. In these figures, the dashed lines indicate the limiting values 3σ= 3 and -3σ= -3 of the normalized innovation sequences. It is evident from the presented graphs that systematic measurement errors lead to a shift in the innovation sequence of the optimal Kalman filter, but do not have a significant effect on the innovation sequence of the difference Kalman filter, which remains unbiased over the entire evaluation interval.

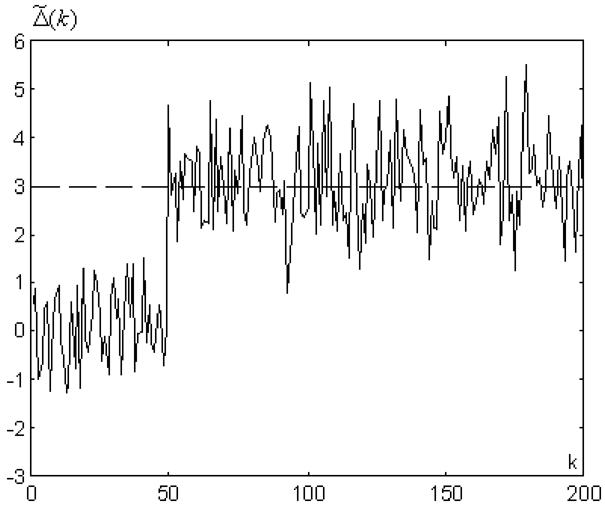

Fig. 2. Normalized innovation sequence of the optimal Kalman filter

Figure 4 shows the graphs of the behavior of the state estimates of the optimal and difference Kalman filters with measurement bias. In the Figures, the estimates of the difference Kalman filter are designated as $\hat{x}_d(k)$, the true state values are shown by a dotted line, the estimates of the optimal Kalman filter are shown by a solid line, and the estimates of the difference Kalman filter are shown by a dashed line. As can be seen from the results, after the appearance of a constant bias in the measurements at the 50th step of estimating, the optimal Kalman filter gives strongly biased estimates, while the estimates of the difference Kalman filter are very close to the true state values.

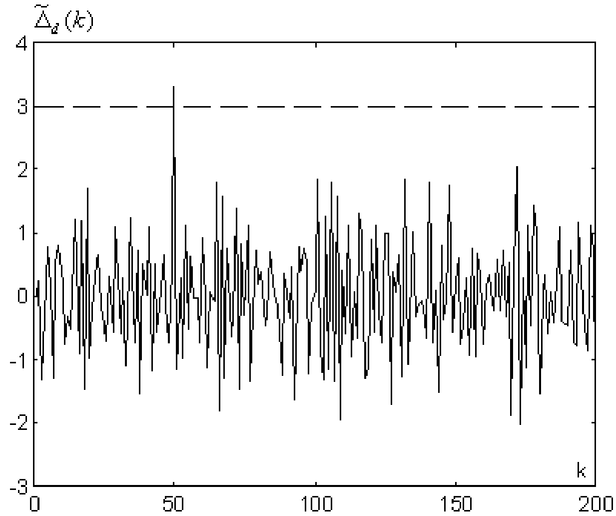

Fig.3. Normalized innovation sequence of the difference Kalman filter

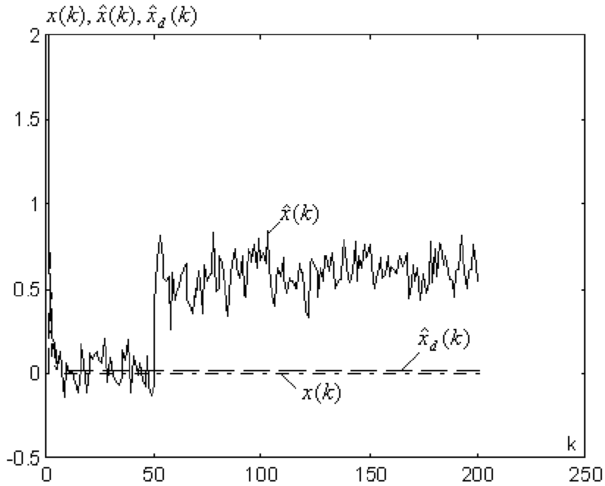

Fig. 4. Results of evaluation with constant measurement bias: $x(k)$ - true values; $\hat{x}(k)$ - estimates of the optimal Kalman filter; $\hat{x}_d(k)$ - estimates of the difference Kalman filter

