# Planning the Experiment to Investigate the Efficiency of an Injection Vortex Burner

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***Abstract* - A full factorial experiment is an empirical technique used to evaluate the relationship between experimental variables and corresponding decisions. A full factorial experiment helps to develop a statistical model of response by performing a minimum number of well-chosen experiments and to determine the optimal values of the process parameters. To master the methodology of experimental research, a complete understanding of the methods of mathematical statistics and regression analysis is necessary. It is also necessary to develop the ability to analyze the object of study, correctly select the optimization parameter and the essential factors of the object of study, build a matrix for planning the experiment to obtain an adequate mathematical model of the object. In this work, a full factorial experiment was performed to determine the influencing operating and design parameters on the efficiency of the combustion device under consideration. The full factorial experiment was conducted following the reviewed and described stages and sequence of its implementation.**

***Keywords* – Combustion, full factorial experiment, efficiency, injected swirling jet.**

## *A. Introduction*

Experimental design combines various analytical procedures that allow for the minimum number of experiments to obtain maximum information about the studied physical or technological process [1], [2], [3].

Experimental planning helps to appropriately select relevant operating and design parameters to optimize combustion devices, on which both the efficiency of gas fuel use and the amount of harmful emissions released into the environment depend [1], [2], [3].

A full factorial experiment involves a series of applied statistical tools used to systematically classify and quantify the cause-and-effect relationships between variables and outcomes in a studied process or phenomenon, which can lead to finding the settings and conditions under which the process is optimized [11].

In accordance with the information data regarding the parameters affecting the efficiency of the combustion device, a full factorial experiment with a $2^k$ type design, i.e. at two levels with four factors (k=4), was selected in the present work [3], [7], [8].

The goal in choosing this plan is to obtain the most complete information possible regarding the influence of the adopted factors on the burner efficiency, minimizing the number of trials and observing the requirements for rotatability and orthogonality of the respective matrix [3].

In order to eliminate the influence of external factors, the experiments were conducted in a randomized manner over time. Using this approach allows to filter out the influence of unaccounted for significant factors and to avoid the accumulation of systematic error [3], [9].

From previous experiments and data from literature sources [4], [5], [6], four main independent parameters can be determined that significantly affect the efficiency of the burner, such as: angle of inclination of the contact nozzles, diameter of the contact nozzle opening, heat load and length of the mixing section.

## *B. Substantive Part*

The aim of this work is to implement mathematical planning of the experiment using previously obtained experimental data, thereby obtaining a regression equation, which will propose a method for studying the influence of the operating and design parameters on the efficiency coefficient [η] of the combustion device.

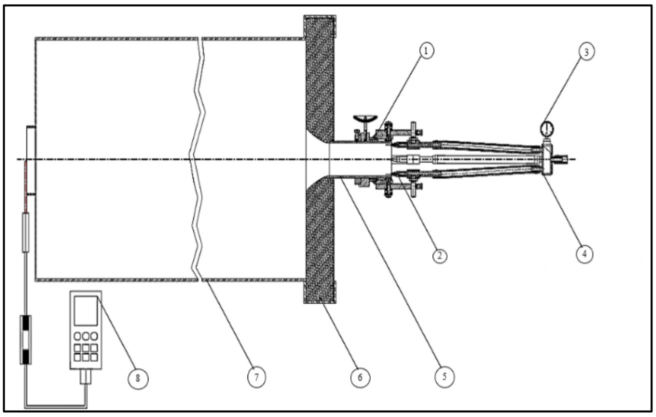


Fig. 1. Diagram of a laboratory installation.

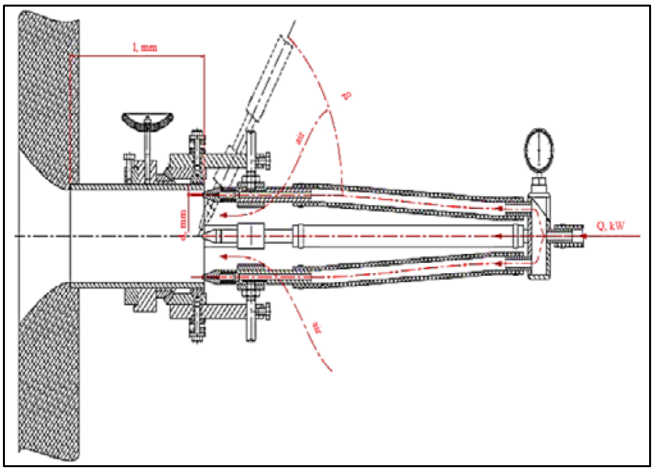


Fig. 2. Diagram of a rotating apparatus.

