Planetary Gear Trains Design Parameters Selection

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Abstract— **There are many benefits that are inherent to planetary gear trains (PGTs) which make them more suitable than classical gear trains. The most important of these advantages is a considerable reduction of mass and dimensions for the same torque rating. Because of that, the application of PGTs has been significantly expanded in various engineering applications. PGTs as a totality, and particularly complex multi-carrier PGTs cover a vast area of technical knowledge. Planetary gear trains the basic parameter is number of teeth of all gears, because the same or close transmission ratio can be achieved by different number of gear teeth. The uniqueness of the planetary gear trains puts on the need to also include the number of satellites. Thus, the number of satellites is also an important design parameter. All dimensions of gears are determined through gear module. It is the reason for selection gear module for significative design parameter. Face width is responsible for uniform distribution of load along the contact line. Center distance and addendum modification coefficients represent the measure of the train size. This paper gives the procedure for planetary gear transmissions design parameters selection which are predicted for using as variables in multicriteria optimization adjusted for basic type of planetary gear train.**

Index Terms— **the numbers of teeth of gears, number of satellites, module, center distance, face width**

I. Introduction

Optimization is defined as science which deals with determination of the best solution for functioning of a system. Optimization tasks for mechanical systems are compound processes of theoretical research which include knowledge of mechanical systems design in general, uniqueness of concrete mechanical system and methods of mathematical optimization. Geared power transmissions with constant transmission ratio are the most frequently used mechanical transmissions. In the scope of this group planetary gear transmissions increase their importance. Design of planetary gear transmissions also means successful application of parameter optimization.

The selection of an optimal transmission of this type which can satisfy specific requirements is complex, and it can be performed by means of multi-criteria optimization. The usage of multi-criteria optimization to gear trains, and particularly planetary gear trains, has not been the topic of many studies, however an overview can be given. The usage of multi-purpose optimization access, predicated on the Pareto optimality concept, to helical gears design was proposed in [1], while the choice of the best optimization parameters for getting the necessary gear quality and the optimization of the design procedure itself was provided in [2].

In [3], design problems of gears with minimal dimensions are indicated. A simple, descriptive, and easy-to-handle method for investigating the transmission ratio, the internal power flows and the efficiency of complex multiplanetary gearings is introduced in [4]. The process of planetary gear transmission optimization is shown in paper [5] as a method which leads to the optimum (housing diameter and gear volume are considered to achieve their minimum). A model for finding a solution to this problem is the application of stochastic methods, where parameter values vary by accidental numbers.

An optimization task is defined by the variables, objective functions and conditions required for the proper functioning of a system determined by the functional constraints [6,7].

Under the mathematical model definition, it is necessary to determine the variables since each objective function is the function of several parameters.

Since the design parameters are often used as variables in mathematical models, the aim of this paper is design parameters for usage in multicriteria optimization determination selection. Procedure presented in the paper is adjusted for basic type of planetary gear train.

II. The Number of Teeth Determination

The basic type of a planetary gear train (PGT), i.e. a design which has a central sun gear (external gearing - 1), central ring gear (internal gearing - 3), satellites (planets - 2) and carrier (h), shown in Fig.1, is the subject of the paper, limited to geared pairs. Planets are in simultaneous contact with the sun gear and the ring gear. This type of planetary gear trains (mark A, acc. to the Russian literature, i.e. AI acc. to the German lit.) have the broadest use in mechanics. This gear train is characterized with simple manufacturing, high grade of efficiency, reliable work, small dimensions, and mass.

This type of a PGT is often used as single stage transmission, as a building block for higher compound planetary gear trains.

The following design parameters are considered: the number of teeth of all gears, the number of planets n_w, the

Received: 01.12.2023
Published: 22.12.2023
https://doi.org/10.47978/TUS.2023.73.04.003

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gear module m_n, the facewidth b, centre distance and addendum modification coefficients.

One of design parameters is the number of teeth of gears. This is an important parameter in optimization in regard that the same or close gear ratio can be achieved by different combinations of the numbers of teeth and optimal will be one which suits the best to all quoted demands.

