PROCEEDINGS OF THE TECHNICAL UNIVERSITY OF SOFIA, ISSN: 2738-8549, 2738-8530 VOL.76, NO. 1, YEAR 2026

Models for Activating of the Participants in Social Networks

Petar Marinov

***Abstract*— The work considers models of centralized, decentralized and distributed management for activating a social network of interacting goal-oriented participants. The behavior of the participants is conformal, i.e. when deciding to change their behavior, each participant takes into account the opinion of other participants in the network. Therefore, a change in the states of some participants at the initial moment of time leads to changes in the states of other participants. The management task consists in finding such a set of participants on which a certain initial impact is exerted, in which the network passes into the required state. An example is the task of information management in social networks. Centralized and distributed management as well as information confrontation are considered.**

***Keywords*— Social networks, behavior models, management**

I. Introduction

This work considers models of centralized, decentralized and distributed management of the activation of a social network by interacting goal-oriented participants.

Problem statement. Let the social network be represented as a set of participants influencing each other in decision-making. The change in the states of some participants at the initial moment of time leads to the fact that under the influence of these changes the states of other participants also change. The nature and character of such dynamics can be different. They include models of the spread of active behavior in the network, failure models in the information-managed space, information security models, consensus models, etc. [1].

The management task (targeted activation of a social network) consists in searching for such a set of participants on which a certain initial impact is exerted, at which the network passes into the required state. The tasks of information management of social networks [2] fit into such a statement. management of collective threshold behavior, etc.

Social network. Let N = {1, 2, ..., n} be a finite set of participants in a social network, described by a directed graph Γ = (N, E), where $E \subseteq N \times N$ is the set of edges. Let each participant be in one of two states, inactive or active. Let us denote by $x_i \in \{0; 1\}$ the state of the ith participant, $i \in N$, with $\bar{x} = (x_1, x_2, \dots, x_n)$ – the vector of their states. We conventionally call the transition from inaction to action activation of the corresponding participant [3].

Behavior of participants. We assume that initially all participants are inactive, and the network dynamics is described by the transformation:

$$\Phi: 2^N \to 2^N,$$

Where $\Phi(S) \subseteq N$ is the set of active participants after the end of the transition process caused by the activation of the network – a change in the initial time of the participants from the set $S \subseteq N$.

It is assumed that the control actions are performed once from inaction to action.

For the image Φ(·) we assume that it has the following properties:

A.1 ("reflexivity") $\forall\ S \subseteq N$ is satisfied $S \subseteq \Phi(S)$;

A.2 ("monotonicity") $\forall\ S,\ U \subseteq N$: is satisfied $S \subseteq \Phi(S)$;

A.3. ("convexity") $\forall\ S,\ U \subseteq N$ such that $S \cap U = \varnothing$, is satisfied $\Phi(S) \cup \Phi(U) \subseteq \Phi(S \cup U)$.

Given a given image Φ(·), a function $\hat{G}: \{0; 1\}^n \to \{0; 1\}^n$, can be defined that maps the vector of the participants' initial states to the vector of their final states:

It is also possible to determine the states of participants that are indirectly activated:

First, the task of centralized control of the activation of the social network is formulated and an example of such control is given for the case of threshold behavior of the participants.

Then, the task of decentralized control is formulated, in which the participants independently make decisions about their own activation and the question of the feasibility (from the point of view of game theory) of effective or given states of the social network is investigated.

As an example, the task of controlling a group of participants in the social network is considered. A model of informational counteraction between centers interested in certain states of the social network and having the opportunity to exert influence on it is described.

II. Problem of centralized control

Let the functions on the sets be given

$$C: 2^N \to \Re^1 \quad H: 2^N \to \Re^1 \qquad S, W \in 2^N$$

C(S) is the cost of the initial change in the state of the participants in group S. Which centers bear the costs is determined by the specific task.

H(W) is profit from the resulting activation of group W.

