A Research and Comparative Analysis of the full Planet Engagement Planetary Gear Train

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Abstract – **This article aims to research and classify the full pinion engagement planetary gear train. It can be described as a type of gear train in which each planet gear is arranged in such a way so that it is engaged with the adjacent planets forming a closed meshed loop. It can be also described as "full planetary engagement gearbox" or "constrained planetary gearbox" An overview of the literature is conducted aiming to compare the "full pinion engagement planetary gear train" with its closest type of planetary gear train designs and other geared systems. The comparison will include planetary gearbox and constrained gear systems specific- assembly and neighbor conditions, center distance relations as well as a short analysis of the characteristics and uses of each design. The article will also include an overview of a manufactured prototype using the full planet engagement concept and a theoretical analysis of the potential uses of such geartrains.**

Index terms – **Planetary gearbox, full engagement, double pinion planetary gearbox, constrained planetary gear system, torque split transmission.**

I. INTRODUCTION

Planetary gear trains are used in variety of fields of mechanical engineering. Their design allows for very compact gear trains with high specific strength and low weight, thanks to load sharing between multiple planet gears and their internal balance of forces leading to greatly reduced loads on bearings, supports, and other components with carry the resulting forces of gear meshing. On the other hand, variety of possible arrangements and types allow for large spectrum of gear ratios and the ability to work as a differential or combinatory device. These properties make them suitable for the fields of robotics, automotive, heavy lifting equipment and so on.

Unfortunately, there isn't a global unified standard for the description and classification of the planetary gear trains. Although the German guideline VDI 2157 is worth noting [1], [2]. In this research a short comparison will be made between the most recognizable basic planetary gear train, the double planet gear train and the full planet engagement planetary. But first the following clarification should be made:

1. Planetary gear train vs pseudo planetary or power branching transmission

The literature describes the planetary gear train as a mechanism consisting of friction or mostly toothed gears in which one or more of the gears geometrical axes is in motion [3]. Other often used term is "Epicyclic gear train" derived from epicycloid – the motion of a circle that rolls without slipping around another fixed circle [4]. However, there are possible scenarios and arrangements in which the carrier is fixed (Fig. 1, b) and there is no epicyclic motion and the geometrical axes are stationary.

While often in practice and in the common engineering such gear trains are considered planetary, because their design and components similarities, the literature is rather strict on this matter and describes the planetary gear trains only as ones with a movable geometric axis. The other type with a fixed carrier is sometimes called pseudo planetary [2] or power branching transmission [4].

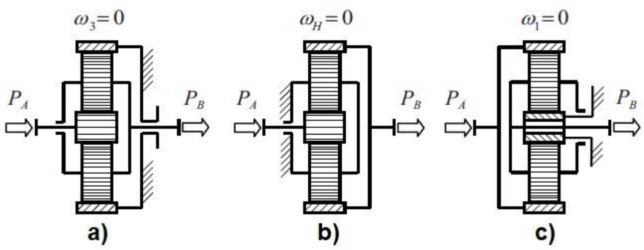

Fig. 1. Working modes of AI-PGTs: With F = 1 degree of freedom. [2] a) Fixed ring gear ($\omega_3 = 0$), b) Fixed carrier ($\omega_H = 0$), c) Fixed sun gear ($\omega_1 = 0$)

2. Positive vs negative and basic speed ratio

The design of the planetary gear trains allows for a variety of configurations and speed ratios depending on which of the components is held stationary and which is the input and output. Working with 2 degrees of freedom, also called three-shaft transmission, is possible as well [2], [4]. This sometimes makes determining the type of transmission, either positive or negative, rather confusing. This is a why the term basic speed ratio is used, defined by the letter $\boldsymbol{i}_0$. It is regarded as the speed ratio when the carrier is immovably, and the transmission resembles the kinematics of a "pseudo planetary"

"This mode of operation completely characterizes each revolving drive train and, therefore, provides a basis for its further analysis" [4].

With that said a positive ratio drive is one that in the conditions of a fixed carrier, the input and output shafts of the transmission have the same direction of rotation and the basic speed ratio is $\boldsymbol{i}_0>0$. A negative ratio drive will have the input and output shaft rotating in opposite directions and the basic speed ration will be $\boldsymbol{i}_0<0$

3. Simple planetary vs compound planetary gear train

Most of the literature regards simple or basic planetary gear trains as ones with only one carrier. Compound or composed [5] gear trains have more than one carrier and can consist of multiple stages and more complex forming of the gear ratios. This type of designs are often found in the automatic gearboxes of vehicles [4], [6], [7].