Figure 1 shows a diagram of the experimental laboratory installation located in the "Fuel Equipment and Technologies" laboratory at the Institute of Physics and Technology-Sliven. It consists of a (1) conical gear rotating apparatus (Fig. 2), four contact nozzles (2) attached to the apparatus, a refractory plate (6), a gas distribution collector

(4) with a pressure gauge (3) mounted on it, which guarantees a constant pressure of the supplied combustible gas, a mixing chamber (5), and a combustion chamber (7). The experimental measurements were performed with a gas analyzer "Testo 300M" (8).

In our previous works [10], [3], the methodological sequence for conducting the full factorial experiment is described in detail.

Table 1 gives the intervals of variation of the factors (independent variables) and their values in natural scale at the basic, upper and lower levels.

Table 2 shows an expanded matrix of the full factorial experiment $2^4$ in coded and natural scale after applying the dependence between the factors, as well as the results of the experiments, with the dependent variable $\bar{y}$ representing the efficiency coefficient [η]. A column of a dummy variable $x_0$ has been added to the table, taking a positive value in all trials, necessary to estimate the free term $b_0$ of the regression equation.

TABLE 1
FACTOR CHANGE INTERVALS

| Factor change intervals | | | | |
|---|---|---|---|---|
| **Factors** | **Basic level ($X_{i0}$)** | **Change interval ($\Delta X_i$)** | **Upper level ($x_i = +1$)** | **Lower level ($x_i = -1$)** |
| ***$X_1$ (β°)*** | 45 | 15 | 60 | 30 |
| ***$X_2$ (d, mm)*** | 0,8 | 0,2 | 1 | 0,6 |
| ***$X_3$ (Q, kW)*** | 90 | 25 | 115 | 65 |
| ***$X_4$ (l, mm)*** | 225 | 75 | 300 | 150 |

The coded values of the factors ($x_i$) are associated with natural ($X_i$) by dependency:

$$x_i = \frac{X_i - X_{i0}}{\Delta X_i} \quad (1)$$

After applying this dependence, we obtain:

$$x_1 = \frac{\beta - 45}{15} \quad (2)$$

$$x_2 = \frac{d - 0{,}8}{0{,}2} \quad (3)$$

$$x_3 = \frac{Q - 90}{25} \quad (4)$$

$$x_4 = \frac{l - 225}{75} \quad (5)$$

When processing the full factorial experiment in the specific case of uniform duplication of trials, a model of the type is constructed:

$$\bar{y} = b_0 + \sum_{1 \le i \le 4} b_i x_i + \sum_{1 \le i \le j \le 4} b_{ij} x_i x_j + \sum_{1 \le i \le j \le l \le 4} b_{ijl} x_i x_j x_l + b_{ijlm} x_i x_j x_l x_m \quad (6)$$

The conducted full factorial experiment is implemented by performing the following checks:

- For homogeneity of the series of variances according to the Cochrane criterion:

$$G^* = \frac{S^2_{y_{umax}}}{\sum_{u=1}^{N} S^2_{y_u}} = 0{,}12 \quad (7)$$

where: $S^2_{y_u}$ - variance for each trial in order; $S^2_{y_{umax}}$ - largest variance in the series; $N$ - number of attempts; $G^*$ - estimated value of the Cochrane criterion, $G^* < G^t$- uniformity of the series of variances.

- For statistical significance of the coefficients from the regression equation according to the Student's t-criterion:

$$S^2_{b_i} = \frac{S^2_y}{n_u \cdot N} \quad (8)$$

$$\Delta b_i = t_{\alpha;f_1} \cdot S_{b_i} = 0{,}00075 \quad (9)$$

where: $S_{b_i}$ - root mean square error; $t_{\alpha;f_1}$ - t-criterion value; $f_1$ - the number of degrees of freedom; α - level of significance; $\Delta b_i$ - confidence interval. Coefficients whose absolute value is equal to or greater than the confidence interval are statistically significant.