The objective function of the control center is the difference between the profit and the cost:

$$v(S) = H(\Phi(S)) - C(S)$$

The task of centralized control is for the center to select such a set of initially activated participants that maximizes its objective function v(S):

$$v(S) = H(\Phi(S)) - C(S) \to \max_{S \subseteq N} \tag{1}$$

In this case, the costs are borne by the center and the profit is received by the center.

The solution $S^* \subseteq N$ of the discrete problem (1) in the general case requires enumerating all 2^n possible groups. The development of efficient methods for solving this problem is of special interest [4].

The state S* of the social network, which maximizes the objective function (1), is called efficient.

In the special case of additivity of the cost and benefit functions over the participants:

$$u(S) = \sum_{i \in \Phi(S)} H_i - \sum_{j \in S} c_j \quad (2)$$

where $(c_i, H_i)_{i \in N}$ are given non-negative constants.

In the centralized control problem, the participants are passive. The participants from the set S are activated by the center, after which this activation is propagated according to the operator $\Phi(\cdot)$.

Another control objective function is also possible, for example, maximizing the profit at limited costs by adding the cost and profit functions between the participants, which essentially represents the knapsack problem, or minimizing the costs for ensuring a given profit, etc.

For large social networks (with a large number of participants), discrete problems of type (1) have high computational complexity, so in these cases it is appropriate to consider the networks as random graphs with given probabilistic characteristics as the control problem is in terms of expected values [5] (minimizing or maximizing the mathematical expectation of the number of active participants [3]).

With threshold behavior of the participants in the social network, the influence of the i-th participant on the j-th is reflected by an arc (i, j). Let $N^{\text{in}}(i)$ is the set of neighbors – participants influencing the ith participant (predecessors), $N^{\text{out}}(i)$ is the set of participants (followers) – influenced by the ith participant

$$N^{\text{in}}(i) = \{j \in N \mid \exists\ (j; i) \in E\},\ N^{\text{out}}(i) = \{j \in N \mid \exists\ (i; j) \in E\}$$

$$n^{\text{out}}(i) = |N^{\text{out}}(i)| \qquad n^{\text{in}}(i) = |N^{\text{in}}(i)|.$$

We describe the process of collective decision-making by participants with a threshold behavior model:

$$x_i^t = \begin{cases} 1, & \dfrac{1}{n-1} \sum_{j \in N^{in}(i)} x_j^{t-1} \geq \theta_i \\ x_i^{t-1}, & \text{otherwise} \end{cases} \quad (3)$$

Where x_i^t is the state of the ith participant at time t, and $\theta_i \in [0; 1]$ is the threshold of this participant, t = 1, 2, …; given initial conditions: $x_i^0 = x_i$, $i \in N$.

Once activated, the participant cannot return to inactivity. Therefore, within the dynamics yp (3): the number of active participants does not decrease over time. The transition ends in no more than n steps. the correspondence between the initial and final states satisfies assumptions A1-A.3.

Of interest is the special case of threshold behavior with single thresholds ($\theta_i = 1$, $i \in N$), i.e. when for a participant to be activated, it is sufficient for at least one of its predecessors to be activated. The corresponding representation $\Phi(\cdot)$ will be linear:

$\forall\ S, U \subseteq N$: $S \cap U = \varnothing$ and $\Phi(S) \cup \Phi(U) = \Phi(S \cup U)$, i.e.

$$\Phi_0(S) = \bigcup_{i \in S} \Phi_0(\{i\}).$$

Function (2) in this case is subadditive, i.e.

$\forall\ S, U \subseteq N$: $S \cap U = \varnothing$ is fulfilled $u(S \cup U) \leq u(S) + u(U)$.

Let the graph G of participants in the social network contain no paths for which the initial and final vertices coincide (acyclicity) and the costs of initial activation of each participant are the same and equal to c (specific costs).

For each participant $i \in N$ we find the set Mi of all its "indirect followers", i.e. to which there is a path from the given participant in the graph.

