Discrete Wiener Filter Application in Wind Turbine Control

T. Puleva, O. Ognyanova, E. Haralanova

***Abstract* — Discrete stochastic control algorithms for wind turbine generators based on Wiener filter theory are explored in this paper. This approach is very effective in operation in a tracking regime. A discrete controller is designed by taking into attention functional and magnitude constraints. The application of Wiener filter design permits to extend the problems range: extrapolation of the input signal and controller direct design based on Diophantine equation instead of functional constraints in the classical design method. This approach simplifies the design procedure. The frequency characteristics of the optimal system and of the controller by variable wind speed are analyzed. The system performance is investigated in the frequency domain as well as by the error standard deviation under variable wind speed.**

***Index Terms* — Wind turbine, tracking regime, Discrete Wiener filter, functional and magnitude constraints, direct design based on Diophantine equation**

I. INTRODUCTION

The wind turbine generators (WTG) as renewable energy sources have an essential role in the “green” energy production. The wind generation capacity has increased rapidly since 2000. Many countries in Europe have achieved high levels of wind power production, for example Denmark (41%), Portugal (24%), Ireland (24%), Germany (21%), and Spain (19%). The increased importance of the green energy production including wind energy sources lead to the improvement of technologies in design and implementation of WTGs and their control systems. Due to variable energy source it is very important for the control systems to ensure high performance of WTG operation in both regimes: “*partial load operation*” or extract the maximum power from the wind and “*operation on a rated power*”. That means the control system of WTG should track a variable speed reference. At wind speed variation in a wide range, it is important to ensure a high value of the power efficiency factor. Its theoretical maximum is $C_{p\,\max} = 0.593$, known as Betz limit. The power factor of modern wind turbines is about 0.45 and it is quite below the theoretical value. The efforts in many research works are focused on the design of impproved control algorithms which satisfy the requirements of the system performance (stability and efficient operation) under variable wind speed.

The control strategies are investigated in detail in [1] - [4]. In the literature a wide range of research related to control algorithms are presented. They vary from classical to adaptive, predictive, multi-variable and robust control systems with different modifications [5]. Most of the commercial wind turbine use proportional–integral (PI) blade-pitch controller [6] to regulate the rotor speed. As stated in [7], a joint operation of two controllers is used to realize a trade-off between speed regulation and load reduction: the main pitch controller is used for speed control while the individual pitch controller is used for load reduction. Both controllers are based on LQR control law with an extra integral state (LQRI) in order to cancel the steady-state error for a step wind speed disturbances and Kalman filter for system states and disturbance estimation. A similar approach is applied in [8]. The requirements to ensure a high system performance at wind variation in wide ranges lead to application of robust control [9] - [13]. In [10] the wind turbine is presented by a model with uncertainty whose parameters depend on the wind speed. By using the technique of μ-synthesis a two degree-of-freedom (2DOF) controller is designed. To suppress the negative effect of the random variation of the wind speed robust control schemes have been investigated in [13]. It is recommended to apply H_2 controller in applications where disturbance rejection and noise attenuation are crucial, and $H_{\inf}$ controller – in applications when the robustness to model uncertainties is important.

In this paper discrete control algorithms based on optimal stochastic filtering theory are explored. Motivation for this is given by the importance of the task to improve the control system performance in the tracking regime in order to keep a high value of the power efficiency factor. Also the stochastic nature of the wind is our motivation to apply the Wiener theory for optimal filtering and forecasting [14] - [16]. This paper is continuation of our research presented in [17]. In section II the discrete controller structure involving functional and magnitude constraints is presented. In section III an application of discrete Wiener filter design by extrapolation of the input signal is presented.

The new idea in this paper is the functional constraints from the classical method to be replaced by direct design based on Diophantine equation. This modification of the classical design method simplifies its application and as a result the controller structure determination.

The author Teofana Puleva, Assoc. Prof., PhD, is from the Faculty of Automatics, Technical University of Sofia, 1000 Sofia, Bulgaria (e-mail: tpuleva@tu-sofia.bg).

The author Elena Haralanova, Assoc. Prof. ,PhD, is from the Faculty of Automatics, Technical University of Sofia, 1000 Sofia, Bulgaria (e-mail: elhar@tu-sofia.bg).

The author Ognyana Ognyanova is a PhD student from the Faculty of Automatics, Technical University of Sofia, 1000 Sofia, Bulgaria (e-mail: ognjna@abv.bg).

https://doi.org/10.47978/TUS.2022.72.02.008

II. DISCRETE WIENER FILTER DESIGN. PROBLEM FORMULATION

The discrete Wiener filter design is based on the minimization on the mean square error of the system $M\left[\varepsilon^2(kT_0)\right]$, which is determined as a difference from the outputs of the desired transformation of a random process $y_0(kT_0)$ and the output of the real system $y(kT_0)$ as it is shown in Fig.1.

$$\varepsilon\left(kT_0\right) = y_0\left(kT_0\right) - y\left(kT_0\right). \tag{1}$$

