# Experimental Determination of the Principal Stresses at a Point of a Beam Loaded Simultaneously in Bending and Torsion

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***Abstract*–The scientific publication examines a beam from an experimental stand, loaded simultaneously in bending and torsion in laboratory conditions. A computational scheme has been drawn up, consistent with the tested real structure. Strain gauge equipment has been used to measure the relative linear deformations caused by the applied combined external load. The principal stresses at different load points of the arm, perpendicular to the longitudinal axis of the beam, have been experimentally calculated and their directions relative to the coordinate system of the considered structure have been determined. The results obtained experimentally have been compared with the theoretically calculated stresses by applying the classical formulas for bending and torsion. The correspondence between the experimental and theoretical results has been analyzed, reflecting the reasons for the obtained deviations. The study shows the importance of experimental verification in determining the real stress state of machine elements subjected to combined loading and confirms the application of strain gauge methods for engineering analyses. The results of the study have practical significance for greater accuracy in the design and assessment of the load-bearing capacity of machine elements in real conditions.**

***Keywords*—beam, bending, experiment, principal stresses, torsion.**

## I. INTRODUCTION

Combined loads, in which structural elements are simultaneously subjected to bending and torsion, are often encountered in modern design and operation of load-bearing structures [1], [2]. Accurate assessment of the stress state under complex loads is essential for ensuring the safety, durability and efficiency of engineering facilities [3]. In the classical theory of the strength of materials, analytical methods have been developed for determining the normal and tangential stresses in bending and torsion separately, as well as stress transformation methods for determining the principal stresses and their directions [1], [4]. However, a number of factors such as real boundary conditions, technological inaccuracies, material inhomogeneities and design features can lead to deviations between the theoretically calculated values and the actual stress state [5]. For this reason, experimental research using strain gauge methods retains its relevance and practical significance for confirming and supplementing theoretical methods [6], [7]. In addition to the strain gauge method for experimentally determining stresses, the photoelastic method is also widely used. In this method, test specimens are made of plexiglass, which are illuminated with polarized white light. Due to the variation in stress across different regions of the specimen, the light is decomposed into colored fringes, with each color corresponding to a specific stress magnitude. In this way, a visual and volumetric representation of the stress distribution at all points of the test specimen is obtained, which is the main advantage of the method over contact measurements [6]. Laboratory exercises in the strength of materials are an important stage in the training of engineering personnel and provide an opportunity for practical experience in measuring and analyzing deformations and stresses [8]–[10].

### *A. Objective*

To experimentally determine the principal stresses and their directions at point K of a beam subjected to combined bending and torsion, and to compare the experimentally obtained results with the theoretically calculated values.

### *B. Tasks*

1. To develop a suitable calculation scheme that reflects the real experimental setup and loading of the beam.
2. To use suitable strain gauge equipment to measure the strains at point K of the beam and to perform experimental testing under predefined external loading.
3. Based on the experimental data obtained, to calculate the principal stresses and their directions.
4. To make a comparison between experimental and theoretical results and to analyze the reasons for possible differences.
5. To make an analysis and conclusions regarding the reliability of the adopted theory and its applicability under combined loads.

## II. THEORETICAL STATEMENT

When examining a beam loaded simultaneously in bending and torsion, basic relationships from the theory of resistance of materials are used. The normal stresses caused by bending are determined by the classical formula:

$$\sigma_x = \frac{M_y}{W_y}, \tag{1}$$

where $M_y$ is the bending moment, and $W_y$ is the section modulus for bending, for a circular ring cross-section:

$$W_y = \frac{\pi D^3}{32}\left(1-\alpha^4\right), \quad \alpha = \frac{d}{D} \tag{2}$$

where $D$ is the outer diameter of the ring, and $d$ is the inner diameter of the ring.

The tangential stresses during torsion are found by:

$$\tau_x = \frac{M_x}{W_c} \tag{3}$$

where $M_x$ is the torsional moment, and $W_c$ is the torsional section modulus, for a circular ring cross-section:

$$W_c = \frac{\pi D^3}{16}\left(1-\alpha^4\right), \quad \alpha = \frac{d}{D} \tag{4}$$

In a two-dimensional stress state, the principal stresses are determined using the following formula [9]:

$$\left.\begin{matrix}\sigma' \\ \sigma''\end{matrix}\right\} = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_x^{\ 2}} \tag{5}$$

In the present formulation, the following simplifications are assumed:

- The material is homogeneous and isotropic.
- The deformations are below the elastic limits.