- For the adequacy of the obtained regression equation according to Fisher's criterion:

$$S^2_{in} = \frac{n \cdot \sum_{u=1}^{N} (y_u - \bar{y}_u)^2}{f_2} \quad (10)$$

$$F^*_{f_2;f_1} = \frac{S^2_{in}}{S^2_y} = 1{,}48 \quad (11)$$

where: $S^2_{in}$ - variance of inadequacy; $F^*$ - calculated value of Fisher's exact criterion. The adequacy hypothesis is accepted when the calculated value of the F-criterion is $F^* \le F^t$.

After implementing the full factorial experiment$2^4$ and taking into account the condition for statistical significance of the coefficients, an equation of the form:

$$\bar{y} = 0{,}9378 + 0{,}0161 x_1 + 0{,}0088 x_2 + 0{,}0109 x_3 - 0{,}0031 x_4 - 0{,}0031 x_1 x_2 - 0{,}0038 x_1 x_3 - 0{,}0021 x_2 x_4 + 0{,}004 x_3 x_4 - 0{,}0013 x_1 x_2 x_3 + 0{,}001 x_1 x_2 x_4 - 0{,}0031 x_1 x_3 x_4 + 0{,}0013 x_2 x_3 x_4 \quad (12)$$

After moving from a coded to a natural scale, the equations take the form:

$$\eta = 0{,}9378 + 0{,}0161 \cdot \frac{\beta-45}{15} + 0{,}0088 \cdot \frac{d-0{,}8}{0{,}2} + 0{,}0109 \cdot \frac{Q-90}{25} - 0{,}0031 \cdot \frac{l-225}{75} - 0{,}0031 \cdot \frac{\beta-45}{15} \cdot \frac{d-0{,}8}{0{,}2} - 0{,}0038 \cdot \frac{\beta-45}{15} \cdot \frac{Q-90}{25} - 0{,}0021 \cdot \frac{d-0{,}8}{0{,}2} \cdot \frac{l-225}{75} + 0{,}004 \cdot \frac{Q-90}{25} \cdot \frac{l-225}{75} - 0{,}0013 \cdot \frac{\beta-45}{15} \cdot \frac{d-0{,}8}{0{,}2} \cdot \frac{Q-90}{25} + 0{,}001 \cdot \frac{\beta-45}{15} \cdot \frac{d-0{,}8}{0{,}2} \cdot \frac{l-225}{75} - 0{,}0031 \cdot \frac{\beta-45}{15} \cdot \frac{Q-90}{25} \cdot \frac{l-225}{75} + 0{,}0013 \cdot \frac{d-0{,}8}{0{,}2} \cdot \frac{Q-90}{25} \cdot \frac{l-225}{75} \quad (13)$$

After conversion, we get:

$$\eta = 0{,}8532 + 0{,}22.10^{-3}\beta + 0{,}163d - 0{,}7.10^{-3}Q - 0{,}13.10^{-3}l - 0{,}58.10^{-3}\beta d + 0{,}28.10^{-4}\beta Q + 0{,}56.10^{-5}\beta l +$$

$$0{,}4.10^{-4}dQ - 0{,}66.10^{-3}dl + 0{,}38.10^{-5}Ql - 0{,}18.10^{-4}\beta dQ + 0{,}5.10^{-5}\beta dl - 0{,}107.10^{-6}\beta Ql + 0{,}35.10^{-5}dQl \quad (14)$$