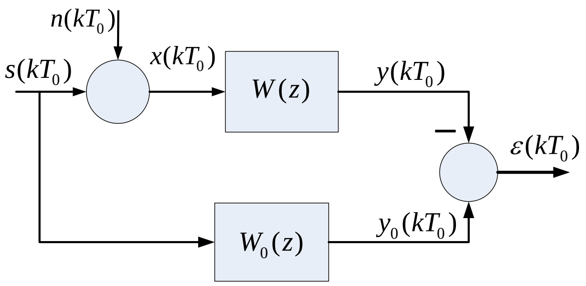


Fig. 1. Generalized error formulation

In this block diagram $W_0(z)$ is the desired transformation of the random process $s(kT_0)$ and $W(z)$ is the discrete transfer function of the real system in presence of a measurement noise $n(kT_0)$. The system error can be expressed by the impulse responses of the real system $w(kT_0)$ and desired transformation $w_0(kT_0)$ presented in the following form:

$$\begin{aligned}\varepsilon(kT_0) &= \sum_{i=1}^{\infty}[w_0(iT_0) - w(iT_0)]s(kT_0 - iT_0) - \\ &-\sum_{i=1}^{\infty} w(iT_0)n(kT_0 - iT_0).\end{aligned} \tag{2}$$

This presentation is essential for the relationship between the mean square error, correlation functions and their spectral density functions [15], [16], [18].

$$\begin{aligned}\mathrm{M}[\varepsilon^2(kT_0)] &= R_{y0}(0) - 2\sum_{i=0}^{\infty} w(iT_0)R_{y0.x}(iT_0) + \\ &+\sum_{i=0}^{\infty}\sum_{q=0}^{\infty} w(iT_0)w_0(qT_0)R_x(qT_0 - iT_0)\end{aligned}. \tag{3}$$

where $R_{y_0}(mT_0)$ is the autocorrelation function of the desired transformation $y_0(kT_0)$; $R_{y_0x}(mT_0)$ is the cross correlation function between the desired output $y_0(kT_0)$ and the input of the real system $x(kT_0)$; $R_x(mT_0)$ is the cross correlation function of the real system input signal $x(kT_0)$ with applied additive noise. In the case of statistically independent desired random signal $s(kT_0)$ and measurement noise $n(kT_0)$, their cross correlation function and spectral density function are zero and the mean square error is determined by the spectral density functions of the input signals:

$$\begin{aligned}\mathrm{M}[\varepsilon^2(kT_0)] &= \frac{1}{2\pi j}\oint_{|z|=1}\left|W_0(z) - W(z)\right|^2 S_s(z)z^{-1}dz + \\ &+\frac{1}{2\pi j}\oint_{|z|=1}\left|W(z)\right|^2 S_n(z)z^{-1}dz,\end{aligned} \tag{4}$$

where the desired transformation of the random signal $s(kT_0)$ is $W_0(z) = 1$ when a tracking regime is required; $W(z)$ is the transfer function of the real system, $S_s(z)$ is the spectral density function of the random signal (for example wind velocity), and $S_n(z)$ - the noise spectral density function.

The optimal transfer function with taking into account functional constraints is [15], [16], [18]:

$$W_{opt}(z) = \frac{E(z)}{\varphi(z)E(z^{-1})}.\left[\frac{W_0(z)S_S(z)E(z^{-1})}{\varphi(z^{-1})}\right]_+, \tag{5}$$

where $\varphi\left(z\right)$ and $\varphi\left(z^{-1}\right)$ are obtained as a result from the factorization of the sum of random signal s and measurement noise n spectral density functions.

$$S_s(z) + S_n(z) = \varphi(z)\varphi(z^{-1}), \tag{6}$$

where the poles and zeros of $\varphi\left(z\right)$ are located in the unit circle, and for $\varphi^{-1}(z)$ - outside it. The expression

$$\left[\frac{W_0(z)S_S(z)E(z^{-1})}{\varphi(z^{-1})}\right]_+ \tag{7}$$

is obtained after separation of the expression in the brackets in physically feasible part and physically unfeasible part (unstable part), $E(z)$ is the non-minimal phase part of the discrete model, and $E(z^{-1})$ - its minimal phase discrete presentation. With taking into account the magnitude constraints the optimal transfer function is modified as follows:

$$W_{opt}(z) = \frac{E(z)}{\varphi(z)E(z^{-1})Y(z)}.\left[\frac{W_0(z)S_S(z)E(z^{-1})}{\varphi(z^{-1})Y(z^{-1})}\right]_+. \tag{8}$$

The function K presents the magnitude constraints of one or several variables, and η is a Lagrange multiplier.

$$K(z) = Y(z)Y(z^{-1}),\ K(z) = 1 + \eta\left|W_n(z)\right|^{-2}. \tag{9}$$

At a given discrete transfer function of the plant model W_n, the controller structure can be obtained:

$$W_c\left(z\right) = \frac{W_{opt}(z)}{W_n\left(z\right)\left(1 - W_{opt}(z)\right)}. \tag{10}$$

III. WIENER PREDICTION FILTER

The discrete Wiener prediction filter is based on the minimization of the generalized mean square error. With taking the both functional and magnitude constraints of the control signal, the mean square error is presented by the expression:

$$Q = M\left[\varepsilon^2\right] + \eta M\left[u^2\right] =, \qquad (11)$$

hence

$$\begin{aligned} Q = &\frac{1}{2\pi j}\oint_{|z|=1}\left|\left[W_0(z) - W_{opt}(z)\right]\right|^2 S_s(z) z^{-1} dz + \\ &+\frac{1}{2\pi j}\oint_{|z|=1}\left|W_{opt}(z)\right|^2 N^2 z^{-1} dz + \\ &+\frac{1}{2\pi j}\eta\oint_{|z|=1}\left|\frac{W_{opt}(z)}{W_n}\right|^2\left(S_s(z) + N^2\right) z^{-1} dz, \end{aligned} \qquad (12)$$

The optimal feasible discrete system with functional and magnitude constraints is:

$$W_{opt}(z) = \frac{E(z)}{\varphi(z)E(z^{-1})Y(z)}.\left[\frac{z^l S_S(z)E(z^{-1})}{\varphi(z^{-1})Y(z^{-1})}\right]_+, \qquad (13)$$

where $\varphi(z)$ and $\varphi(z^{-1})$ are obtained in accordance with (6).