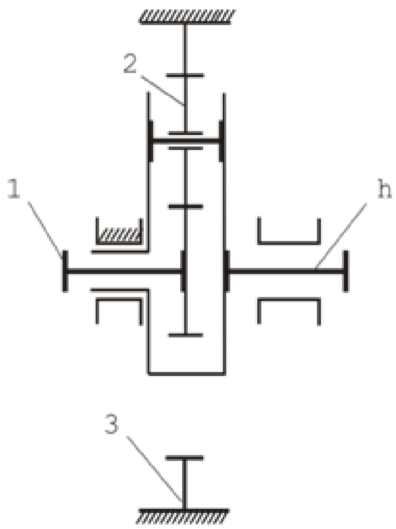

Fig. 1. Basic type of planetary gear train (1- sun gear; 2 – planet; 3 – ring gear; h – carrier)

Specifics of planetary gears imposes the need to also include the number of satellites.

Procedure applied here is for immobile ring gear. Procedure would be the same for the other types of planetary gear trains, only mathematical expressions would be different.

Taking into consideration that the procedure of optimization means selection of the "best", i.e. "optimal" solution for assigned criteria from large number of solutions, and it begins with forming of groups of solutions including determination of the number of teeth and the number of satellites.

Most frequently, the choice of the numbers of teeth of all gears starts with establishing a relation between the number of teeth of central sun gear and number of teeth of other gears. The initial data for the numbers of teeth determination is the number of central sun gear (z_1) definition. This datum is adopted, according to recommendation, taking into consideration the danger of undercutting and the strength problems occurrence, either in case of low or high number of teeth.

In this paper, for number of teeth of the sun gear the adopted range is: z_1=12…36 [8,9]. The number of satellites is in relation with condition of adjacency. It is predicted to select number of satellites from recommended range n_W=2…6 and after that confirmation of adjacency condition is performed.

When the number of satellites and the number of teeth of central sun gear are adopted, the next step is calculation of the number of teeth of central ring gear z_3, according to the equation:

$$z_3 = z_1 \cdot (1 - i). \qquad (1)$$

where i is assigned gear ratio.

Next step is, after integer rounding if it is necessary, checking of conjugation condition:

$$\frac{z_1 - z_3}{n_w} = C. \qquad (2)$$

where C is integer value.

If the condition of conjugation is not fulfilled, correction of number z_3 is performed. Regarding that for the purposes of optimization is necessary to find as many combinations of teeth number as possible, and in the case of condition of conjugation fulfillment, number z_3 is corrected by increasing and decreasing for 1 and 2 (±1,2) where the condition of conjugation is checked according to the equation (2).

The actual transmission ratio is calculating according to the equation (3):

$$i' = 1 - \frac{z_3}{z_1}. \qquad (3)$$

and compared with assigned value i.

$$\Delta i = \frac{|i - i'|}{i} 100 < \Delta i_{doz}$$

On that way, by varying the values for z_1 and n_w from predicted groups of numbers, considerable number of variants z_1, z_3, n_w is obtained.

Thereafter, it is necessary to determine the number of teeth of the satellite. It is defined from the coaxiallity condition, providing for the uniformity of all the axis distances in the conjugated gear pairs:

$$z_2 = \frac{z_3 - z_1}{2}. \qquad (4)$$

If the number of teeth of satellite determined according to the expression (4) is obtained as integer value that means all gears are made without addendum modification and without standardizing of axial distance, or the sum of addendum modification coefficients is equal to zero. Rounding on integer value, when decimal number is obtained and by adding and subtraction of one, i.e. ±1, addendum modification is introduced. The number of teeth of satellite should satisfy the adjacency condition expressed by relation (5):

$$z_2 + 2 < (z_1 + z_2) \sin \frac{\pi}{n_w}. \qquad (5)$$

On this way, it is possible to obtain for one combination of $z_1/z_3/n_w$ different values of z_2.

For assigned gear ratio, according to described procedure shown on the short algorithm in Fig. 2., combinations of the teeth numbers of all gears and number of satellites $z_1/z_2/z_3/n_w$ are determined. The number of these combinations is large. In the next steps of optimization, by introducing limit condition from the aspects of geometry and strength, number is decreasing. The example of combinations of the number of teeth of all gears for only one number of satellites n_w=3 is given in Table 1. Assigned gear ration is i=4.5 and obtained is marked as i'.

