A Research and Comparative Analysis of the full Planet Engagement Planetary Gear Train

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Abstract – This article aims to research and classify the full pinion engagement planetary gear train. It can be described as a type of gear train in which each planet gear is arranged in such a way so that it is engaged with the adjacent planets forming a closed meshed loop. It can be also described as “full planetary engagement gearbox” or “constrained planetary gearbox.”

An overview of the literature is conducted aiming to compare the “full pinion engagement planetary gear train” with its closest type of planetary gear train designs and other geared systems. The comparison will include planetary gearbox and constrained gear systems specific-assembly and neighbor conditions, center distance relations as well as a short analysis of the characteristics and uses of each design. The article will also include an overview of a manufactured prototype using the full planet engagement concept and a theoretical analysis of the potential uses of such geartrains.

Index terms – Planetary gearbox, full engagement, double pinion planetary gearbox, constrained planetary gear system, torque split transmission.

I. INTRODUCTION

Planetary gear trains are used in variety of fields of mechanical engineering. Their design allows for very compact gear trains with high specific strength and low weight, thanks to load sharing between multiple planet gears and their internal balance of forces leading to greatly reduced loads on bearings, supports, and other components with carry the resulting forces of gear meshing. On the other hand, variety of possible arrangements and types allow for large spectrum of gear ratios and the ability to work as a differential or combinatorial device. These properties make them suitable for the fields of robotics, automotive, heavy lifting equipment and so on.

Unfortunately, there isn’t a global unified standard for the description and classification of the planetary gear trains. Although the German guideline VDI 2157 is worth noting [1], [2]. In this research a short comparison will be made between the most recognizable basic planetary gear train, the double planet gear train and the full planet engagement planetary. But first the following clarification should be made:

1. Planetary gear train vs pseudo planetary or power branching transmission

The literature describes the planetary gear train as a mechanism consisting of friction or mostly toothed gears in which one or more of the gears geometrical axes is in motion [3]. Other often used term is “Epicyclic gear train” derived from epicyclid – the motion of a circle that rolls without slipping around another fixed circle [4]. However, there are possible scenarios and arrangements in which the carrier is fixed (Fig. 1, b) and there is no epicyclic motion and the geometrical axes are stationary.

While often in practice and in the common engineering such gear trains are considered planetary, because their design and components similarities, the literature is rather strict on this matter and describes the planetary gear trains only as ones with a movable geometric axis. The other type with a fixed carrier is sometimes called pseudo planetary [2] or power branching transmission [4].

Fig. 1. Working modes of AI-PGTs: With F = 1 degree of freedom. [2] a) Fixed ring gear (ω1 = 0), b) Fixed carrier (ω0 = 0), c) Fixed sun gear (ω0 = 0)

2. Positive vs negative and basic speed ratio

The design of the planetary gear trains allows for a variety of configurations and speed ratios depending on which of the components is held stationary and which is the input and output. Working with 2 degrees of freedom, also called threeshift transmission, is possible as well [2], [4]. This sometimes makes determining the type of transmission, either positive or negative, rather confusing. This is a why the term basic speed ratio is used, defined by the letter \( i_b \). It is regarded as the speed ratio when the carrier is immovably, and the transmission resembles the kinematics of a “pseudo planetary.”

"This mode of operation completely characterizes each revolving drive train and, therefore, provides a basis for its further analysis” [4].

With that said a positive ratio drive is one that in the conditions of a fixed carrier, the input and output shafts of the transmission have the same direction of rotation and the basic speed ratio is \( i_b > 0 \). A negative ratio drive will have the input and output shaft rotating in opposite directions and the basic speed ration will be \( i_b < 0 \). 

3. Simple planetary vs compound planetary gear train

Most of the literature regards simple or basic planetary gear trains as ones with only one carrier. Compound or composed [5] gear trains have more than one carrier and can consist of multiple stages and more complex forming of the gear ratios. This type of designs are often found in the automatic gearboxes of vehicles [4], [6], [7].
II. COMPARISON

The comparison between the different gear train designs will be made with respect to their type, speed ratio, assembly condition, neighbor condition and coaxially condition.