The prediction time is presented by a number of samples $l = \frac{\tau}{T_0}$. The magnitude constraints of the control signal are presented by (9), where η is a Lagrange multiplier, which is obtained from the constraint $D_u \le D_d$ concerning the variance of the control signal and its admissible values.

IV. DYNAMICS DESCRIPTION OF THE WIND ENERGY CONVERSION SYSTEM (WECS)

The wind turbine generators comprise three dynamical subsystems. They are aero dynamical, mechanical and electrical subsystems (Fig. 2). The basic elements of the aero dynamical subsystem are the turbine rotor (wind wheel), hub and blade pitching system. The main components of the mechanical subsystem are the drive train (shaft) and the gearbox. The turbine rotor and generator converting the turbine mechanical power into electrical power and electronic converters are elements of the electro-mechanical subsystem.

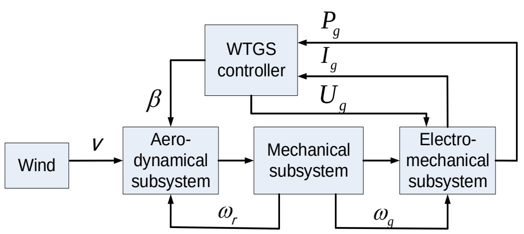


Fig. 2. Block diagram of the WECS subsystems

A. Wind dynamics modeling

The wind speed is regarded as a non-stationary random process, which consist of two components [3], [9]:

$$v(t) = v_s(t) + v_t(t)\ , \qquad (14)$$

where $v_s(t)$ is a low-frequency component which describes the slow variations in the wind speed, and $v_t(t)$ - high-frequency component which is the turbulence model. The turbulence comprises all wind speed variations in the frequency band above the spectral gap. There are two widely accepted wind turbulence models. Von Karman wind turbulence model is characterized by irrational power spectral density function [3], [4], [9]:

$$S(\omega) = \frac{K_v}{\left[1 + (\omega T_v)^2\right]^{5/6}}\ , \qquad (15)$$

and the Kaimal spectrum

$$S(\omega) = \frac{K_v}{\left[1 + (\omega T_v)^2\right]^{5/3}}\ . \qquad (16)$$

The time constant T_V determines the turbulence frequency band, while K_V refers to the turbulence power. In the time domain T_V is a measure of the correlation time. These two parameters depend on the average wind speed and on the terrain topology. For example, the coefficients in (15) can be approximated by the expressions [3]:

$$K_V = 0.475\sigma_v^2\frac{L_v}{V_m(z)}\ ,\ T_V = \frac{L_v}{V_m(z)} \qquad (17)$$

where L_v is correlation length of the turbulence, σ_v is the turbulence intensity. To generate a random process with a Karman spectrum a shaping filter should be designed having the transfer function:

$$W_f(s) = \frac{\sqrt{k_V}}{\left(1 + sT_V\right)^{5/6}}\ , \qquad (18)$$

This transfer function is not a rational function and the standard approach for a shaping filter design cannot be applied. In [19] an approach based on the relationship between the real frequencies characteristics $P(\omega)$ and the step response is proposed. In many applications the auto correlation function can be approximated by the expressions:

$$K(\tau) = \sum_{i=1}^{m}\sigma_i^2 e^{-\alpha_i|\tau|}\cos\beta_i\tau\ , \qquad (19)$$

or

$$K(\tau) = \sum_{i=1}^{m}\sigma_i^2 e^{-\alpha_i|\tau|}\ . \qquad (20)$$

We suppose the following simple approximation by (20)

$$K_x(\tau) = a_1 e^{-\alpha_1|\tau|} + a_2 e^{-\alpha_2|\tau|}, \qquad (21)$$

where the parameters $a_1, a_2, \alpha_1, \alpha_2$ are obtained from the optimization procedure for minimization of the integral of square error (ISE) criterion between spectral density function (15) and Furrier transform of the approximation (21) for a given wind speed:

$$F\{K_x(\tau)\} = \frac{2\alpha_1 a_1}{\alpha_1^2+\omega^2} + \frac{2\alpha_2 a_2}{\alpha_2^2+\omega^2}.$$

The model parameters for different values for the wind speed are given in Table 1.

TABLE I
AUTO CORRELATION FUNCTION PARAMETERS

Wind speed V_m [m/s]	a_1	a_2	α_1	α_2
6	0.0138	0.0220	0.0420	0.0420
8	0.0117	0.0220	0.0478	0.0478
10	0.0077	0.0242	0.0533	0.0532
12	0.0100	0.0204	0.0592	0.0591
16	0.0170	0.0234	0.3574	0.0635
18	0.0201	0.0204	0.0648	0.3068

The discrete spectral density function is:

$$S_s(z) = R_x^*(z) + R_x^*(z^{-1}) - R_x(0), \tag{22}$$

where $R_x^*(z)$ is the *Z*-transform of the discrete auto correlation function (21). For the wind speed model approximation, the following expression can be obtained:

$$S_s(z) = \frac{(a_1 - a_1 d_1^2)z}{(z-d_1)(1-d_1 z)} + \frac{(a_2 - a_2 d_2^2)z}{(z-d_2)(1-d_2 z)}, \tag{23}$$

where $d_1 = e^{-\alpha_1 T_0}$, $d_2 = e^{-\alpha_2 T_0}$.