## III. EXPERIMENTAL METHODOLOGY

A steel cantilever beam, fixed at one end, is considered. It is made of 37Cr4 steel according to DIN standards (fig. 1) and represents a tubular element 1 with length $l_1 = 28\ cm$ . At the free end of the beam, a steel arm 2 with length $l_2 = 41\ cm.$ is attached perpendicularly to its longitudinal axis in order to create a combined loading. A weight 3 is suspended at the end of the arm, resulting in a static force $G = 25\ N$ applied to the structure. This loading generates simultaneous bending and torsion. The magnitude of the torsional moment can be adjusted by changing the length of the arm, while the bending moment remains constant.

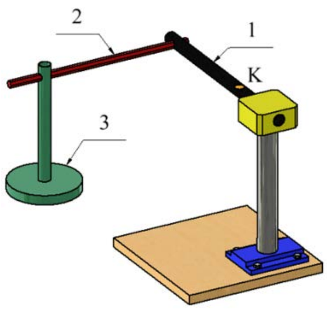


Fig. 1 Experimental model

A case of a two-dimensional stress state with unknown principal stress directions is considered. To experimentally determine these directions, deformations are measured along three directions - $a$ , $b$, and $c$ in the vicinity of the examined point K on the beam, located near the fixed support, as shown in the figure. A strain gauge method is applied for this purpose. The surface where the strain gauges are to be mounted is first mechanically cleaned using sandpaper up to grit № 200, starting with coarser grades. The surface is then degreased multiple times (3–4 times) using acetone. Adhesive X 60 is used for bonding. The adhesive consists of two components - a powder and a liquid which are mixed and left to react for about 5 minutes. The prepared adhesive is applied as a thin layer to the cleaned surface of the test specimen. The strain gauge, attached to an acetate strip, is pressed with the soft part of the thumb for 5 to 10 minutes, depending on ambient temperature. After bonding, the strain gauges are checked using a multimeter to ensure their functionality. The gauges are connected in a full or half-bridge circuit depending on the application. Once the gauges are connected, the test setup is calibrated. Practical measurements are conducted under static loading at three different points along the arm of the tested structure, taking into account the relative linear deformation $\varepsilon$, as $\varepsilon = \Delta l / l$, where $l$ is the length of the beam before deformation, and $\Delta l$ - the absolute linear deformation.

When the metal is deformed, the bonded strain gauge also undergoes deformation. As the test specimen elongates, its cross-sectional area changes, leading to a variation in the electrical resistance of the gauge. Measuring this change allows for the determination of the magnitude of the strain. By measuring the strain in the test specimen and the strain gauge, the stresses can be calculated using the simple or generalized form of Hooke's law, depending on the type of stress state.

In the present study, the measurements are conducted using a half-bridge configuration consisting of three active strain gauges bonded to the surface of the tubular element of the cantilever structure, arranged in a rectangular rosette and three compensating gauges are placed externally. The strain gauges are positioned so that they form angles $\alpha_a = -45^\circ$, $\alpha_b = 0^\circ$, $\alpha_c = +45^\circ$

with respect to the longitudinal axis of the beam. The gauge bonded along the $x$ -axis measures the relative linear strain $\varepsilon_{0^\circ}$ .The other two gauges, bonded at an angle $\pm 45^\circ$, measure the bending strains $\varepsilon_{45^\circ}$ , which are positive and also indicate tension. A single-channel Hottinger strain gauge measuring device is used to record the deformations. Prior to conducting the practical measurements, the wiring of the gauges in the rosette is prepared and connected to the equipment. After zeroing the device, the structure is loaded, creating both bending and torsional moments. Three different loading scenarios are analyzed, in which a $2{,}5\ kg$ weight is applied at three characteristic points along the arm, positioned perpendicular to the longitudinal axis of the beam. In this way, a combined bending and torsion loading is created. The magnitude of the torsional moment can be adjusted by changing the point of force application, while the bending moment remains constant in all cases.

