Analysis of the Shifting Capabilities of Planetary Gear Trains with Coplanar Planet Gearsets

Željko Vrcan, Sanjin Troha, Kristina Marković, Milan Tica, Miroslav Milutinović

***Abstract*— This paper deals with the comparison of two different simple planetary gearsets. The first type is a 2k-h, variant A geartrain, while the other is a simple planetary gearset with coplanar paired planets. Methods for the kinematic synthesis of both gear trains are explored, together with the analysis of the transmission ratios that can be achieved by the gearsets. Both gear trains are analysed for the possibility of operating as a reversing gear train within their design limitations. The type with paired planets is of particular interest, as it can be built in a configuration that will have a transmission ratio of -1. This also enables compound gear trains having equal transmission ratios in both directions of rotation of the output shaft to be built.**

***Index Terms*— planetary gear train, simple gear train, coplanar paired planet gears, reversing gear train, compound planetary gear train**

I. INTRODUCTION

The application of planetary gear trains (PGTs) is expanding due to their advantages in relation to conventional gear trains, notably reduced size, and weight. The compact build and the positive effects of power distribution across several satellites enable the transmission of high powers and high torques by a relatively compact gearbox. On the other side, the great variance of kinematic schemes and the need for considerably more complex calculations when compared to conventional gear trains points to the need for a systematic research plan which will enable the benefits of PGT application to be fully utilised.

Some applications, such as railway vehicles, watercraft, tracked vehicles and machine tools, require a transmission with the same transmission ratios in both directions of rotation of the output shaft [1-5].

In conventional planetary transmissions this is achieved by a stage that transmits power from sun to ring gear (or vice versa) with a locked carrier, however this solution is restricted to a single gear ratio only. The PGT with coplanar paired planets [6,7] will be explored for this purpose, as its potential use as an input or output gearbox enables the main gearbox to be built without a reverse gear, potentially expanding the total number of available gear ratios.

II. 2K-H, VARIANT A PLANETARY GEAR TRAIN

The 2k-h, variant A, according to Kudryavtsev, or 1AI according to the German classification, PGT has imposed itself over other PGTs as it offers the most obvious benefits of PGT application when related to other PGT solutions (Fig. 1).

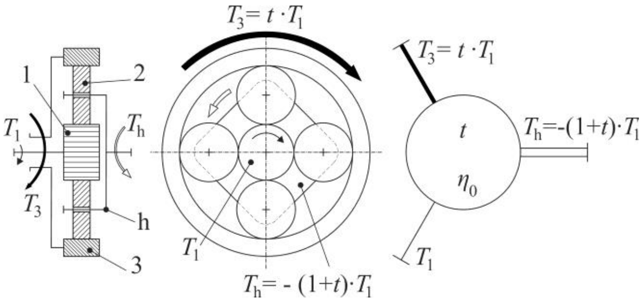

Fig. 1. Simple 2k-h, variant A (or 1AI) PGT.

The internal gear pairs of planet and internal gear ensure lower contact stresses and reduced size as all gears must fit inside the ring gear [6,7]. When using simple planet gears (sun to planet and planet to ring meshing in a single plane), which is standard for 2k-h, variant A PGTs, the gearbox build is extremely compact, and easy to manufacture as many equal parts are used. All these properties mean that the PGT is in very common use either as a standalone or a component of compound gearboxes [8-10]. The basic parts of this PGT are the sun gear 1, planet gears 2, ring gear 3 and planet carrier h. It should also be mentioned that Wolf-Arnaudov symbols are commonly used to depict PGTs (Figure 1, right) [11].

The ideal torque ratio is defined by:

$$t = \frac{T_3}{T_1} = \frac{T_{\mathrm{D\,max}}}{T_{\mathrm{D\,min}}} = \frac{|z_3|}{z_1} > +1 . \tag{1}$$

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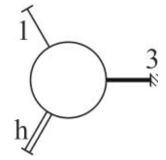
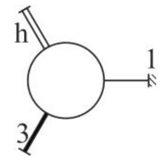
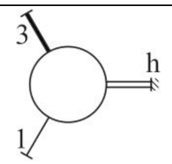
$$T_1 : T_3 : T_h = +1 : +t : -(1+t). \qquad (2)$$

The main operating modes of the PGT are laid out in Table 1. Any of the three shafts of the PGT may be locked, with the two remaining shafts operating as the input and output, resulting in a total of six operating modes.