The noise *n* applied on the system input describes all measurement inaccuracies and the influence of all additional disturbances which cannot be measured. We suppose this signal is a white noise with a spectral density function:

$$S_n(z) = N / T_0^2 . \tag{24}$$

With taking into account a constant component (slow variation) in the random process (wind speed), we obtain the spectral density function of the random signal *s*:

$$S_s(z) = \frac{V_m^2 z}{(z-1)(1-z)} + \frac{(a_1 - a_1 d_1^2)z}{(z-d_1)(1-z.d_1)} + \frac{(a_2 - a_2 d_2^2)z}{(z-d_2)(1-z.d_2)} . \tag{25}$$

B. Aero dynamical subsystem modelling

The aero dynamical torque of the wind turbine is a non-linear function with respect to the wind speed *v* and the torque coefficient $C_T(v,\beta,\omega)$. The last one depends on the pitch angle β, the wind speed and the turbine angular velocity ω

$$T_a = 0.5\pi\rho R^3 v^2 C_T(v,\beta,\omega). \tag{26}$$

where ρ is the air density $[kg/m^3]$ and *R* – radius of the turbine rotor [*m*].

The turbine mechanical power is

$$P_W = 0.5\pi\rho R^2 v^3 C_T(v,\beta,\omega), \tag{27}$$

where the torque coefficient $C_T(v,\beta,\omega)$ depends on the power efficiency factor $C_P(v,\beta,\omega)$ in accordance with the relationship

$$C_T(\lambda,\beta) = \frac{C_P(\lambda,\beta)}{\lambda}, \tag{28}$$

where λ is the turbine tip-speed ratio which may be determined from the expression

$$\lambda = \frac{\omega R}{v}. \tag{29}$$

Hence the aero dynamic torque acting on the turbine blades is

$$T_a(\lambda,\beta) = \frac{0.5\pi\rho R^3 v^2 C_P(\lambda,\beta)}{\lambda}. \tag{30}$$

More detailed descriptions of this subsystem can be found in [3], [4], [9].

C. Mechanical subsystem modeling

The most commonly used realization of the drive train model utilized as an element of the WECS model in the power system operation analysis is based on a two-mass model [3]-[5],[9]. The turbine and its hub are modeled as a first part, the generator and the gear as the other mass. The rotor shaft model is represented by the damping and stiffness coefficients.

$$J_w \frac{d\Delta\omega_w}{dt} = T_a - K(\delta_w - \delta_g/\upsilon) - D_e(\Delta\omega_w - \Delta\omega_g/\upsilon), \tag{31}$$

$$J_g \frac{d\Delta\omega_g}{dt} = T_g + (K(\delta_w - \delta_g/\upsilon) + D_e(\Delta\omega_w - \Delta\omega_g/\upsilon))/\upsilon, \tag{32}$$

$$\frac{d\delta_w}{dt} = \Delta\omega_w, \frac{d\delta_g}{dt} = \Delta\omega_g, \Delta\omega_w = \omega_w - \omega_{w0}, \Delta\omega_g = \omega_g - \omega_{g0},$$

where ω_{w0} and ω_{g0} are the wind wheel (turbine rotor) and the generator rotor speed [rad/s] in the steady state, T_a is the rotor torque, T_g - the generator torque [Nm], J_w and J_g - turbine and generator moment of inertia [kgm^2] respectively, *K* – stiffness coefficient [Nm/rad], and D_e is the damping coefficient [Nms/rad], υ - gear ratio coefficient. More detailed models of WTG subsystems are presented in [3], [4], [9], [27].

The complicated nonlinear model of WTG as well as the non-stationary stochastic process describing the dynamic behavior of the wind with irrational spectral function make the controller design a non-trivial task. The approximation by first order plus time delay model (FOPTD) is widely used simple model in many industrial applications. They lead to simple implementation of conventional controllers. In many cases concerning dynamical modeling in aero-dynamics, thermodynamics, mechanical and electromecha-nical engineering, chemical industry etc., these models cannot present the special features of the plant. A nonlinear simulation model of the WTG system is created in this research assuming a two-mass model. The model parameters are shown in Table II.

TABLE II
WTGS PARAMETERS

Rotor radius	R=40 m
Shaft damping coefficient	$D_e = 0.95\,10^6\ Nm/rad^2$
Shaft stiffness	$K = 2.5\,10^7\ Nm/rad$
Gear ratio coefficient	$\upsilon = 67.5$
Wind speed cut-out	$v_{cut\ out} = 25\ m/s$
Wind speed cut-in	$v_{cut\ in} = 3\ m/s$
Air density	$\rho = 1.25\ kg/m^3$
Rotor inertia moment	$J_w = 49.5\,10^5\ kg\ m^2$
Generator inertia moment	$J_g = 35.18\ kg\ m^2$

The plant dynamics can be approximated by a third-order non-minimal phase model with oscillating behavior [23]:

$$W_{pl}(s) = \frac{K_m(-T_{3m}s+1)}{(T_{1m}^2 s^2 + 2T_{1m}\xi s + 1)(T_{2m}s+1)}. \qquad (33)$$

The parameters of the model (33) are determined by using an optimization procedure based on the minimization of the ISE criterion. The error is determined on the base of the responses of the linear model (33) and the nonlinear plant model created in accordance with (26) - (32) for a given control signal (pitch angle) and wind dynamics presented by (25). A set of models (33) is obtained for different values of the average wind speed shown in Table III.