The profit from activating the i-th participant is:

$$h_i = \sum_{j \in M_i} H_j$$

Due to the assumptions about the participant graph, it makes sense to activate only participants that have no predecessors (the set $M \subseteq N$).). Then task (1) will take the form:

$$\sum_{j \in \bigcup_{i \in S} M_i} H_j - c\,|S| \to \max_{S \subseteq N}.$$

Despite many simplifying assumptions (threshold behavior, uniqueness of thresholds, equality of costs, acyclicity of the graph), the resulting centralized control problem still does not allow an analytical solution and requires enumerating all subsets of the set M, which is computationally intensive.

Heuristics can be used to solve the problem - for example, consider as optimal the activation of those participants for whom the profit exceeds the costs as $S^* = \{i \in M \mid h_i \geq c\}$.

Thus, the tasks of centralized control of the activation of a social network rarely allow simple solutions.

III. PROBLEM OF DECENTRALIZED CONTROL

Let us consider possible formulations and methods for solving the tasks of decentralized control - when participants make activation decisions independently.

Let the mechanism for making management decisions be:

$$\sigma = \{\sigma_i(G(\bar{x})) \geq 0\}_{i \in N}$$

the distribution of profits between participants and additional profits can be obtained only by those participants who acted at the initial moment of time (profit from indirect activation of other participants is subject to distribution) [6].

Let us assume that the participants are active, i.e. at the initial moment with the mechanism σ they simultaneously and independently decide on their state – to act or not.

The further dynamics of the states of the participants is described by the operator $\Phi(\cdot)$.

The objective function $f_i(\bar{x})$ of the i-th participant is equal to the difference between its profit and costs:

$$f_i(\bar{x}) = \sigma_i(G(\bar{x})) + (H_i - c_i)\, x_i,\ i \in N \quad (4)$$

Here, if the participant decides to activate, then he bears the costs of his performance and receives the profit H_i, plus what he receives from the center according to the mechanism. "Self-activation" is the benefit to a given participant of activating, regardless of the mechanism used.

From (4) it follows that if $H_i > c_i$ for the i-th participant, then he self-activates. To exclude “self-activation” of active participants, we assume that:

$$H_i < c_i,\ i \in N.$$

Different mechanisms for distributing profits between participants are possible. A typical example is the equal distribution mechanism:

$$\sigma_i(G(\bar{x})) = \frac{\sum_{j\in N} H_j G_j(\bar{x})}{\sum_{j\in N} x_j}\, x_i,\ i \in N \quad (5)$$

In accordance with eq. (4) and eq. (7), the control center redistributes among those participants who themselves initially activated, the profit from their indirect activation of other participants. At the same time, no one has any costs for indirect activation.

A. *Equilibrium and efficient states of the social network*

The vector y* is by definition a Nash equilibrium of the game of participants (in game theory) [7], if

$$\sigma_i(G(\bar{x}^*)) + (H_i - c_i)\ x_i^* \ge \sigma_i(G(\bar{x}^*_{-i},\ 1 - x_i^*)) + (H_i - c_i)\ (1 - x_i^*),\ i \in N \quad (8)$$

In the Nash equilibrium, it is disadvantageous for active participants to switch to a state of inaction (provided that others do not change their states) and it is disadvantageous for inactive participants to change their state to action. Therefore, for active participants (for whom $x_i^* = 1$, $i \in N$,) it is satisfied

$$\sigma_i(G(\bar{x}^*)) + H_i \ge c_i \quad (9)$$

and for inactive participants ($x_i^* = 0$, $i \in N$) it is fulfilled

$$\sigma_i(G(x^*_{-i}, 1)) + H_i \le c_i \quad (10)$$

Conditions (9)-(10) for mechanism (5) of uniform distribution for participants in equilibrium have a form

$$\frac{\sum_{j\in N} H_j G_j(\bar{x}^*)}{\sum_{j\in N} x_j^*} + H_i \ge c_i \quad (11)$$

and for participants inactive in equilibrium have the form

$$c_i \ge \frac{\sum_{j\in N} H_j G_j(x^*_{-i}, 1)}{\sum_{j\in N} x_j^* + 1} + H_i \quad (12)$$

The question arises as to how the set of solutions to the centralized control problem and the set of Nash equilibria are related, i.e. in what cases can an efficient (according to criterion (1) or (2)) state of the network be realized in a decentralized manner.