TABLE I
EXAMPLE OF COMBINATIONS OF GEAR TOOTH NUMBERS FOR NUMBER OF PLANET GEARS n_w=3

i	i'	z_1	z_2	z_3
4.50	4.500	12	15	42
4.50	4.615	13	17	47
4.50	4.500	14	17	49
4.50	4.500	14	18	49
4.50	4.400	15	18	51
4.50	4.500	16	20	56
4.50	4.588	17	22	61
4.50	4.500	18	22	63
4.50	4.500	18	23	63
4.50	4.500	19	23	65
4.50	4.500	20	25	70
4.50	4.571	21	27	75
4.50	4.500	22	27	77
4.50	4.500	22	28	77
4.50	4.435	23	28	79
4.50	4.500	24	30	84
4.50	4.560	25	32	89
4.50	4.500	26	32	91
4.50	4.500	26	33	91
4.50	4.444	27	33	93
4.50	4.500	28	35	98
4.50	4.552	29	37	103
4.50	4.500	30	37	105
4.50	4.500	30	38	105
4.50	4.452	31	38	107
4.50	4.500	32	40	112
4.50	4.545	33	42	117
4.50	4.500	34	42	119
4.50	4.500	34	43	119
4.50	4.457	35	43	121
4.50	4.500	36	45	126

The number of satellites influences the planetary gear trains structure compactness. Besides that, the number of teeth of central sun gear and the number of satellites determine the maximum transmission ratio according to equation (6):

$$i \leq \frac{2-4/z_1}{1-sin(\pi/n_w)}. \quad (6)$$

Equation (6) is obtained from the condition of adjacency (5). Maximal transmission ratio for z_1=14…35 and n_w=3…6 is given in Table 2.

TABLE II
MAXIMAL TRANSMISSION RATIOS FOR z_1=14…35 AND n_w=3…6

z_1	n_w			
	3	4	5	6
14	12.796	5.853	4.159	3.428
17	13.172	6.025	4.281	3.428
20	13.435	6.145	4.362	3.6
23	13.630	6.235	4.430	3.652
26	13.780	6.303	4.479	3.692
30	13.933	6.373	4.528	3.733
35	14.182	6.487	4.609	3.8

By increasing the number of satellites with the constant number of teeth of central sun gear, transmission ratio decreases. By increasing the number of teeth of central sun gear with constant number of satellites, transmission ratio increases. The transmission ratio limit value can be increased a little by a satellite teeth profile modification, equation (7):

$$i \leq i\frac{2-4(1+x_2+\Delta/m)/z_1}{1-sin\left(\frac{\pi}{n_w}\right)cos\,\alpha_n/\,cos\,\alpha_{w12}}_{max}. \quad (7)$$

Whereby the condition $cos\,\alpha_n\,/\,cos\,\alpha_{w12} \leq 1.08$ is satisfied.

In mathematical expression for mentioned condition:

- i_{max} is the maximum permitted transmission ratio related to the adjacency condition,
- Δ/m is relative permitted clearance between the tip diameter of the adjacent satellites,
- x_2 is the satellite addendum modification coefficient.
- α_n is pressure angle,
- α_{w12} is a working pressure angle.

The values of the transmission ratios defined according to the equation (6) may be used as the starting point in making such transmission gear construction concept. The experiences show that in the planetary gear train families developing, the number of teeth of sun gears decreased with the transmission ratio increasing due to the limited number of teeth of the ring gear.

III. MODULE

Gear module represents the important parameter of gear through which all dimensions of teeth and gears at all are determined. Along with the number of teeth, this is the most frequently used parameter for optimization of gear trains. For limitation of number of tools and control equipment of gear trains, values for modules are standardized according to ISO 54 [12], arranged in two ranges of priority. However, module is in relation to other parameters and values for gear trains. This makes it possible, for the same center distance, by changing the number of teeth and addendum modification, to also vary value of gear train module. Gears with bigger modules have bigger teeth, and with that, a bigger load width compared with gears with smaller modules. Due to that module values are given depending on gear width. On the other side, for helical gears by increasing module values, overlap ratio decreases. Considering those facts, it is necessary to define limitation, i.e., minimum module value for such gears.