TABLE I
OPERATING MODES OF 2K-H, VARIANT A PGT

Locked member	Power flow	Transmission ratio as a function of the ideal torque ratio	Extreme transmission ratio values for t = 2…12	Ideal torque ratio as a function of the transmission ratio	Symbolic view
3	$1 \rightarrow h$	$i_{1h(3)} = t+1$	3…13	$t = i_{1h(3)} - 1$	
	$h \rightarrow 1$	$i_{h1(3)} = \frac{1}{t+1}$	0,333…0,077	$t = \frac{1 - i_{h1(3)}}{i_{h1(3)}}$	
1	$3 \rightarrow h$	$i_{3h(1)} = \frac{1+t}{t}$	1,5…1,083	$t = \frac{1}{i_{3h(1)} - 1}$	
	$h \rightarrow 3$	$i_{h3(1)} = \frac{t}{t+1}$	0,666…0,923	$t = \frac{i_{h3(1)}}{1 - i_{h3(1)}}$	
h	$1 \rightarrow 3$	$i_{13(h)} = -t$	-2…-12	$t = -i_{13(h)}$	
	$3 \rightarrow 1$	$i_{31(h)} = -\frac{1}{t}$	-0,5…-0,083	$t = -\frac{1}{i_{31(h)}}$	

Of these modes, four are reduction modes, and two are multiplier modes. It should be also noted that it is impossible for this PGT to achieve a transmission ratio of -1. However, it is possible to achieve a transmission ratio of 1 by creating a solid connection between two elements of the PGT (Fig. 2). In this example, this is achieved by adding brake C1 and clutch C2. With brake C1 on, the gear train operates in its standard sun-to-carrier mode with the ring gear locked, while releasing the brake and applying clutch C2 connects the sun and ring gears together causing the gear train to rotate as a block.

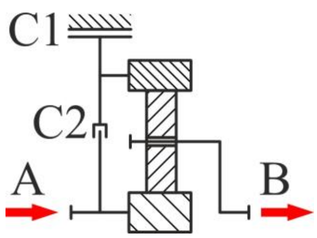


Fig. 2. 2k-h variant A PGT with two control elements.

III. PLANETARY GEAR TRAIN WITH COPLANAR PAIRED PLANETS

As their name says, PGTs with coplanar paired planets have pairs of planets in mesh with each other, with all gears meshing in the same plane (Fig. 3). Otherwise, they are like the previously discussed 1AI PGT. A modified Wolf-Arnaudov symbol with the centre greyed out is used to distinguish this PGT from the 1AI.

The planet gears 2 are 2' are usually made with the same number of teeth [8]. The conditions for the synthesis of the PGT are given in [Orlić], while the assembly conditions are defined by 5 inequations [7,9].

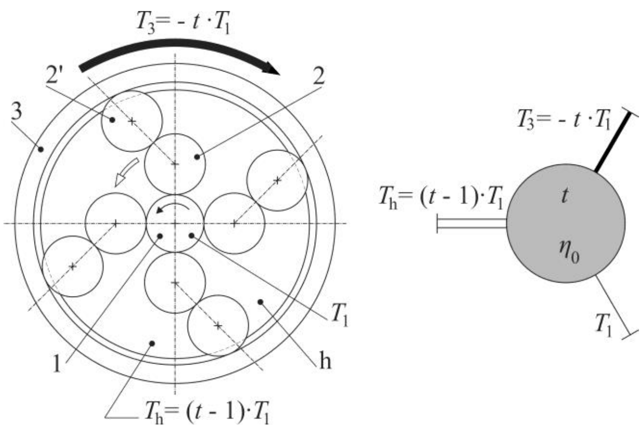


Fig. 3. Simple PGT with coplanar paired planets.

The planet gears 2 are 2' are usually made with the same number of teeth [8]. The conditions for the synthesis of the PGT are given in [Orlić], while the assembly conditions are defined by 5 inequations [7,9].

To calculate the transmission ratios, all internal losses of the PGT will be disregarded in this article:

$$\eta_0 = \eta_{13(h)} = \eta_{31(h)} = 1. \qquad (3)$$

The ideal torque ratio is defined by:

$$t = \frac{T_3}{T_1} = \frac{T_\Sigma}{T_{D\min}} = -\left|\frac{z_3}{z_1}\right| < -2\,. \qquad (4)$$

The torque ratios are defined as:

$$\begin{aligned} T_1 : T_h : T_3 &= T_{D\min} : T_{D\max} : T_\Sigma = \\ &= T_1 : (t-1)T_1 : -tT_1 = +1 : +(t-1) : -t \end{aligned}\,. \qquad (5)$$

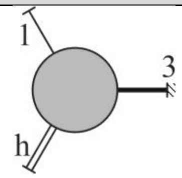
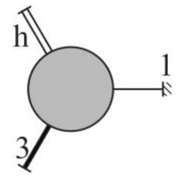
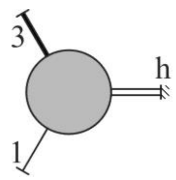
Subject to the conditions of:

$$T_1 \equiv T_{D\min} < T_h \equiv T_{D\max} < |T_3| \equiv |T_\Sigma|\,. \qquad (6)$$

As in the case of 2k-h, variant A, this PGT can be also observed in six operating modes (Table 2).

The calculation presented here will cover only the transmission ratio function for the case in which the ring gear is locked, and the power that is transmitted from sun to planet carrier.