1. 2K-H, \(\overline{AI}\) Gear train

![Fig. 2. Schematics of a 2k-H gear train](image)

Perhaps the most recognizable type of planetary gear train. Consisting of a sun gear with external teeth, ring gear with internal teeth and planets with external teeth mounted on a carrier. Most widespread abbreviation of this gear train is 2K-H. 2K means two central gears, H denotes one carrier [8]. However, this abbreviation is not exclusive to single design and may refer to several different types arrangements. \(\overline{AI}\) is the abbreviation used by prof. Tkachenko [9], [10] and adopted by prof. Arnaudov [2], which is clearer. The German VDI 2157 [1] also have similarities with this one. A – stand for external meshing, I – stand for internal meshing [11]. The dash on top indicates that the planets are not stepped. Gear trains with stepped planets (consisting of two coaxial gear wheels) do not have a dash over the abbreviation [2], [9].

![Fig. 3. Model of 2k-H planetary gear train](image)

Basic speed ratio is negative:

\[
i_0 = \frac{\omega_1}{\omega_3} = -\frac{Z_2}{Z_1} < 0
\]  

(1)

A. Assembly condition

The number of teeth in the different links of a planetary gear train cannot be chosen randomly. Failure to comply with this condition will result in teeth interfering with each other and preventing the assembly of the gear train and its operation. Number of teeth of sun gear, ring gear and planets have to be chosen with respect to each other and to the number of planets as well. For sake of simplicity the case where all the planets are spaced evenly is considered. Meaning that the angle between the planets axis is equal with respect to the central axis. The assembly condition is as follows:

\[
\frac{Z_1 + Z_3}{k} = \text{Integer number}
\]  

(2)

With \(Z_1\) being the number of teeth of sun gear, \(Z_3\) being the number of teeth of ring gear and \(k\) being the number of planets.

B. Neighbor condition

Neighbor condition is associated with the fact that the planets should have adequate space between them so that their addendum diameters do not interfere. This has an effect on the maximum possible gear ratio when there is, more than two planet gears. With the possible ratio getting smaller with the increase of the number of planets. In order to comply with this condition, the following criteria should be met:

\[
da_2 < 2a_w \sin \frac{180^\circ}{k}
\]  

(3)

C. Coaxiality condition

This condition is met when the center distance of the sun gear (1) and planet gear (2) is equal to the center distance of the planet gear (2) and ring gear (3). Number of teeth and profile shift coefficient need to be chosen accordingly. This can be expressed with the following equation:

\[
\frac{Z_1 + Z_2}{\cos \alpha_{w12}} = \frac{Z_3 - Z_2}{\cos \alpha_{w23}}
\]  

(4)

With \(\cos \alpha_{w12}\) being the working pressure angle of \(Z_1\) and \(Z_2\) pair and \(\cos \alpha_{w23}\) being the working pressure angle of \(Z_3\) and \(Z_2\).

2. Dual pin planetary gear train

Can also be seen as “double pinion planetary”, “double planet planetary” or 2K-H +. Again, different authors use different designations. 2K-H type D [3], \(\overline{AI}\) [2], [9] 2K-H positive by some German authors [4], [6]. While some articles [12] describe this as a compound planetary, its
construction is more related to the simple gear trains with just one carrier.

It has two central gears sun\((Z_1)\) and ring gear\((Z_4)\), but 2 rows of planet gears \(Z_2, Z_3\) mounted on one carrier\((H)\). In the condition of a fixed carrier, this makes the sun gear and ring gears rotate in the same direction. So for the basic speed ratio we have:

\[
i_0 = \frac{\omega_1}{\omega_4} = \frac{Z_4}{Z_1} > 0
\]  

**A. Assembly condition**

\[
\frac{Z_4 - Z_1}{k} = f \in \text{int} > 0
\]

\[
\delta = \frac{f \cdot 360^\circ}{Z_4 - Z_1}
\]

Where by \(\delta\) is denoted the spacing angle between the planet pairs \((k)\).