Taking into consideration gear ratio range which can be achieved by this planetary gear trains (i=3…9, [9]) module can take values in the range of 2…8 mm from mentioned standard, i.e. m_n takes value from set of values: 2, 2.25, 2.5, 2.75, 3, 3.5, 4, 4.5, 5, 5.5, 6, 7, 8 (in mm).

Regarding introducing of module as parameter for optimization, in this moment two approaches are possible. First, for each combination of the number of teeth and number of satellites determined in previous phase, each module value is assigned, and after that, geometric values have been determined with a simultaneous check of geometrical conditions. On this way, many combinations are obtained for further analysis. This makes the optimization process overly complex and prolongs time for calculations, which is not a problem if we use computer support.

Nevertheless, because optimization is predicted also for conditions from aspects of loads, speed and shaft torque and together with defining of material for gears, it is possible to speed up process by module limiting. This can be achieved by diameter of central sun gear determination d_1, obtained from previous calculation conditions [11]:

$$m \geq \frac{d_1}{z_1}. \tag{8}$$

The value of standard module have to be taken based on this relation.

IV. Center Distance

Knowing the number of teeth of all gears in gear train and module makes determination of a center distance. Center distance is the measure of the train size. This is a particularly important train parameter, and it influences other train elements such as overall measures, housing, etc.

Regarding the center distance two approaches are also possible. In case that is needed to develop gear trains family, center distance and module are parameters that must be taken into consideration, making standardization of center distance necessary. Consequence can be large values of addendum modification coefficient because it is needed to match more conditions.

For the purposes of optimization, standardization of center distances of all combinations $z_1/z_2/z_3/n_w/m$ is insignificant, which means that according to literature recommendation [9] center distance is obtained on the following way:

If: $a_{d12}=a_{d23}$, $a= a_{d12}=a_{d23}$, else:

$$a = a_{bigger} - (0.2 \dots 0.4) \cdot |\Delta a| \tag{9}$$

When reference center distance and center distance are known, it is necessary to determine coefficients of addendum modification. There are many approaches for distribution of these coefficients, depending on what is expected to achieve.

V. Center Distance

The influence of addendum modification on changes of teeth shape and load capabilities of gears is significant. Suitable selection of addendum modification has major influence on gear load-carrying capacity. It has been developed the full range of system for selection of addendum modification, where criteria can be:

- The equal sliding speed at pinion and wheel tip
- The equal scuffing resistance for pinion and wheel
- The equal teeth root stress of pinion and wheel

Distribution of modification coefficients sum is the basic thing that takes significant attention in literature.

Reliable results for gear load-carrying capacity can be achieved by selection of addendum modification for external geared couples according to DIN 3992. According to DIN 3992 by selection of addendum modification sum can be achieved:

- High load-carrying capacity
  $x_1+x_2=0.7...1.2$
- High level of contact ratio
  $x_1+x_2=-0.4...0$
- Uniform tooth system
  $x_1+x_2=0.2...0.4$

Distribution of the sum on x_1 and x_2 is done according to diagrams given in this standard.

As well it can be found other recommendations for this distribution. According to TGL 10545 [12], it is possible to realize coefficient sum distribution on several ways:

- According to condition for achievement of approximately the same local tooth root stress (especially suitable for strengthen gears)

$$x_1 \approx \frac{x_1+x_2}{u+1} + 0.5\frac{u-1}{u+1}$$

- Through distribution

$$x_1 \approx \frac{x_1+x_2}{u+1} \cdot \frac{z_1+12}{z_1+2} + \frac{8}{z_1+2}$$

can be achieved approximately the same flank loading; this is suitable for use at improved and unstrengthen flanks with $u>2$

- Distribution

$$x_1 \approx \frac{x_1+x_2}{u+1} + \frac{u-1}{u+1+0.4z_2}$$

gives almost the same sliding speeds on tooth tips for driving and driven gear; it is recommended for gears with big velocities, at u<2 (u is kinematic gear ratio)

According to MAAG recommendations, limits for modification coefficients are given together with mean interval value.

$$x_1 = \frac{\Sigma x}{2} + \left(A - \frac{\Sigma x}{2}\right) \cdot \frac{\ln u}{\ln\frac{z_{n1} \cdot z_{n2}}{100}}.$$

Where u is kinematic gear ratio and A is constant which depends on tools.