Is it true, for example, that one of the efficient states is an equilibrium state or that among the equilibria there is at least one efficient one?

Let us consider two examples of the relationship between the set of efficient states and Nash equilibria being nontrivial and both hypotheses are generally false.

Example 1. Let there be three participants with unit thresholds eq. (3), whose costs and benefits are shown in Fig. 1. In this case, the condition $c_i > H_i$, $i \in N$ is not satisfied.

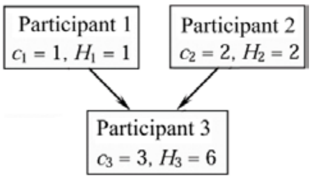

Fig 1. Example 1.

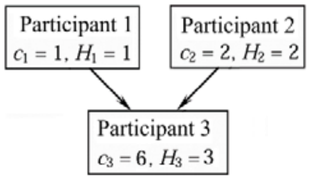

Fig 2. Example 2.

In this example, three state vectors of the participants are efficient: (0;1;0), (1;0;0), and (1;1;0). The Nash equilibria, regardless of the mechanism for distributing profits, are the four remaining state vectors other than (0;0;0). In this example, none of the four Nash equilibria is efficient, and none of the efficient states is an equilibrium.

Example 2. In the conditions of example 1. only the costs and profits of participant 3 have changed Fig. 2. Here the same three vectors of participant states are effective: (0;1;0), (1;0;0) and (1;1;0). The Nash equilibrium – the vector (1;1;0), is unique and effective.

Thus, the choice of mechanisms for distributing benefits in decentralized implementation of effective states of the social network remains open in the general case.

Mechanism (5) is balanced – the controlling center distributes all the profit from “indirect activation of the social network” among the initially acting participants. This type of mechanism is conditionally a mechanism of motivational management – the center encourages participants to choose certain states as an equilibrium of their behavior [8].

The alternative to motivational management is a more rigid institutional management, i.e. the center sets restrictions and norms for the activities of the participants.

B. *Institutional management*

Let the objective functions of the participants have the following form:

$$f_i(\bar{x}) = s_i(\bar{x}) + (H_i - c_i)\ x_i \quad (13)$$

where $s_i(\bar{x}) \ge 0$ is the reward paid by the center to the i-th participant. The set of dependencies of the participants' rewards on the vector of participants' actions will be called the vector function of the incentive of the participants by the controlling center [6]. This is a special case of the distribution of profits among the participants, introduced above.

Let us fix a certain set of participants V. Let us consider the following problem of "institutional management" - to find the minimum (in the sense of the total costs of the center for incentive) vector-function of incentive, which activates the fixed set V of participants as an equilibrium of Nash $\bar{x}^*(s(\cdot))$ of their behavior. The problem is written as follows:

$$\begin{cases} \sum_{i\in N} s_i(x^*) \to \min\limits_{s(\cdot)} \\ x_i^*(s(\cdot)) = 1,\ i \in V \\ x_j^*(s(\cdot)) = 0,\ j \notin V \end{cases} \quad (14)$$

It is solved using a decomposition of the participants' game

[8]. Let us consider the following vector incentive function:

$$s_i^*(\overline{x}) = \begin{cases} c_i - H_i + \varepsilon_i, & i \in V \text{ и } x_i = 1 \\ 0, & \text{otherwise} \end{cases} \quad (15)$$

where $\varepsilon_i > 0$ are arbitrarily small constants. $i \in V$.

When the center uses mechanism (15), the choice of single actions by participants from the set V is the only equilibrium in the dominant strategies of the game of participants with objective functions (13) [9].