TABLE III
MODEL PARAMETERS

$V_m\,[m/s]$	8	10	12	16	18
K_m	1.05	1.62	2.47	3.15	3.63
$T_{3m}\,[s]$	8.375	5.453	5.463	5.003	1.524
$T_{1m}\,[s]$	2.942	5.212	6.015	4.961	2.831
ξ	1.1	1.3	1.3	1.6	1.37
$T_{2m}\,[s]$	200	294	325	200	205

Discrete models are obtained assuming a sample time $T_0 = 6$ s and ZOH method. In Fig. 3 their frequency characteristics are shown. It can be seen the magnitude response varies in the low-frequency range with about 10dB.

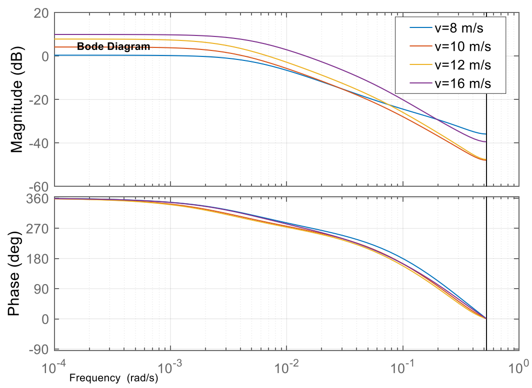


Fig. 3. Bode diagram for a set of models

V. DISCRETE STOCHASTIC DESIGN METHODS

The main goal of control system design of WTG is to ensure stability and optimal operation in both regimes:

- partial load operation;
- operation at a rated power

at all operating points. The references in these regimes are determined as follows: $\omega_{ref} = \frac{\lambda_{opt}}{R} v$ for "*partial load operation*", where λ_{opt} is the optimal value of the tip-speed ratio, and $\omega_{ref} = \frac{P_n}{T_a}$ for a "*rated power operation*". These control strategies change depending on the wind conditions It is very important to improve the control system performance in the regime "*partial load operation*" in order to keep the power efficiency factor close to its maximum value. One approach to ensure a high performance in a tracking regime is to apply Wiener filter design theory. Following the design procedure presented in section 3, a set of discrete Wiener filter for different values of the average wind speed are designed. In Fig. 4 the frequency characteristics of the optimal system are shown.

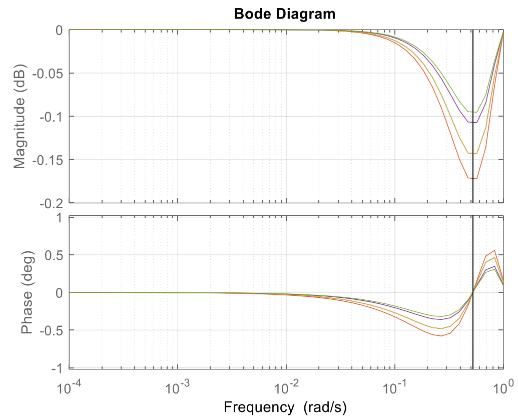
The following constraints are considered in the design procedure:

a. functional constraints of the type $E(z) = z - N_1$, where $abs(N_1) > 1$;
b. magnitude constraint of the control signal variance $D_u \le D_{u\max}$

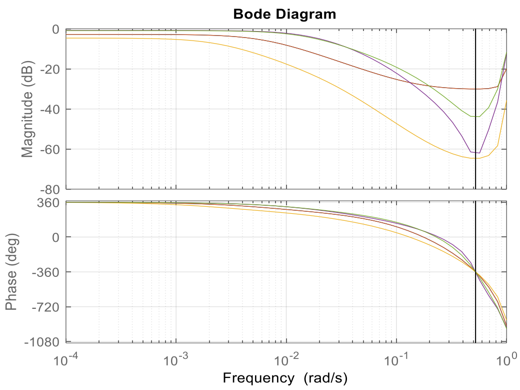
The transfer function of the optimal system is obtained by desired transformation of the input stochastic signal $W_0 = 1$ and constraints concerning non-minimal phase properties of the model and control signal limitation (the variance of the blades pitch angle is limited to 16 deg^2).

The controller structure is obtained applying (10). The existence of a non-minimal phase term in the plant model leads to unstable controller structure. In this case in design procedure should be obligatory taken into account functional constraints involved from the plant model. In Fig. 4a Bode plot of the optimal system by variable wind speed $V_m = [6, 18]\ m/s$ is shown, and in Fig.4b – the corresponding frequency characteristics of the optimal system with functional and magnitude constraints. It can be noticed the system keeps its tracking performance in the frequency range up to 4.10^{-2} rad/s. The values of the magnitude response in the low-frequency range become not equal to one when constraints are involved in the design procedure. In this case the optimal system has a non-zero steady-state error. The frequency characteristics of the controllers shown in Fig.4c have properties of lead-lag controller. The coefficient varies in the range of 12 dB depending of the wind speed.