It should be noted that the critical moment is the beginning of the operation of the difference filter $k = j$. Since the random error of the estimate $\hat{x}(j-1)$ that enters into the difference filter estimation equation $\hat{x}_d(j) = \hat{x}(j-1) + \Delta\hat{x}(j)$ will be included in subsequent estimates as a systematic error, i.e. all estimates $\hat{x}_d(k)$, $k \geq j$ will contain a constant systematic error $e(j-1) = \hat{x}(j-1) - x(j-1)$. It follows that the estimates of the state of the difference Kalman filter $\hat{x}_d(k)$ will be biased (although insignificantly). The results shown in Fig. 4 clearly demonstrate the indicated position. Therefore, when using the difference Kalman filter in order to reduce the bias of estimates, a fairly accurate estimate $\hat{x}(j-1)$ should be obtained. This can be achieved by combining measuring instruments and various data processing algorithms. It is also possible to carry out a measurement at the specified point using a more accurate (although expensive) measuring instrument.

Scenario 2. The same dynamic system (1) – (2) is considered with the same parameters and Kalman filters as in Scenario 1. However, unlike Scenario 1, here the average value of the measurement error was changed by introducing a bias from the beginning of the system's operation (i.e. $\tau = 0$) a slowly growing bias $\Delta x = 0.001k^2$. This case is typical for inertial navigation systems. The simulation results are shown in Figs.5-8.

The behavior graph of the difference Kalman filter estimates $\Delta\hat{x}(k)$ is shown in Fig. 5. In the figure, the true values of the difference between two successive states $\Delta x(k)$ are shown by a solid line, and the estimates $\Delta\hat{x}(k)$ by a dashed line. As can be seen from the graphs, the estimates of the difference filter are quite close to the true values.

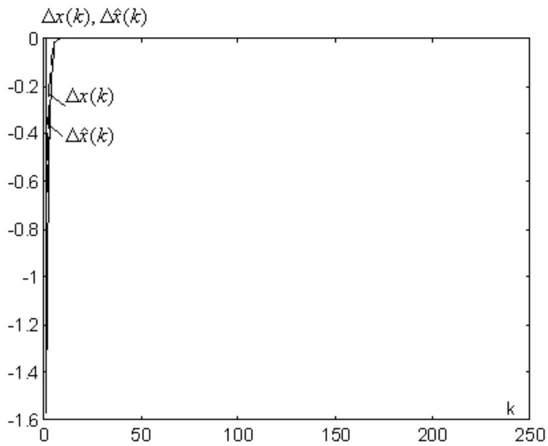

Fig.5. Behavior of the estimates $\Delta\hat{x}(k)$ of the difference Kalman filter

The normalized innovation sequences of the optimal and difference Kalman filters are shown in Fig. 6 and Fig. 7, respectively. It is evident from the graphs that slowly changing systematic measurement errors lead to a shift in the innovation sequence of the optimal Kalman filter. At the same time, the innovation sequence of the difference Kalman filter ignores the indicated shifts in measurements and gives very good estimates over the entire estimation interval.