Although formally the problem (14) is one of stimulation, its solution (15) can be interpreted more as institutional management - the participants are given quite strict norms of activity: for any deviation from the actions prescribed by the controlling center, they are severely punished (their stimulation is equal to zero).

We choose the objective function of the center F(•) to be equal to the difference between the profit from activating the participants from the set Φ(V) and the total cost of stimulation (15), realizing the activation of the participants from the set V (also eq. (2)), i.e.

$$F(V) = \sum_{i \in \Phi(V)} H_i - \sum_{j \in V} s_j(y^*) = \sum_{i \in \Phi(V)} H_i + \sum_{j \in V} H_j - \sum_{i \in V} c_i - \varepsilon_V \quad (16)$$

Comparing Eq. (16) and Eq. (2), we obtain that for sufficiently small values of the parameter ε_V the inequality $F(V) \geq u(V)$ is satisfied.

So far, the decentralization problem has been solved only partially, since in mechanism (5) the control center does not explicitly indicate to the participants what actions it expects from them, providing an opportunity for independent behavior in the situation. And the main idea of decentralization of control is precisely to build such a procedure for autonomous interaction of the participants, which would lead to their choice of an effective, in the sense of a centralized criterion, vector of actions.

In mechanism (15) there is an explicit instruction to the control center, the choice of which actions is expected from which of the participants. In addition, the center is still forced to solve the problem with centralized control of type (1), that is, knowing what its optimal (minimum costs of stimulating participants to activate them) profit (16), it must determine which group of participants it should activate initially:

$$F(V) \to \max_{V \subseteq N} \quad (17)$$

Thus, the advantage of decentralized mechanisms is that their participants may have incomplete information (each of them is not required to calculate the Nash equilibrium (8) or solve discrete optimization problems of type (1) or (17)). The disadvantage is the complexity (even impossibility in the general case) of solving the problem of finding an efficient decentralized mechanism.

The advantage of the centralized mechanism is that all information and computational costs are borne by the management center. But these costs can be very high.

Example: "activating" participants. Above we considered a model of managing participants who make decisions about their actions or inactions depending on the number of already active participants. In this case, the number of active participants is of essential importance for the center (efficiency criterion). From the perspective of the model considered in this section, the management task can be formulated as choosing a set of initially activated participants, in which the number of indirectly activated participants will be maximum (or minimum, equal to a given number, etc.), and the management costs will not exceed the budget constraint C_0, i.e.:

$$\begin{cases} |\Phi(S)| \to \max_{S \subseteq N} \\ C(S) \leq C_0 \end{cases} \quad (18)$$

Let the behavior of the participants be described by expression (3) and the graph of connections between the participants is complete. Let us renumber the participants in ascending order of their threshold values: $\theta_1 \leq \theta_2 \leq \ldots \leq \theta_n$. Let us denote by

$$P(x) = \frac{1}{n}\left|\{i \in N : \theta_i < x\}\right|$$

the distribution function of the thresholds of the participants and c – the sequence of shares of the active participants (in discrete time, where t denotes the number of the moment in time).

Suppose that the fraction x_0 of the participants acting at step zero is known. The fraction of participants whose thresholds do not exceed x_0 is $P(x_0)$, so that at the first step

$$x_1 = \max\{x_0; P(x_0)\}$$

At the next step, such a part of participants x_2 will act that the thresholds of the participants included in it do not exceed x_1, i.e.