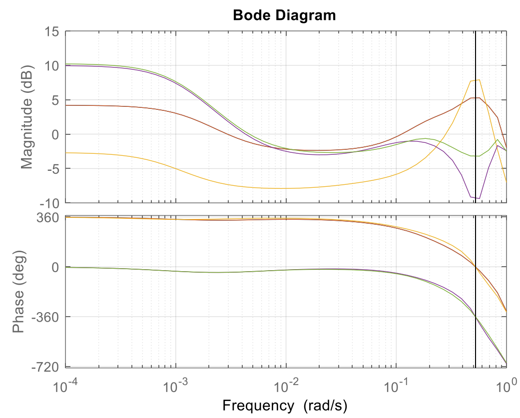
In this paper we propose a modification of the classical design method. It consists of optimal filter design with magnitude constraints only. The controller design is determined by a direct design based on Diophantine equation [21], [22], [25], [26] by taking into attention functional constraints involved from the model.



(a)



(b)



(c)

Fig. 4. Bode plot by variable wind speed: (a) of the optimal system, (b) of the optimal system with constraints, (c) of the controller

The control signal is obtained from the following polynomial equation:

$$R(z)u(z) = T(z)r(z) - S(z)y(z), \qquad (34)$$

where *R*, *T* and *S* are appropriate degree polynomials and *r* and *y* are the system reference and the output respectively. The transfer function $y(z)/r(z)$ is the desired (optimal) transfer function $W(z) = B(z)/A(z)$, obtained by Wiener filter design method. In this case we obtain a simpler Wiener filter since the functional constraints are included in the procedure of selection of polynomials *R*, *S* and *T*.

If the plant model has a non-minimal phase part $E(z)$, the numerator has to be presented in the following form:

$$B(z) = B^{+}(z)E(z). \qquad (35)$$

The numerator of the desired system is presented in the form $B_d = EB_d^{'}$, which includes the non-minimal phase of the plant model, and the polynomial *R* comprises the rest of terms which contain minimal phase terms of the polynomial *B* i.e. $R = B^{+}R'$. The equation associated with the polynomials determination in (36) takes the form:

$$\left(AR^{'} + ES\right) = B^{+}A_0 A_d, \quad T = B_d^{'}A_0, \qquad (36)$$

where A_0 is a characteristic polynomial of an additional filter. Thus the characteristic equation of the closed-loop system has the form:

$$AR + BS = B^{+}A_0 A_d. \qquad (37)$$

The following expressions for the polynomials *R*, *S* и *T* are determined:

$$R(z) = z^5 - 2.779z^4 + 2.863z^3 - 1.292z^2 + 0.216z + 0.0002$$

$$S(z) = 0.361z^2 - 0.473z + 0.141$$

$$T(z) = 5.32z^5 - 22.789z^4 + 30.219z^3 - 6.432z^2 - 12.489z + 6.241$$

They are established from the already found optimal transfer function from (14), and separated non-minimal phase part of the plant model $E(z) = z - 1.642$.

The input random signal can be predicted for a given time interval $\tau = lT_0$ if in the design procedure $W_0(s) = z^l$ and by using (13) [24].

In Fig. 5 the power efficiency factor by a wind speed 18 m/s is shown. It can be noticed the control system ensures high values of the power efficiency, but quite below to the theoretical maximum of 0.593. In Fig. 6 the tip-speed ratio by variable wind speed is presented. Remember the tip-speed ratio λ depends on the current values of the wind speed and the turbine rotational speed. To keep the optimal value of λ is a necessary condition to obtain a high value of the power efficiency factor.