TABLE II
OPERATING MODES OF PGT WITH COPLANAR PAIRED PLANETS

Locked member	Power flow	Transmission ratio as a function of the ideal torque ratio	Extreme transmission ratio values for t = 2…7	Ideal torque ratio as a function of the transmission ratio	Symbolic
3	$1 \rightarrow h$	$i_{1h(3)} = 1 - t$	-1…-6	$t = 1 - i_{1h(3)}$	
	$h \rightarrow 1$	$i_{h1(3)} = \frac{1}{1-t}$	-1…-0,166	$t = \frac{i_{h1(3)} - 1}{i_{h1(3)}}$	
1	$3 \rightarrow h$	$i_{3h(1)} = \frac{t-1}{t}$	0,5…0,857	$t = \frac{1}{1 - i_{3h(1)}}$	
	$h \rightarrow 3$	$i_{h3(1)} = \frac{t}{t-1}$	1…1,166	$t = \frac{i_{h3(1)}}{i_{h3(1)} - 1}$	
h	$1 \rightarrow 3$	$i_{13(h)} = t$	2…7	$t = i_{13(h)}$	
	$3 \rightarrow 1$	$i_{31(h)} = \frac{1}{t}$	0,5…0,143	$t = \frac{1}{i_{31(h)}}$	

The procedure for the other five cases is similar and based on the law of energy preservation:

$$T_1\omega_1 + T_3\omega_3 + T_h\omega_h = 0\,. \qquad (7)$$

If $\omega_3 = 0$:

$$T_1\omega_1 + T_h\omega_h = 0\,. \qquad (8)$$

$$\frac{\omega_1}{\omega_h} = -\frac{T_h}{T_1}\,. \qquad (9)$$

It is possible to derive from (8) and (9):

$$i_{1h(3)} = \frac{\omega_1}{\omega_h} = -\frac{t-1}{1} = 1 - t\,. \qquad (10)$$

The ideal torque ratio is then easily calculated from (10):

$$t = -i_{1h(3)} + 1\,. \qquad (11)$$

Therefore, for a particular application which requires $i_{1h(3)} = -1$, the ideal torque ratio equals $t = 2$. It should be also noted that for this PGT, the summary element is not the planet carrier, but the ring gear.

IV. REVERSING GEARBOX

For applications that require a reversing gearbox with the transmission ratio $i_{c1/c2} = +/- 1$, the following kinematic scheme may be used with a single brake C2 and a single clutch C1 as the control elements (Fig. 4).

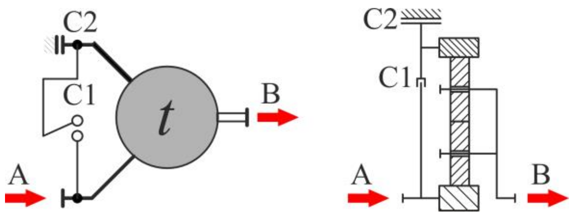


Fig. 4. Wolf-Arnaudov symbol (left) and kinematic scheme (right) of PGT based reversing gearbox.

This PGT can be combined with a common 2k-h, variant

A PGT to obtain several gearbox variants (Fig. 5) with four single external shafts.

For example, if the planet carrier of the PGT from Figure 4 is connected to the sun gear of a standard 2k-h variant A PGT, a reversing gearbox is obtained with its transmission ratios defined by the 2k-h PGT (Fig. 6). By variating the ideal torque ratio t_{II} of the simple 1AI PGT it is possible to create a wide spectrum of reversing gear ratios.

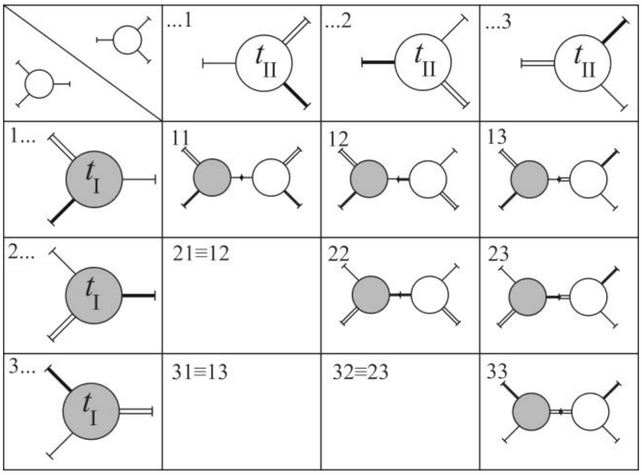


Fig. 5. Possibilities of connection of the component single-carrier PGTs in a compound two-carrier PGT with four single external shafts.

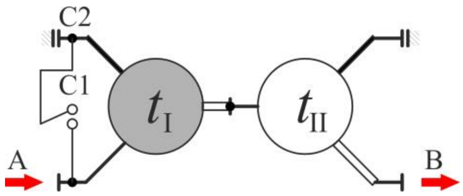


Fig. 6. Symbolic scheme of simple reversing gearbox.

V. Conclusion

The transmission ratio ranges of the 2k-h variant A PGT have been analysed within its design limits. This type of PGT can achieve a transmission ratio equal to 1 by simultaneously locking two components to the input member but cannot achieve a transmission ratio equal to -1. Because of this, the PGT with coplanar meshing of paired planets was analysed within its design limits. Unlike the 2k-h, this type can achieve a transmission ratio of -1 when the PGT is built with the ideal torque ratio $t = 2$.

It can be concluded that by combining these two types of PGT a wide palette of PGTs may be created with broad shifting capabilities and numerous possibilities for application.

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