TABLE 2
EXTENDED MATRIX OF THE FULL FACTORIAL EXPERIMEN

| **Extended matrix of the full factorial experiment $2^4$** | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **№** | **Code scale** | | | | | | | | | | | | | | | | **Natural scale** | | | | **η, %** |
| | $x_0$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_1x_2$ | $x_1x_3$ | $x_1x_4$ | $x_2x_3$ | $x_2x_4$ | $x_3x_4$ | $x_1x_2x_3$ | $x_1x_2x_4$ | $x_1x_3x_4$ | $x_2x_3x_4$ | $x_1x_2x_3x_4$ | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $\bar{y}$ |
| 1 | +1 | +1 | +1 | +1 | +1 | +1 | +1 | +1 | +1 | +1 | +1 | +1 | +1 | +1 | +1 | +1 | 60 | 1 | 115 | 300 | 0,962 |
| 2 | +1 | -1 | +1 | +1 | +1 | -1 | -1 | -1 | +1 | +1 | +1 | -1 | -1 | -1 | +1 | -1 | 30 | 1 | 115 | 300 | 0,952 |
| 3 | +1 | +1 | -1 | +1 | +1 | -1 | +1 | +1 | -1 | -1 | +1 | -1 | -1 | +1 | -1 | -1 | 60 | 0,6 | 115 | 300 | 0,955 |
| 4 | +1 | -1 | -1 | +1 | +1 | +1 | -1 | -1 | -1 | -1 | +1 | +1 | +1 | -1 | -1 | +1 | 30 | 0,6 | 115 | 300 | 0,929 |
| 5 | +1 | +1 | +1 | -1 | +1 | +1 | -1 | +1 | -1 | +1 | -1 | -1 | +1 | -1 | -1 | -1 | 60 | 1 | 65 | 300 | 0,948 |
| 6 | +1 | -1 | +1 | -1 | +1 | -1 | +1 | -1 | -1 | +1 | -1 | +1 | -1 | +1 | -1 | +1 | 30 | 1 | 65 | 300 | 0,903 |
| 7 | +1 | +1 | -1 | -1 | +1 | -1 | -1 | +1 | +1 | -1 | -1 | +1 | -1 | -1 | +1 | +1 | 60 | 0,6 | 65 | 300 | 0,937 |
| 8 | +1 | -1 | -1 | -1 | +1 | +1 | +1 | -1 | +1 | -1 | -1 | -1 | +1 | +1 | +1 | -1 | 30 | 0,6 | 65 | 300 | 0,891 |
| 9 | +1 | +1 | +1 | +1 | -1 | +1 | +1 | -1 | +1 | -1 | -1 | +1 | -1 | -1 | -1 | -1 | 60 | 1 | 115 | 150 | 0,968 |
| 10 | +1 | -1 | +1 | +1 | -1 | -1 | -1 | +1 | +1 | -1 | -1 | -1 | +1 | +1 | -1 | +1 | 30 | 1 | 115 | 150 | 0,946 |
| 11 | +1 | +1 | -1 | +1 | -1 | -1 | +1 | -1 | -1 | +1 | -1 | -1 | +1 | -1 | +1 | +1 | 60 | 0,6 | 115 | 150 | 0,959 |
| 12 | +1 | -1 | -1 | +1 | -1 | +1 | -1 | +1 | -1 | +1 | -1 | +1 | -1 | +1 | +1 | -1 | 30 | 0,6 | 115 | 150 | 0,918 |
| 13 | +1 | +1 | +1 | -1 | -1 | +1 | -1 | -1 | -1 | -1 | +1 | -1 | -1 | +1 | +1 | +1 | 60 | 1 | 65 | 150 | 0,96 |
| 14 | +1 | -1 | +1 | -1 | -1 | -1 | +1 | +1 | -1 | -1 | +1 | +1 | +1 | -1 | +1 | -1 | 30 | 1 | 65 | 150 | 0,933 |
| 15 | +1 | +1 | -1 | -1 | -1 | -1 | -1 | -1 | +1 | +1 | +1 | +1 | +1 | +1 | -1 | -1 | 60 | 0,6 | 65 | 150 | 0,942 |
| 16 | +1 | -1 | -1 | -1 | -1 | +1 | +1 | +1 | +1 | +1 | +1 | -1 | -1 | -1 | -1 | +1 | 30 | 0,6 | 65 | 150 | 0,901 |

*C. Results*

After conducting a full factorial experiment and processing the results obtained from it, a diagram (Fig. 7) is shown of the relative magnitude of influence of the factors and their interactions. As well as graphs (figures 3 and 4) of the change in the efficiency coefficient with changing the angle of inclination of the gas nozzles and the thermal load of the burner.

Graphically (Figures 5, 6, 8 and 9) the reaction surfaces and their sections for the burner efficiency coefficient are depicted, due to the influencing interactions between the selected independent factors, with each of the factors fixed at a zero (base) level.: *β = 45°; d = 0,8mm; Q = 90kW; l = 225mm.*