$$x_2 = \max\{x_1; P(x_1)\}$$

Reasoning in a similar way, for the next steps we can write the following recurrence relation describing the dynamics of the behavior of a set of participants [10]:

$$x_{k+1} = \max\{x_k; P(x_k)\} \quad (19)$$

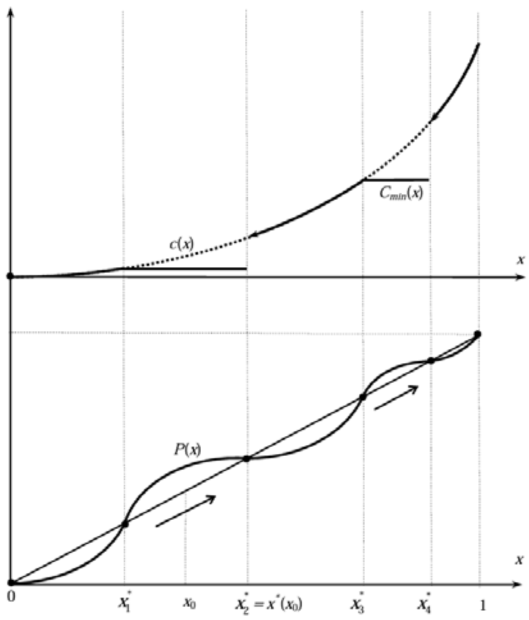

Fig. 3. Threshold distribution function and cost functions

The equilibrium positions of the system (19) are determined by the initial point x_0 and the intersection points of the graph of the function P(•) with the bisector of the first quadrant: P(x) = x. The equilibrium points can be stable if the graph of the function P(•) intersects the bisector, approaching it "from above" (Fig.3 (bottom graph)). In the continuous case, the points x_2^* and x_4^* are stable.

Let us denote by $\Psi(P(\cdot)) \subseteq [0;1]$ the set of roots of the equation P(x) = x . The unit is always a root. Let us define

$$x^*(x_0) = \min\ \{y \in \Psi(P(\cdot)):\ \ y > x_0\}.$$

For example, for the point x_0 shown in Fig. 3, $x^*(x_0) = x_2^*$ is satisfied. From Eq. (19) and the conditions of equilibrium stability, it follows that

$$\Phi(x_0) = \begin{cases} x_0, & P(x_0) \le x_0 \\ x^*(x_0), & P(x_0) > x_0 \end{cases} \tag{20}$$

Let $c(x_0)$ be the cost of initial activation of a given share x0 ϵ [0;1] participants. $c(x_0)$ is a non-decreasing function. Participants with lower thresholds are subject to activation first. Taking into account Eq. (20), problem (18) takes the form:

$$\begin{cases} \Phi(x_0) \to \max\limits_{x_0 \in [0;1]} \\ c(x_0) \le C_0 \end{cases} \tag{21}$$

Let us denote by $x^+ = \max\ \{x \in [0;1]:\ \ c(x) \le C_0\}$. Due to the non-decreasing nature of function (20), the solution to problem (21) is obvious: the maximally permissible (from the point of view of cost constraints) action x^+ should be applied. If, other things being equal, one should strive for cost minimization, then the optimal solution x_0^*.

$$x_0^* = \begin{cases} x^+, & P(x^+) \le x^+ \\ \max\{y \in \Psi(P(\cdot)) : y < x^+\}, & P(x^+) > x^+ \end{cases} \tag{22}$$

The inverse problem can also be solved (with respect to Eq. (21)) – to find the minimum costs $C_{min}(x)$ for implementing the initial activation, leading to the final share of activated participants in the social network not less than the set value x ϵ [0;1]. An example of solving this problem is shown in Fig. 3.

In the case where the goal of the control center is to minimize the activity of the participants, the corresponding problem is solved in a similar way, since the analysis of stable states and their dependence on the model parameters (Fig. 3) give a constructive characterization of the dependence of the obtained states on the initial ones.

C. *Informational counteraction*

Above, the social network is considered as an object of control, carried out by one subject – the center. In the case where there are, for example, two control centers, informational interaction occurs between these subjects. Such situations are described by a game in normal form between the centers, and the strategies chosen by the centers determine the parameters of the game between the participants.