Distribution of coefficients, according to FZG recommendations, should provide great resistance, minimal teeth thickness on top and the same sliding speed (i is transmission ratio):

$$x_1 + x_2 \approx 3.5 - \frac{80}{z_{n1} + z_{n2} + 10}$$

$$x_1 \approx \frac{x_1+x_2}{i+1} + \frac{i-1}{i+1+0.4z_{n2}}$$

Together with this approach, where sum of modification coefficients is determined, and then distribution of that sum, it is possible to introduce modification coefficients for gears.

Here also exist several approaches. The most frequently used are:

- According to Belgian norms [11] for z<30 $x = 0.03 \cdot (30 - z)$

- Than for $15 \leq z \leq 34$, there is also a recommendation $x = 1 - \frac{z}{2} \cdot \left[\frac{1}{\cos\beta} - \frac{1}{\cos 3^{\circ} \sqrt{\cos^2_{\beta} + tg^2_{\alpha_n}}} \right]$, and for right tooth $x = 1 - 0.0294z$ which can be obtained from the condition that pressure angle at the beginning of active part of profile is less than 3° providing that use of evolvent is not to the final point, which can cause unstable work and contact kinematics disturbances.
- 0.5 ($x_1 = x_2 = 0.5$) and V-zero ($x_1 = x_2 = 0.5$) gearing is known according to DIN 3994 and DIN 3995.

Other recommendations for selection of modification coefficients can be found in literature. Mostly, coefficient is then determined from the condition to avoid occurrence of teeth under cutting, peak shaped teeth, etc. In addition, minimal and maximal values of modification coefficient should be taken into consideration: $x_{min} \dots x_{max} = -0.5 \dots 1.0$, however the actual interval is a little narrower $x_{min} \dots x_{max} = -0.25 \dots 0.7$.

Profile modification introducing for planetary gear trains is performed in such way that first are defined modifications for gears with external gearing, and accordingly to them and defined center distance, modification coefficient for ring gear.

For the optimization purposes at this type of transmission, it is suitable decision for determination of modification coefficients sum based on center distance and sum distribution according to MAAG recommendations. By such distribution, balanced resistance of both gears is provided which is suitable when both gears are made of the same material (at planetary trains is common for external gears to be made of the same material). These recommendations give distribution values close to DIN 3992 recommendations. Selection of modification coefficients at this moment is not a key question, regarding that next phases are checking of geometrical conditions and strength conditions.

VI. Face width

A basic criterion for selection of face width is the possibility for uniform distribution of load along the contact line. To achieve this, the basic condition is accurate manufacturing of teeth and shaft. For those reasons, face width, if there are no other conditions, must be adopted according to possibilities to realize uniform load distribution, which is hard to obtain. As a measure for face width *b*, ratio of width to the diameter of smaller gear, b/d_1 is accepted.

Selection of face width is usually assigned regarding reference radius of pinion (b/d_1), to module (b/m_n) or regarding center distance (b/a). For selection of face width in regard to reference radius of pinion, recommendations where this ration is given depending of material type and bearing are used. Connection between minimal module value and face width is given depending on construction and quality of gearing.