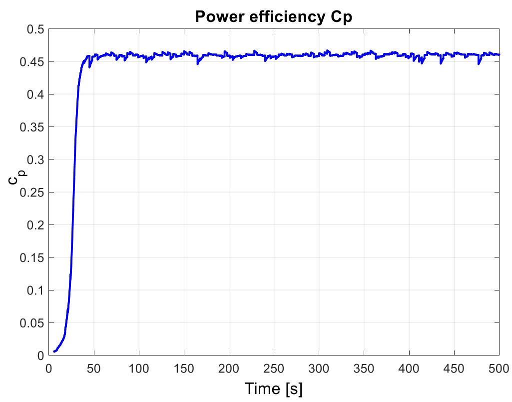


Fig. 5. Power efficiency factor at wind speed 18 m/s

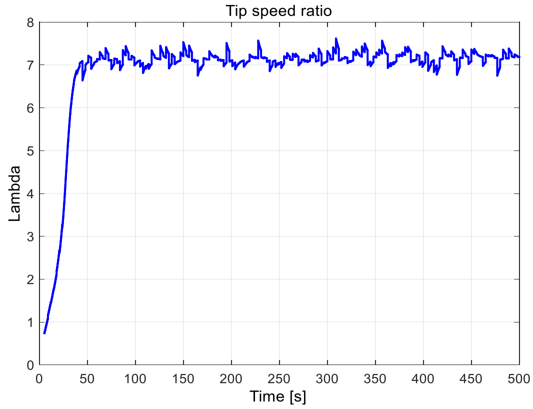


Fig. 6. Tip-speed ratio at wind speed 18 m/s

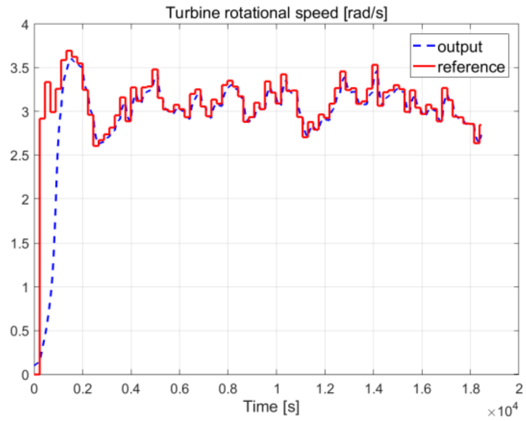


Fig. 7. Turbine rotational speed [rad/s] at wind speed 18 m/s

The control system performance in the tracking regime is shown in Fig. 7 and in Fig.8 and Fig.9 - the blade pitch angle and the turbine mechanical power at a wind speed 18 m/s. These simulation results are obtained by using a nonlinear plant model. The turbine rotational speed tracks the variable reference very well and the system performance can be evaluated by the standard deviation of the error from Fig. 1.

TABLE IV
STANDARD DEVIATION OF THE ERROR AND CONTROL SIGNAL (WIENER FILTER)

Wind speed [m/s]	8	10	12	16	18
σ_{opt_c}	0.1702	0.2469	0.1527	0.2006	0.3031
σ_{opt}	$7.4 10^{-3}$	$7.4 10^{-3}$	$7.5 10^{-3}$	$7.7 10^{-3}$	$7.8 10^{-3}$
σ_u	0.0437	0.0413	0.0519	0.0607	0.0931

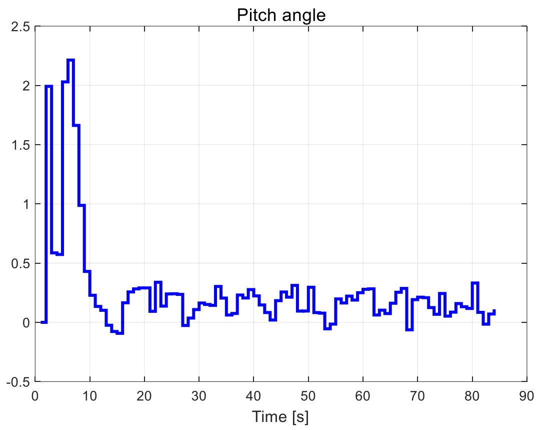


Fig. 8. Blade pitch angle at wind speed 18 m/s

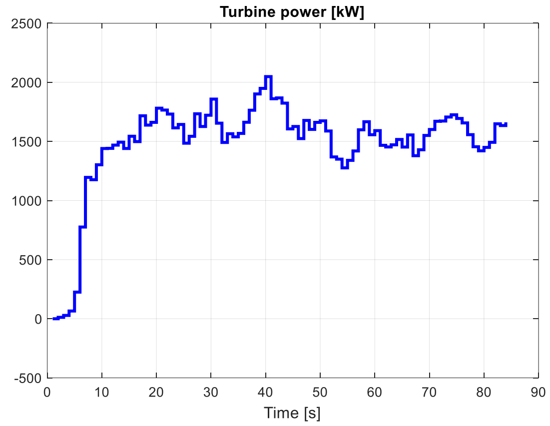


Fig. 9. Turbine power at wind speed 18 m/s

The values of the error standard deviation for the optimal system without constraints σ_{opt} and with constraints σ_{opt_c} for different values of the average wind speed are presented in Table IV. It can be seen the error standard deviation increases significantly when constraints are taken into account.

It is interesting to compare the obtained results for the error and control signal standard deviation with the corresponding characteristics obtained by using a conventional controller. By application a discrete PID controller with parameters tuned by a minimization a cost function, the system performance evaluated by standard deviation is presented in Table V. It can be noticed the standard deviation of the error as well of the control signal increase significantly in the case of discrete PID application. The reason for this is the Wiener filter design and the regulator based on it implies the statistical characteristics of the random process (the wind speed). This fact convincingly confirms the advantages of the proposed approach of Wiener filter application in the control systems operating in presence of stochastic processes.

TABLE V
STANDARD DEVIATION OF THE ERROR AND CONTROL SIGNAL (PID CONTROLLER)

Wind speed [m/s]	8	10	12	16	18
$\sigma_{\varepsilon\ PID}$	0.2049	0.1368	0.0943	0.0753	0.0378
σ_u	0.5959	0.4998	0.4629	0.2319	0.3588

VI. CONCLUSION

In this paper we explore the task of speed and power control of wind turbine in the regime "partial load operation". In this regime it is very important to improve the control system performance by tracking the variable reference signal in order to keep the power efficiency factor close to its maximum value. One approach to ensure a high performance in the tracking regime is to apply Wiener filter design theory. Discrete control algorithms based on Wiener filter design are investigated. They are focused on the stochastic nature of the input signal (wind speed) as well as functional and magnitude constraints involved with the plant model. The extrapolation of the input signal based on Wiener filter design is discussed. We propose a modification of the classical design method applying a discrete controller design by solving a Diophantine equation instead of functional constraints. This modification simplifies Wiener filter design. The analysis of the control system performance is based on the frequency characteristics of the optimal system as well as on the error standard deviation. The error and the control signal standard deviation are compared with the corresponding results by application a discrete PID controller which emphasizes the advantages of the proposed approach.