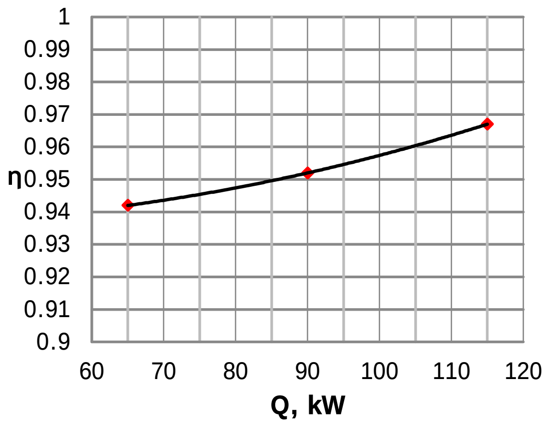


Fig. 3. Change in efficiency at different thermal loads

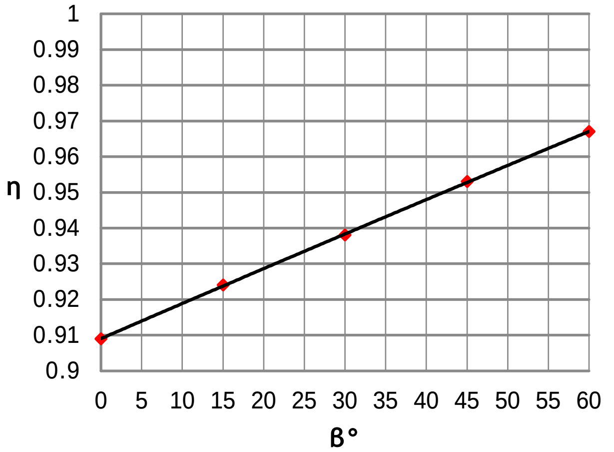


Fig. 4. The change in the efficiency coefficient depending on the change in the inclination angle of the gas nozzles

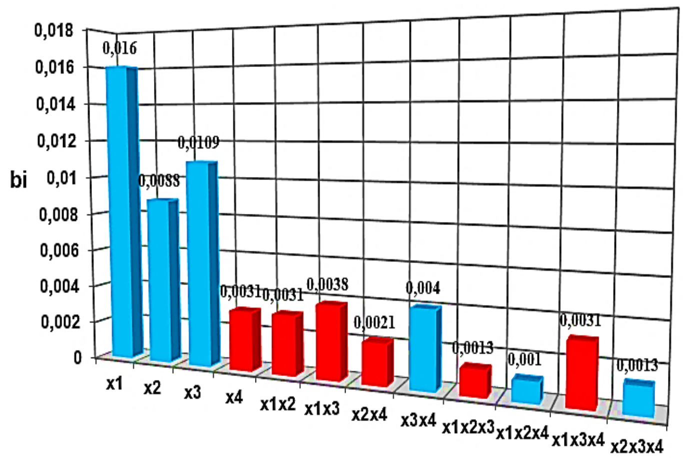


Fig. 7. Relative magnitude of influence of factors and their interactions

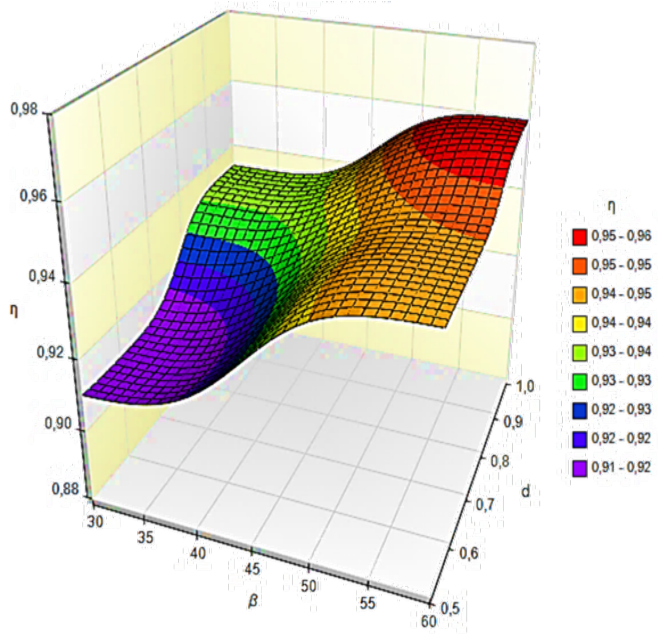


Fig. 5. Fixed factors: Q = 90kW; l = 225mm.