Face width *b* is here introduced through parameter

$$\psi_{bd} = \frac{b}{d_1}.$$

It is determined:

- If teeth are case-hardened, hardened and nitrided:

$$\psi_{bd} = (0.1\dots0.3\dots0.5) + \frac{u}{20}$$

- If teeth are through hardened, soft:

$$\psi_{bd} = (0.2\dots0.5\dots0.8) + \frac{u}{10}$$

Planetary transmissions mostly with symmetric bearing gears make possible to take greater values for *Ψ*. For approximate calculations it could be taken from the range $\Psi=(0.1\div0.18)\cdot i_0$ [9], where i_0 is basic transmission ratio $i_0=z_3/z_1$ Parameter *Ψ is* used for determination of active contact width for two gears, i.e. $b = \psi \cdot d_1$. In the aim of compensation of inaccurate work at manufacturing and mounting, face width of pinion is taken slightly bigger compared to the face width of wheel. That can increase tooth root strength of pinion. That follows $b_2=b$, where the calculated width of sun pinion is slightly bigger $b_1=b_2+(5\dots10)$., It could be found limitations for active width *b* in literature. These limitations are mostly given in relation to reference diameter of pinion [11]:

$$b_{min} = 0.2(0.4) \cdot d_1$$
$$b_{max} = 1.2(1.6) \cdot d_1$$

Numbered design parameters for optimization of geared couples of planetary transmissions are not independent variables, i.e. changes of one cause change on another. Particularly important is the dependence of the number of teeth of pinion and module. By selection of bigger number of teeth z_1 smaller modules are obtained, which means more accurate manufacturing, smaller deviations, and smaller intrinsic dynamic stresses. In such cases when relative profile curve radii are bigger, contact Hertzian stress is smaller. Besides that, gears with smaller module cause smaller sliding, followed by smaller wearing, finally at such gears, quantity of striped material (from space between teeth) is smaller, which means less energy for manufacturing. Nevertheless, module reduction is limited by reduction in teeth root strength. Namely, at some diameter of pinion, larger number of teeth, i.e., smaller module means also smaller dimensions of tooth cross section, causing stress to increase. This is especially clear for gears with case hardened flanks, which on calculations give smaller diameters. Such connections of designed parameters point out to justifying of optimization in regard of these parameters.

The short overview of the procedure of design parameter selection is shown in the Fig 2.

Next step is defining the functional constraints and objective functions to complete mathematical model.

Since gear trains often demand many objective functions, it is required application of multi criteria optimization.

The definition of multi-criteria optimization problems is unwell determinate. Because of that, it is necessary to include a concept for making choice. The *Pareto optimality concept* is the well-known concept for making choice "equally good" solutions [6,7] and it is often applied in gear train optimization.

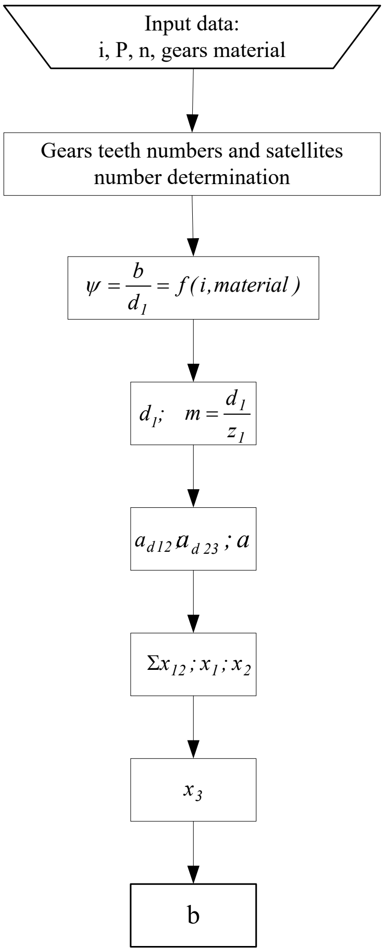

Fig. 2. Short algorithm for design selection

VII. CONCLUSION

The base of optimization process in mechanical systems is comparison of mechanical systems with different parameters in the same conditions and selection of the best variant. With that, the optimization process begins with forming solution groups of design parameters for assigned starting conditions. Based on established criteria and limitations, an "optimal" solution is selected, determined by designed parameters.

Design parameter selection procedure for optimization of planetary transmission of basic type is pointed in this article. The following design parameters is considered: the number of teeth of all gears, module and face width which are suitable for variable in mathematical model for multicriteria optimization. Also, center distance and addendum modification coefficients are considered as important design parameters.

ACKNOWLEDGMENT

This research was financially supported by the Ministry of Education, Science and Technological Development of the Republic of Serbia (Contract No. 451-03-9/2021-14/200109).

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