REFERENCES

[1] Buehring, I. et al., Control Strategies for wind energy conversion systems, Generation, Transmission and Distribution, IEE Proc. C, Vol.128, Issue 5, pp. 253 – 261, 1981, https://doi.org/10.1049/ip-c.1981.0043

[2] Connor, C., Leithead. W., Control strategies for variable speed stall regulated wind turbines, Proc. of European Wind Energy Conference, 1994.

[3] Lubosny, Z., Wind Turbine Operation in Electric Power Systems, Springer, 2003, https://doi.org/10.1007/978-3-662-10944-1

[4] Munteanu, I., Bratcu, A., Cutululis N., Ceanga, E., Optimal Control of Wind Energy System, Springer, 2008.

[5] Jackson, G., Njiri, D. Söffker, State-of-the-art in wind turbine control: Trends and challenges, Renewable and Sustainable Energy Reviews, Vol. 60, 2016, pp.377-393, https://doi.org/10.1016/j.rser.2016.01.110

[6] Hwas A, Katebi R. Wind turbine control using PI pitch angle controller. IFAC Conference on Advances in PID Control Brescia (Italy); 2012, https://doi.org/10.3182/20120328-3-IT-3014.00041

[7] Park, S., Y. Nam, Two LQRI based Blade Pitch Controls for Wind Turbines, Energies, Vol. 5 (6), 2012, pp. 1998-2016, https://doi.org/10.3390/en5061998

[8] Girsang, I., J. Dhupia, Collective pitch control of wind turbines using stochastic disturbance accommodating control, Wind Engineering, Vol.37, Issue 5, 2013, pp. 517-534, https://doi.org/10.1260/0309-524X.37.5.517

[9] Bianchi, F., De Battista H., Mantz R., Wind Turbine Control System, Springer 2003, Berlin.

[10] Ognyanova, O., Petkov, P., et all, Robust Control of a Wind Turbine, MedPower 2010, Agia Napa, Cyprus, 2010, https://doi.org/10.1049/cp.2010.0906

[11] Connor, B., Leithead, W., Robertson, S., Robust two-level control design approach to variable speed stall regulated wind turbines. European Wind Energy Conference'97, pp.551-554, 1998.

[12] Vihriala, H. Control of Variable Speed Wind Turbine, Springer, 2002. ISSN 1221-454X.

[13] Rocha R, Martins Filho LS, Bortolus MV., Optimal multivariable control for wind energy conversion system a comparison between H2 and Hinf controllers. In:44th IEEE conference on Decision and Control of the European Control Conference, Seville, Spain; 2005, pp.7906–11.

[14] Horowitz, B. LIà and J. W. Mc Cormick. Wiener filter-based Minimum Variance Self-tuning Regulation, Automatica, Vol. 34, No. 5, pp. 531-545, 1998, https://doi.org/10.1016/S0005-1098(97)00190-8

[15] Kolos M., I. Kolos. Methods of linear optimal filtration, Lomonosov Moskow State University Press, 2000

[16] Puri, N.N., Fundamentals of Linear Systems for Physical Scientists and Engineers, CRC Press, 2010, https://doi.org/10.1201/b12535

[17] Puleva, T., Ognyanova, O. Stochastic algorithms for speed and power control of wind turbine generator, Proceedings of the 30th International Conference on Cybernetics and Informatics, (K&I, 2020), Velke Karlovice, Czech Republic, 29 Jan.-1 Feb. 2020, 9039848, https://doi.org/10.1109/KI48306.2020.9039848

[18] Haralanova, E., T. Puleva. Control Theory-Part 2, TU-Sofia Publishing, 2019, (in Bulgarian)

[19] Ruzhekov, G., T. Puleva, Experimental test bench for wind modeling Imt. Conference Automatics and Informatics, 2015, Conf. Proceedings (in Bulgarian)

[20] Welfonder, E., R. Neifer, M. Spanner. Development and Experimental Identification of dynamics models for Wind Turbine. Control Engineering Practice, 1997, Vol. 5, No. 1, pp. 63-73.

[21] Astrom, K., B. Wittenmark, Computer controlled systems. Theory and design, Prentice Hall, 3rd ed., 1996, https://doi.org/10.1016/S0967-0661(96)00208-0

[22] Lund M., Pole placement design, Toolbox for MATLAB

[23] Ognyanova, O., E. Haralanova, T. Puleva. Wind Energy Conversion System Modeling, International Conference Automatics and Informatics'09, 29 Sept. - 4 Oct. 2009, Sofia, pp. VI-1-VI-4. (in Bulgarian)

[24] Ognyanova O., E. Haralanova, Speed and power predictive control of wind turbine generator by Wiener filter, International Conference Automatics and Informatics'09, 29 Sept. - 4 Oct. 2009, Sofia. (in Bulgarian)

[25] Franklin, G., J. D. Powell, Emami-Naeini, Feedback Contol of Dynamical Systems, 5th ed., Pearson Prentice Hall, 2006.

[26] M. Sami Fadali, Antonio Visioli, Digital Control Engineering. Analisys and Design, Elsevier, Academic Press, 3rd ed., 2019.

[27] Leithead, W.E., B. Connor. Control of variable speed wind turbines: Dynamic Models, Int. Journal of Control, Vol.73, №13, pp.1173-1188, 2000, https://doi.org/10.1080/002071700417830