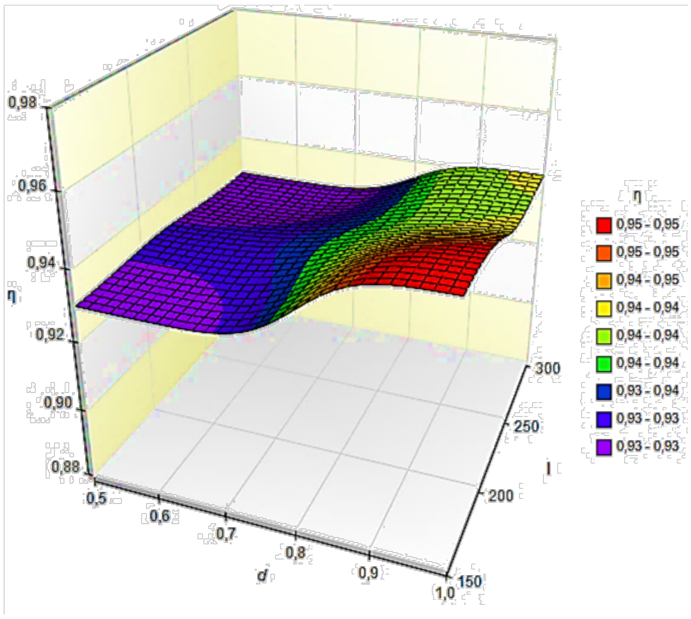


Fig. 8. Fixed factors: β = 45°; Q = 90kW.

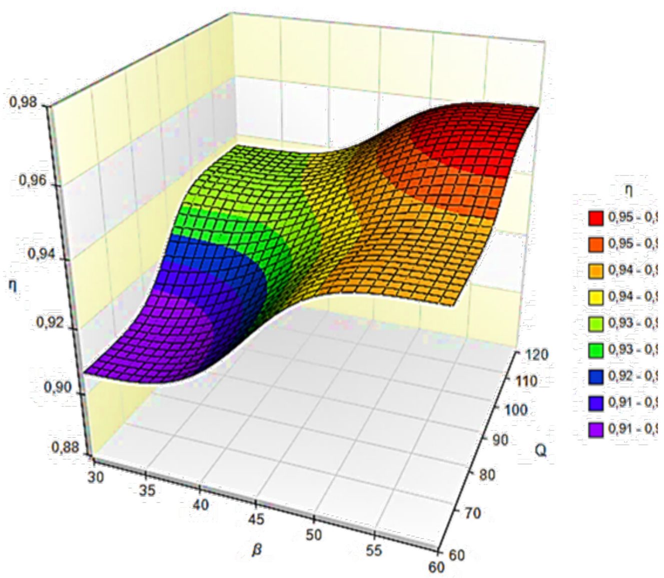


Fig. 6. Fixed factors: d = 0,8mm; l = 225mm.

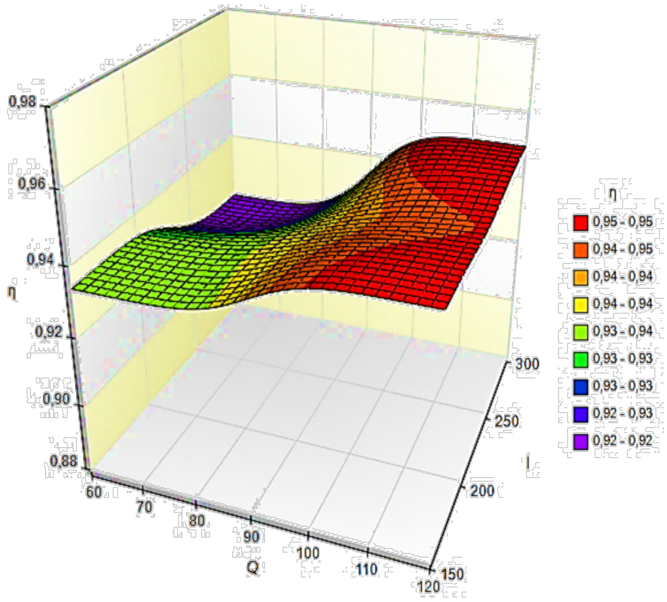


Fig. 9. Fixed factors: β = 45°; d = 0,8mm.

### *D. Conclusion*

- Of the independent variables selected as factors, the angle of inclination of the gas nozzles has a strong influence on the efficiency of the burner, to a lesser extent the diameter of the gas nozzles, the thermal load and the interaction between them.
- The influence of the length of the mixing chamber is weakly expressed, with a noticeable decrease in the efficiency coefficient with increasing length of the mixing chamber.
- A maximum efficiency coefficient is observed in the interval 200 ÷ 250mm along the length of the mixing tract.
- In accordance with the planning matrix and the resulting regression equation, appropriate levels of the factors can be selected to obtain the desired efficiency of the combustion device.

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