The gauge bonded along the $x$ -axis, i.e. $\varepsilon_{0^\circ}$ , measures only the strain due to bending and is not affected by the magnitude of the torsional moment, but only by the bending moment.

The gauge placed at an angle of +45° registers the tensile strain from bending and adds the tensile strain from torsion, i.e. $\varepsilon_{+45^\circ} = \varepsilon_b + \varepsilon_t$ .

The gauge bonded at an angle of –45° registers the tensile strain from bending and subtracts the compressive strain from torsion, i.e. $\varepsilon_{-45^\circ} = \varepsilon_b - \varepsilon_t$ .

Both ±45° gauges respond to increases or decreases in the torsional moment, while the bending moment remains constant.

Based on the measured strains, the principal strains $\varepsilon'$ and $\varepsilon''$, as well as the angle $\alpha'$ of the principal direction for the rectangular rosette, are calculated. The following expressions are used for the calculations [10]:

$$\left.\begin{matrix}\varepsilon' \\ \varepsilon''\end{matrix}\right\} = \frac{\varepsilon_a + \varepsilon_c}{2} \pm \frac{\sqrt{2}}{2}\sqrt{\left(\varepsilon_a - \varepsilon_b\right)^2 + \left(\varepsilon_b - \varepsilon_c\right)^2}, \quad tg\alpha' = \frac{2\left(\varepsilon' - \varepsilon_a\right)}{2\varepsilon_b - \varepsilon_a - \varepsilon_c}. \tag{6}$$

For the experimental determination of the principal stresses in a two-dimensional stress state, the generalized Hooke's law is applied [9]:

$$\sigma' = \frac{E}{1-\mu^2}\left(\varepsilon' + \mu\varepsilon''\right); \ \sigma'' = \frac{E}{1-\mu^2}\left(\varepsilon'' + \mu\varepsilon'\right). \quad (7)$$

After calculating the final values of the principal stresses, they are arranged in descending order such that $\sigma_1 > \sigma_2 > \sigma_3$. For all three loading scenarios considered, the calculations were performed based on pre-developed analytical models (fig.2, fig.3, and fig.4), which accurately represent the actual experimental setup.

For the theoretical determination of the principal stresses in the vicinity of point K of the tested setup, the third strength theory was applied. Finally, the experimentally obtained principal stress values were compared with the theoretically calculated ones, and the relative error was determined.

## IV. NUMERICAL EXAMPLES

### *A. Static Loading at the First Characteristic Point of the Arm in the Experimental Setup*

Input Data (fig. 2):

$l_1 = 28 \ cm = 0,28 \ m$ - Length of the beam (tubular element);

$D = 25 \ mm = 2,5 \ cm$ - Outer diameter of the tube;

$d = 23 \ mm = 2,3 \ cm$ - Inner diameter of the tube;

$l_2 = 10 \ cm = 0,1 \ m$ - Length of the arm to the first loading point;

$G = 2,5 \ kg = 25 \ N$ - Force applied to the arm;

$E = 2,1.10^{11} Pa$ - Elastic modulus of the material;

$\mu = 0,3$ - Poisson's ratio.

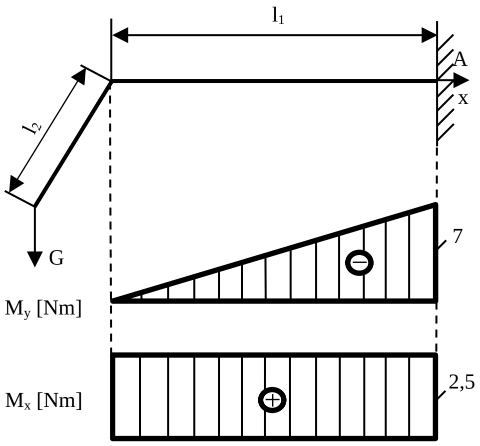


Fig. 2 Computational model for the first investigated case

Experimental determination of the principal stresses:

The following values of relative linear strain were recorded using the employed strain gauge equipment:

$$\begin{aligned} \varepsilon_a &= \varepsilon_{-45^\circ} = 15.10^{-6} \\ \varepsilon_b &= \varepsilon_{0^\circ} = 80.10^{-6} \\ \varepsilon_c &= \varepsilon_{+45^\circ} = 40.10^{-6} \end{aligned} \quad (8)$$

After substituting the measured values (8) into equations (6) and performing the necessary calculations, the principal strains and the angle $\alpha'$ of the principal direction for the rectangular strain gauge rosette were determined:

$$\varepsilon' = 83,28.10^{-6}, \ \varepsilon'' = -28,88.10^{-6} \ u \ \alpha' = +53^\circ, \ \alpha'' = -37^\circ. \quad (9)$$

The resulting principal stresses were calculated by substituting the results from (9) into equation (7):

$$\sigma' = 17,21 MPa; \ \sigma'' = -0,899 MPa. \quad (10)$$

where $\sigma' = \sigma_1; \ \sigma'' = \sigma_3$.

For the first case under consideration, they take the following form:

$$\sigma_1 = 17,21 \ MPa > \sigma_2 = 0 > \sigma_3 = -0,899 \ MPa. \quad (11)$$

Theoretical determination of the principal stresses:

It is known that in simultaneous bending and torsion, the most stressed points are those in which both normal and tangential stresses act simultaneously. In these sections, the normal and tangential stresses reach maximum values. For the case under consideration, the following values for the bending and torsional moments were obtained:

$$\begin{aligned} M_y &= G.l_1 = 7 Nm \\ M_x &= G.l_2 = 2,5 Nm \end{aligned} \quad (12)$$

In order to determine the values of the principal stresses theoretically, it is necessary to calculate the normal bending stress and the tangential torsional stress. For this purpose, the bending moment and the torsional moment of resistance were calculated. After substituting the input data in formulas (2) and (4), the following values were obtained:

$$\begin{aligned} W_y &= 0,435.10^{-6} m^3 \\ W_c &= 0,870.10^{-6} m^3 \end{aligned} \quad (13)$$

After substituting (12) and (13) into expressions (1) and (3), the values of the bending and torsional stresses at point K of the beam are obtained:

$$\sigma_x = 16,09 \ MPa; \ \tau_x = 2,87 \ MPa. \quad (14)$$

The theoretically calculated values of the principal stresses are obtained after substituting (14) into formula (5):

$$\sigma' = \sigma_1 = 16,58 \ MPa; \ \sigma'' = \sigma_3 = -0,5 \ MPa \quad (15)$$

Comparison of results and calculation of the error:

After comparing the values of the stresses σ1 obtained experimentally with those calculated theoretically, the relative error was determined:

$$\Delta = \frac{\sigma_{exp} - \sigma_{theor}}{\sigma_{theor}}.100 = 3,7\% \quad (16)$$

### *B. Static Loading at the Second Characteristic Point of the Arm in the Experimental Setup*

Input data (fig. 3):

$l_1 = 28\ cm = 0,28\ m$

$l_2 = 20\ cm = 0,2\ m$ $\quad D = 25\ mm = 2,5\ cm$

$G = 2,5\ kg = 25\ N$ $\quad d = 23\ mm = 2,3\ cm$

$E = 2,1.10^{11}\,Pa$ $\quad \mu = 0,3$

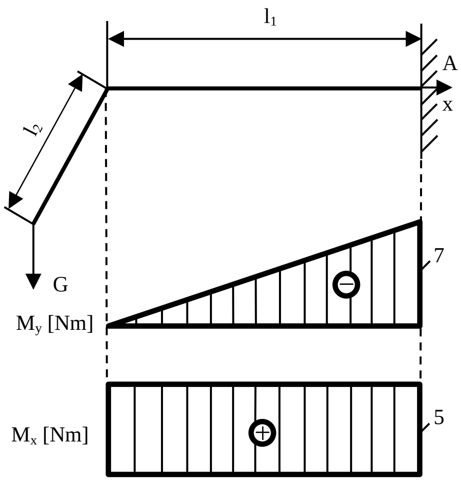


Fig. 3 Computational model for the second investigated case

Experimental determination of the principal stresses:

The values of the relative deformations recorded by the measuring equipment for the second studied case are:

$$\begin{aligned} \varepsilon_a &= \varepsilon_{-45^\circ} = -3.10^{-6} \\ \varepsilon_b &= \varepsilon_{0^\circ} = 80.10^{-6} \\ \varepsilon_c &= \varepsilon_{+45^\circ} = 60.10^{-6} \end{aligned} \tag{17}$$

After substituting the measured values (17) into equations (6) and performing the corresponding calculations, the principal strains and the angle $\alpha'$ of the principal direction for the rectangular strain gauge rosette were determined:

$$\varepsilon' = 88,86.10^{-6}, \varepsilon'' = -31,86.10^{-6} \ и \ \alpha' = +60^\circ 67', \alpha'' = -29^\circ 23'. \tag{18}$$

The obtained values for the principal stresses are calculated by substituting the results from (18) into equation (7):

$$\sigma' = 18,29\ MPa;\ \ \sigma'' = -1,206\ MPa. \tag{19}$$

For the second case under consideration, the final values are:

$$\sigma_1 = 18,29\ MPa > \sigma_2 = 0 > \sigma_3 = -1,206\ MPa. \tag{20}$$

Theoretical determination of the principal stresses:

The calculated values for the bending moment and the torsional moment in this case are:

$$\begin{aligned} M_y &= G.l_1 = 7Nm \\ M_x &= G.l_2 = 5Nm \end{aligned} \tag{21}$$

After substituting (21) and (13) into expressions (1) and (3), the values of the normal bending stress and the tangential torsional stress at point K of the beam are obtained:

$$\sigma_x = 16,09\ MPa;\ \ \tau_x = 5,75\ MPa. \tag{22}$$

The values of the principal stresses are obtained after substituting (22) into formula (5):

$$\sigma' = \sigma_1 = 17,93\ MPa;\ \ \sigma'' = \sigma_3 = -1,84\ MPa \tag{23}$$

Comparison of results and calculation of error:

After comparing the stress values obtained experimentally with those calculated theoretically for the second case under investigation, the relative error was determined as follows:

$$\Delta = \frac{\sigma_{\exp} - \sigma_{theor}}{\sigma_{theor}}.100 = 2\,\% \tag{24}$$

*C. Static Loading at the Third Characteristic Point of the Arm in the Experimental Setup*

Input data (fig. 4):

$l_1 = 28\ cm = 0,28\ m$

$l_2 = 30\ cm = 0,3\ m$ $\quad D = 25\ mm = 2,5\ cm$

$G = 2,5\ kg = 25\ N$ $\quad d = 23\ mm = 2,3\ cm$

$E = 2,1.10^{11}\,Pa$ $\quad \mu = 0,3$

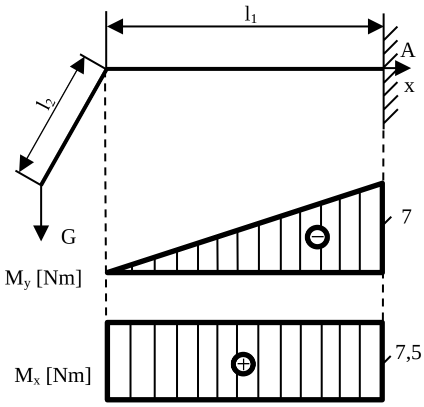


Fig. 4 Computational model for the third investigated case

Experimental determination of the principal stresses:

The following values of relative deformations were recorded for the third studied case:

$$\begin{aligned} \varepsilon_a &= \varepsilon_{-45^\circ} = -21.10^{-6} \\ \varepsilon_b &= \varepsilon_{0^\circ} = 80.10^{-6} \\ \varepsilon_c &= \varepsilon_{+45^\circ} = 82.10^{-6} \end{aligned} \tag{25}$$

After substituting the measured values (25) into formulas (6) and their corresponding calculation, the main deformations and the angle $\alpha'$ of the main direction for the rectangular rosette were calculated:

$$\varepsilon' = 101,92.10^{-6}, \varepsilon'' = -40,92.10^{-6} \ и \ \alpha' = +68^\circ, \alpha'' = -22^\circ. \tag{26}$$