Let there be two centers that at the initial moment of time can perform control actions on the network – once, simultaneously and independently changing the initial states of participants from the sets and, respectively. Let us assume that the dependence of the final state of the network on these control actions is known. Let the objective functions of the centers be differences between their profits and costs:

$$v_i(S_1, S_2) = H_i(\hat{\Phi}(S_1, S_2)\ ) - C_i(S_i),\ i = 1, 2.$$

We arrive at a normal form game between two centers (from Game Theory). If one of the controlling centers has the right to the first move, then we get a Stackelberg game or a game that can be interpreted as a "defense-attack" or "information influence-counteraction" situation.

IV. Conclusion

Designing models for centralized management of a social network as well as seeking a simple and analytical solutions is an interesting and real task for the practice.

The centralized control problems almost always do not allow analytical solutions (regardless of many possible simplifying assumptions) and require enumerating all subsets of participants, which is computationally intensive.

Heuristics can be used to solve these problems - for example, consider as optimal the activation of some of the participants under different conditions.

Thus, the tasks of centralized control of the activation of a social network rarely allow simple solutions.

The considered formulation of the problem of "activating" a social network is general, the non-triviality of the decentralized implementation of an effective equilibrium is established, and an effective mechanism for institutional management is proposed.

In the considered models, both factors (the structure of interaction between participants and the models of their collective behavior) are "hidden" in the operator Φ(•).

Secondly, the three typical types of management are institutional, motivational, and informational. The first two of them have already received some attention, although insufficiently. Therefore, they continue to be of interest, especially for practical tasks. The analysis of the third type – information models for management in social network activation tasks is poorly studied and is also very promising.

Third, the considered tasks for social networks can be extended to other types of networks of participants making strategic decisions, which suggests the possibility of using the results of the study and the ideas for using cooperative games on graphs (in which the graph of communications between participants limits the possibilities for forming coalitions and interactions between them).

Fourth, it is necessary to further study the mechanisms of profit distribution (such as mechanisms (5)) and characterize their classes that possess certain properties (e.g., realization of efficient states).

Fifth, further study of the decentralization of the social network activation task looks promising. Distributed optimization suggests that it is necessary to search for simple procedures of local behavior of participants that would lead to a state that is optimal within the framework of the initial task (tasks (1) and (17)), with high initial computational complexity.

REFERENCES

[1] K. Li, L. Zhang, H. Huang, “Social influence analysis: models, methods, and evaluation”, *Engineering*, vol. 4, 1, pp. 40-46, 2018, doi.org/10.1016/j.eng.2018.02.004.

[2] S. Boyd, L. Vandenberghe, “Convex optimization”, Cambridge, England, Cambridge University Press, 2004.

[3] P.V. Marinov, “Mathematical approach to social network modelling”, 32nd National Conf. with International Participation *Telecom 2024*, Sofia, Bulgaria, November 2024, https://ieeexplore.ieee.org/xpl/conhome/1835184/all-proceedings.

[4] D. Gomez, E. Gonzalez-Aranguena, C. Manuel et al, “Centrality and power in social networks: a game theoretic approach”, *Mathematical Social Sciences*, vol. 46, pp. 27-54, 2003.

[5] B. Bollobas, “Random graphs”. Cambridge, England, Cambridge University Press, 2001.

[6] A. Bonato, N. Hadi, P. Horn, P. Prałat, C. Wang. “Models of online social networks”, *Internet Mathematics*, vol. 6, no. 3, pp. 285–313, 2009, https://doi.org/10.1080/15427951.2009.10390642

[7] R. Myerson, “Game theory: analysis of conflict”, Cambridge, Massachusetts, London: Harvard University, Press, 2001.

[8] J.L. Pfaltz, “A mathematical model of dynamic social networks”, Soc. Netw. Anal. Min., vol. 3, pp. 863–872, 2013. https://doi.org/10.1007/s13278-013-0109-9

[9] S. Borkotokey, R. Kumar, D. Mukherjee, K. S. Rao, S. Sarangi, “Game theory and networks - new perspectives and directions”, INSIS, Springer, 2021.

[10] M. Granovetter, “Threshold models of collective behavior, AJS, vol. 83, no. 6. pp. 1420-1443, 1978.