**B. Neighbor condition**

This condition is more complicated in the case of dual pin planetary.

\[
d_{a2} < l_2 = 2a_{w12} \sin(180^\circ/k)
\]

\[
0,5(d_{a2} + d_{a3}) < l_{23} = \left[\frac{a_{w12}^2 + a_{w34}^2 - 2a_{w12}a_{w34} \cos(360^\circ/k - \phi)}{2}\right]
\]

\[
0,5(d_{a3} + d_{a4}) < l_{24} = a_{w12}
\]

**C. Coaxiality condition**

It can be described as a sum of vectors [2]

\[
a_{w12} + a_{w23} = a_{w34}
\]

The conditions described above, can also be expressed using the number of teeth as described in article [13].

While this type of design doesn’t have the ability to create large ratios, compared to other type of planetary arrangements, there are particular qualities that make it useful in certain applications. Being a positive drive \(i_0 > 0\), the sun gear and the ring gear rotate in the same direction. However, the sun gear and the carrier rotate in opposite directions thanks to the second row of planets that shift the motion of the carrier. This also affects the ratio of the gear train compared to the \(2k\)-\(H\) negative (fig.2, fig.3). In the condition of a fixed ring gear the torque and speed ratio is as follows:

\[
T_H = \frac{Z_4}{Z_1} - 1 = i_0 - 1, \quad \omega_H = \frac{\omega_1}{\omega_4} - 1 = i_0 - 1
\]

Subtracting “1” from the ratio gives interesting opportunities:

- Gear train with ratios closer to 1 can be designed which is rather difficult with the \(2k\)-\(H\) negative.
- When a gear train with \(i_0 = 2\) is designed, according to formula (10) the ratio of the carrier is \(i_0 - 1 = 2 - 1 = 1\). This makes the ratio between the sun gear and the carrier equal.

This gives the opportunity to create an equal power split\((\text{differential})\) or power combinatory (fig 6) transmission.

\[
\delta = \frac{f \cdot 360^\circ}{Z_4 - Z_1}
\]

Such an arrangement can be seen in symmetrical automotive differentials [2], [6], [14]. Being very axially compact and thanks to the internal balance of forces a very lightweight and small casing can be designed, resulting in reduced overall weight and size. Hence, it’s use in high performance automotive and motorsport applications [15], as well as some designs of automatic gearboxes and transfer cases of vehicles [6], [7].
Fig. 8. High performance motorsport differential [15]

However, adding a second row of planet gears inevitably affects the dynamic loads in the gear train [16], [17] as well as the load sharing ratio of the planets [11], [18]. Number and spacing of the planets also affect load bearing characteristic [19], [20].

3. Full planet engagement planetary gear train

There is a surprisingly little evidence for this kind of arrangement in the literature. Honda technical journal 2014 has an article that describes the use of such planetary mechanism for a prototype of a differential for their 2009 season F1 car [21]. In this article they explain the idea of using an arrangement called “full pinion engagement” in which all of the planets mesh with each other (Fig. 5). Authors claim reduced weight and radial dimensions of this arrangement compared to the dual pin planetary $\mathcal{A\mathcal{A}}$, $2k - \mathcal{H}$ positive. Both of these features contributing to the performance of the car.

There are patents [22] that have some similarity with such an arrangement, as well as some developments for military aviation that use the idea of a closed loop meshing [23]. Yet there is little evidence and research on the use and characteristics of such gear train. In can easily be seen that this arrangement can be considered a derivative of the $\mathcal{A\mathcal{A}}$. However, while being kinematically similar, the assembly and neighbor conditions differ. A CAD simulation was done as a test.

A. Assembly condition

Applying formulas (6) and (7) a successful engagement between the sun gear and planet gears and planet gears and ring gears is established. However, this condition is not enough to prevent interference between the planets themselves (Fig. 10). Further simulation shows that not only the number of teeth, but also the angle between the center distances has to be of specific value.