After substituting (26) into formula (7), the following values for the principal stresses were obtained:

$$\sigma' = 20,68 \ \ MPa; \ \ \sigma'' = -2,38 \ \ MPa. \tag{27}$$

For the third case under consideration, the final values are:

$$\sigma_1 = 20,68 \ \ MPa > \sigma_2 = 0 > \sigma_3 = -2,38 \ \ MPa. \tag{28}$$

Theoretical determination of the principal stresses:

The calculated values for the bending moment and the torsional moment in this case are:

$$\begin{aligned} M_y &= G.l_1 = 7Nm \\ M_x &= G.l_2 = 7,5Nm \end{aligned} \tag{29}$$

After substituting (29) and (13) into expressions (1) and (3), the values of the bending and torsional stresses at point K of the beam are determined:

$$\sigma_x = 16,09 \ \ MPa; \ \ \tau_x = 8,63 \ \ MPa. \tag{30}$$

After substituting (30) into formula (5), the values of the principal stresses are obtained:

$$\sigma' = \sigma_1 = 19,845 \ \ MPa; \ \ \sigma'' = \sigma_3 = -3,75 \ \ MPa \tag{31}$$

Comparison of results and calculation of the error:

After comparing the values of the stresses obtained experimentally and theoretically for the third case under investigation, the relative error was determined:

$$\Delta = \frac{\sigma_{\exp} - \sigma_{theor}}{\sigma_{theor}}.100 = 4,2\% \tag{32}$$

## V. RESULTS AND DISCUSSION

### *A. Analysis of the Measured Strains*

Table 1 presents the recorded values of the relative linear strains at different values of the torsional moment. The data confirm that when the torsional moment increases or decreases, the sensor positioned along the x-axis does not respond and its readings remain unchanged. This corresponds to the theoretical expectation that the maximum shear stresses under torsion act at an angle of ±45° relative to the longitudinal axis of the beam. The recorded constant values demonstrate that no shear stresses occur along the x-axis, which supports the hypothesis τx=0.

TABLE 1
EXPERIMENTALLY MEASURED VALUES OF RELATIVE LINEAR DEFORMATIONS WHEN CHANGING THE TORSIONAL MOMENT

| **Deformations in directions** $a$, $b$, $c$ | **arm,** $m$ | **Measured values** $.10^{-6}$ |
|---|---|---|
| $\varepsilon_a = \varepsilon_{-45^\circ}$ | 0,1 | +15 |
| | 0,2 | −3 |
| | 0,3 | −21 |
| $\varepsilon_b = \varepsilon_{0^\circ}$ | 0,1 | +80 |
| | 0,2 | +80 |
| | 0,3 | +80 |
| $\varepsilon_c = \varepsilon_{+45^\circ}$ | 0,1 | +40 |
| | 0,2 | +60 |
| | 0,3 | +82 |

### *B. Analysis of the Results Obtained through Experimental and Theoretical Methods*

The comparison between the experimentally measured and theoretically calculated values is presented in Table 2. It is evident that the difference in the values of the principal stresses is within 5 %, which is entirely acceptable in engineering practice and confirms the reliability of the chosen method.

TABLE 2
EXPERIMENTAL AND THEORETICAL RESULTS FOR THE THREE LOAD CASES

| | First load case | Second load case | Third load case |
|---|---|---|---|
| $\varepsilon'$ | $83,28.10^{-6}$ | $88,86.10^{-6}$ | $101,92.10^{-6}$ |
| $\varepsilon''$ | $-28,88.10^{-6}$ | $-31,86.10^{-6}$ | $-40,92.10^{-6}$ |
| $\sigma'_{\exp}, MPa$ | 17,21 | 18,29 | 20,68 |
| $\sigma''_{\exp}, MPa$ | -0,899 | -1,206 | -2,38 |
| $\alpha'$, deg | +53 | +60°67' | +68° |
| $\alpha''$, deg | -37 | -29°23' | -22 |
| $\sigma'_{theor}, MPa$ | 16,58 | 17,93 | 19,845 |
| $\sigma''_{theor}, MPa$ | -0,5 | -1,84 | -3,75 |
| $\Delta$, % | 3,7 | 2 | 4,2 |