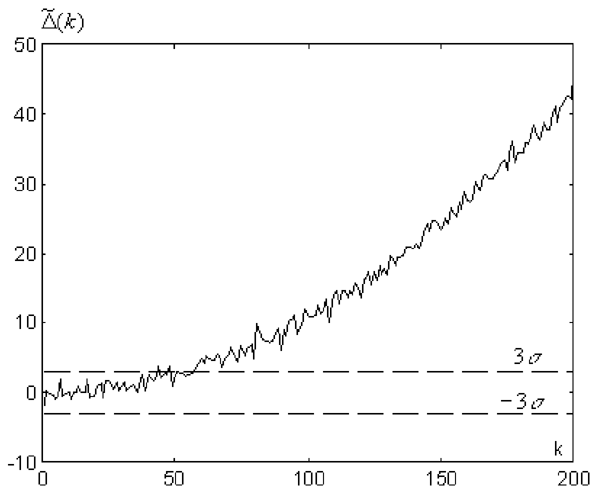

Fig. 6. Normalized innovation sequence of the optimal Kalman filter

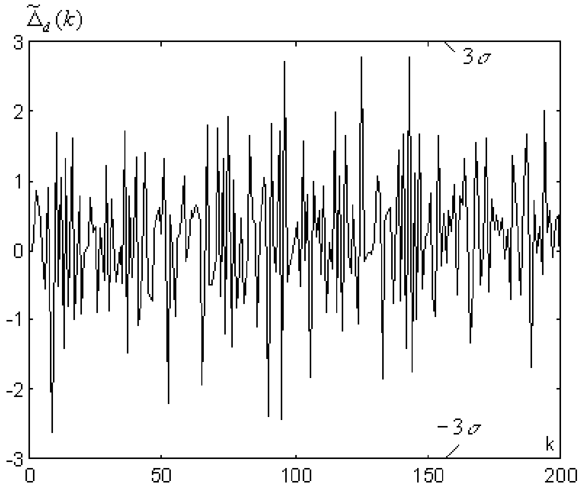

Fig. 7. Normalized innovation sequence of the difference Kalman filter

Fig. 8 shows the graphs of the behavior of the state estimates of the optimal and difference Kalman filters with a slowly growing measurement bias. As can be seen from the results, after the appearance of a slowly growing bias in the measurements at the 50th step of the estimation, the optimal Kalman filter diverges, while the estimates of the difference Kalman filter $\hat{x}_d(k)$ are very close to the true state values $x(k)$.

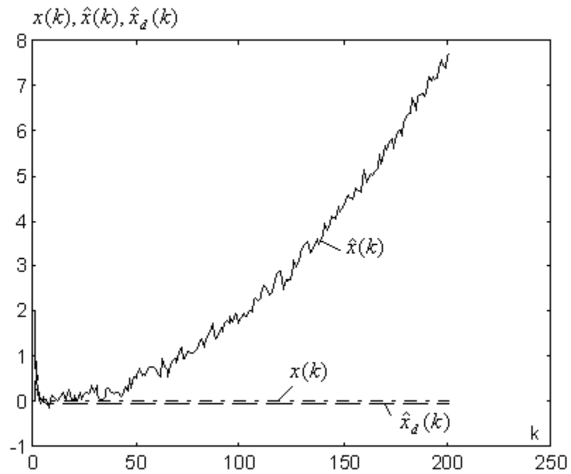

Fig.8. Results of estimation with slowly changing measurement bias: $x(k)$ - true values; $\hat{x}(k)$ - estimates of the optimal Kalman filter; $\hat{x}_d(k)$ - estimates of the difference Kalman filter

VII. CONCLUSION

A new discrete Kalman filter is proposed that takes into account unknown systematic measurement errors. In this case, in order to eliminate the bias of estimates due to systematic measurement errors, it is proposed to use the Kalman filter to estimate not the state, but the differences between two successive states. The differences between two successive measurements are used as measurements, as a result of which the systematic measurement errors mutually exclude (or reduce) each other.

Corresponding expressions for the state estimate, the variance of the estimation errors, and the gain of the difference filter are derived. The specified filter significantly improves the quality of the estimate in the presence of constant and slowly changing biases in the measurements. The advantage of this approach is that no information about the magnitude and sign of systematic errors is required when using it. However, this approach is applicable when the systematic errors are additive.

The developed filter can be widely used in practice, in particular, when processing information in inertial navigation systems, in which errors slowly grow over time and these errors are unlimited. The proposed difference Kalman filter in this case can significantly reduce the impact of systematic error on the accuracy of estimation.

In further research, the proposed idea will be developed taking into account the realities of inertial navigation systems. Considering that the INS is the main measuring system of most existing aircraft, the proposed approach can be widely used in this area.

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