B. Neighbor condition

While most of the neighbor condition rules in other planetary mechanism ensure that the addendum diameters of the planet gears are not touching, quite the opposite is required here. The pitch diameters of all the planet gears have to be tangent for successful meshing. Even more their addendum diameters have to be the right size so that they fit inside the space between the sun gear and ring gear and also meet all the assembly requirements.

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can be found in the synthesis of so-called torque split transmissions [23]. Constrained gear systems used in some braiding machines [24], and portal axles of off-road vehicles [25]. In these designs, gear meshing forms a closed loop similar to the full planet engagement planetary gear train. The assembly condition is given as follows:

\[
\frac{z_1\theta_1}{180} + \frac{z_2(180^\circ + \theta_1 + \theta_2)}{180^\circ} + \frac{z_3\theta_2}{180^\circ} = \text{integer} \quad (11)
\]

A more detailed information on the assembly conditions of a split torque gear system can be found in [27]. Again, as with planetary gear trains and other power branching transmissions there is an unequal load sharing between the members affected by many parameters [24], [28], [29], [30], [31].

III. REVERSE ENGINEERING OF THE HONDA DIFFERENTIAL

A prototype based on the Honda differential was developed by the Bulgarian FSAE team of the Technical University of Sofia for season 2018. During the development the planet arrangement and number of teeth of the model described in [21] was used as a starting point. The gear module and general design was chosen according to the specific case – to be used in a small motorcycle engine car with roller chain final drive. During the development 2 sets of gear were manufactured, with slightly different teeth correction and tolerances (Fig. 14).

During the test assembly the following was observed:
- This type of arrangement is very sensitive to geometrical error especially in the planet gears. With radial runout, and deviation from cylindricality being the worst affecting factors. One of the gear sets had poor geometrical tolerances after heat treatment and resulted in very rough meshing. The second gear set was left untreated and instead of changing the backlash or addendum truncation, a slightly smaller value of the profile shift coefficient was used. Both of these resulted in a significantly better meshing.
- A floating mounting of the planet gears has a positive effect on the gear train.

The prototype model (Fig. 15) worked kinematically but was not integrated in the car and was never tested during intended operation.

IV. POTENTIAL USE

Other than the use described in the cited article [21] as a high-performance automotive differential this arrangement can rarely be seen. However, taking into account the characteristics of other types of planetary and power branching transmissions a theoretical analysis can be made. In the case of the 2k-H setup and not only, the torque split is proportional to the number of planets. Theoretically speaking the higher the number of planets, the higher the specific load bearing capacity of the transmission. Meaning that more performance can be extracted from a given weight and size. Yet the number of planets is limited by the neighbor and assembly conditions. Full planet engagement allows for highest potential load split with regards to size and available space in a 2K-H setup. With
this in mind, it seems that these kinds of transmissions have potential use in highly loaded low speed applications, as well as the already mentioned high performance automotive use as a differentials and combinatorial transmissions and other applications where high specific load bearing capacity is needed and the size envelope is limited.

V. CONCLUSION

The current literature review resulted in very little information about the full planet engagement planetary gear train. While similarities and relations with other types of gear mechanisms can be found, a guide for synthesis and characteristics of such gear train is lacking or incomplete.

With regards to the design itself the following theoretical conclusions can be made:

- Using the full planet engagement design yields the highest potential power split with respect to neighbor and assembly conditions
- In comparison with the AAT there is a further power split and mesh load reduction between the second row of planets because each one is meshing with two adjacent. Similar to the torque split transmissions

Nonetheless there are certain concerns that may affect these advantages:

- The effect of the mutual engagement of the planets on the load sharing ratio is unknown.
- CAD simulations and attempted virtual synthesis show that the possible gear ratios of such designs is strictly limited.
- As proven with the reverse engineered differential project, this arrangement is sensitive to manufacturing errors and requires a carefully designed tolerance
- The effect of high speeds, high heat expansions and high dynamic loads in general is unknown.

One important area that is not covered in this article is the load bearing characteristics of such a gear train. Of particular interest would be the effect of the full planet engagement on the load sharing ratio. This leaves room for studies, as well as empirical experiments on the topic of load bearing characteristics.

REFERENCES