Figures 5, 6, and 7 illustrate how the principal directions and the corresponding stresses change with variations in the load application point and the magnitude of the torsional moment, where a, b, and c represent the orientations of the three active sensors. The theoretical concept that the principal stresses always act at specific angles, which do not coincide with the beam axis and result from the combined effect of bending and torsion, is confirmed in practice. It can be observed that the directions of σ1 and σ3 are mutually perpendicular (complementary to 90°).

If the sign of the torsional moment is changed, the bending stresses retain their values, while the tangential torsional stresses change their directions. This means that the principal stresses σ1 and σ3 exchange their respective directions, but their values remain valid with respect to the applied load.

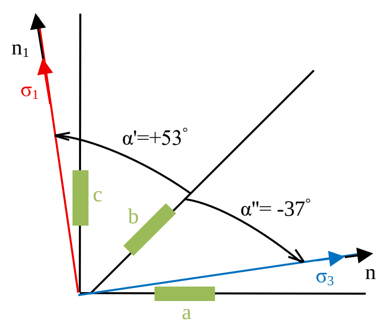


Fig. 5 Directions of the principal stresses σ1 and σ3 at the first characteristic point of the arm (first case) in combined bending and torsion

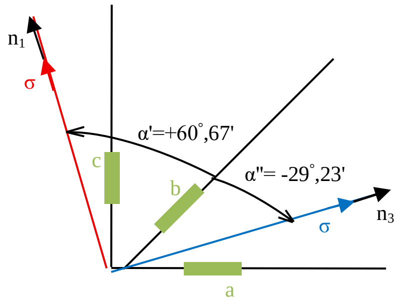


Fig. 6 Directions of the principal stresses σ1 and σ3 at the second characteristic point of the arm (second case) in combined bending and torsion

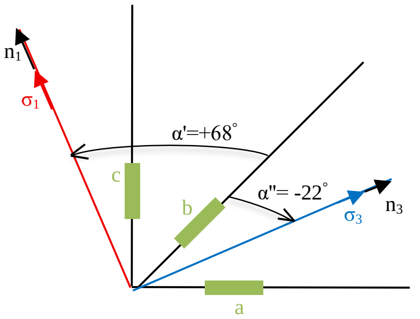


Fig. 7 Directions of the principal stresses σ1 and σ3 at the third characteristic point of the arm (third case) in combined bending and torsion

## VI. CONCLUSIONS FROM THE ANALYSIS

The analysis shows that the difference between the experimental and theoretical results remains within the limits of engineering accuracy - 3,7 %, 2 % and 4,2 % for the three investigated cases, respectively. This confirms the reliability of the strain gauge method for determining principal stresses under combined bending and torsion, and demonstrates the practical applicability of the proposed computational models.

## VII. CONCLUSION

In the present work, a method has been developed and applied for the experimental determination of the principal stresses and their directions at a point on a beam subjected to combined bending and torsion. Using strain gauge equipment and proper bonding of sensors in a rectangular rosette configuration, relative linear strains were measured along three directions. The obtained strain values were processed in accordance with the generalized Hooke's law for a two-dimensional stress state, and the principal stresses were determined.

The comparison between experimentally recorded and theoretically calculated values for the three characteristic points shows discrepancies within the range of 2–5%, which is within the acceptable limits for engineering laboratory investigations. The results confirm the reliability of the employed methodology and demonstrate its applicability for analyzing complex stress states in rods and beams under combined loading. The analysis also shows that the location of the applied load on the arm affects both the magnitude and direction of the principal stresses, which is in full agreement with theoretical relationships for combined loading.

The observed differences between the experimental and theoretical values are mainly due to technological inaccuracies during the bonding of the strain gauges and the strain measurements, as well as possible deviations in material processing. The obtained results have practical value for engineers and students, demonstrating the effectiveness of combining experimental measurements with analytical calculations in the design of structures subjected to combined loads.

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