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II. COMPARISON

The comparison between the different gear train designs will be made with respect to their type, speed ratio, assembly condition, neighbor condition and coaxially condition.

1. 2K-H, $\overline{AI}$ Gear train

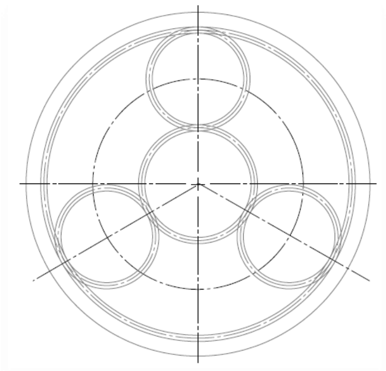

Fig. 2. Schematics of a 2k-H gear train

Perhaps the most recognizable type of planetary gear train. Consisting of a sun gear with external teeth, ring gear with internal teeth and planets with external teeth mounted on a carrier. Most widespread abbreviation of this gear train is 2K-H

2K means two central gears, H denotes one carrier [8]. However, this abbreviation is not exclusive to single design and may refer to several different types arrangements.

$\overline{AI}$ is the abbreviation used by prof. Tkachenko [9], [10] and adopted by prof. Arnaudov [2], which is clearer. The German VDI 2157 [1] also have similarities with this one.

A – stand for external meshing, I – stand for internal meshing [11]. The dash on top indicates that the planets are not stepped. Gear trains with stepped planets (consisting of two coaxial gear wheels) do not have a dash over the abbreviation [2], [9].

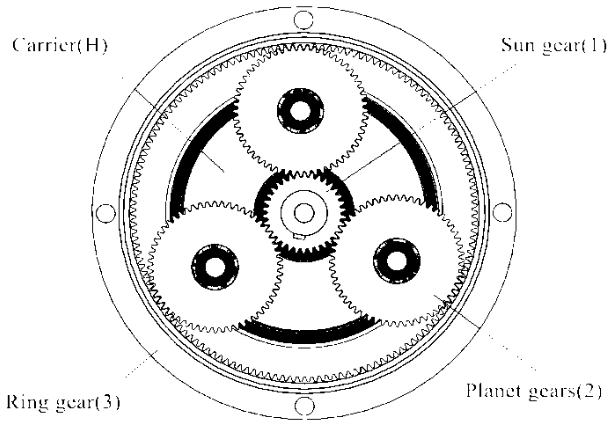

Fig. 3. Model of 2k-H planetary gear train

Basic speed ratio is negative:

$$i_0 = \frac{\omega_1}{\omega_3} = -\frac{Z_3}{Z_1} < 0 \tag{1}$$

A. Assembly condition

The number of teeth in the different links of a planetary gear train cannot be chosen randomly. Failure to comply with this condition will result in teeth interfering with each other and preventing the assembly of the gear train and its operation. Number of teeth of sun gear, ring gear and planets have to be chosen with respect to each other and to the number of planets as well. For sake of simplicity the case where all the planets are spaced evenly is considered. Meaning that the angle between the planets axis is equal with respect to the central axis. The assembly condition is as follows:

$$\frac{Z_1+Z_3}{k} = \text{Integer number} \tag{2}$$

With $\mathbf{Z_1}$ being the number of teeth of sun gear, Z_3 being the number of teeth of ring gear and k being the number of planets.

B. Neighbor condition

Neighbor condition is associated with the fact that the planets should have adequate space between them so that their addendum diameters do not interfere. This has an effect on the maximum possible gear ratio when there is, more than two planet gears. With the possible ratio getting smaller with the increase of the number of planets. In order to comply with this condition, the following criteria should be met:

$$da_2 < 2a_w \sin\frac{180^\circ}{k} \tag{3}$$

C. Coaxiality condition

This condition is met when the center distance of the sun gear (1) and planet gear (2) is equal to the center distance of the planet gear (2) and ring gear (3). Number of teeth and profile shift coefficient need to be chosen accordingly. This can be expressed with the following equation:

$$\frac{Z_1 + Z_2}{\cos\alpha_{w12}} = \frac{Z_3 - Z_2}{\cos\alpha_{w23}} \tag{4}$$

With $\cos\alpha_{w12}$ being the working pressure angle of Z_1 and Z_2 pair and $\cos\alpha_{w23}$ being the working pressure angle of Z_3 and Z_2

2. Dual pin planetary gear train

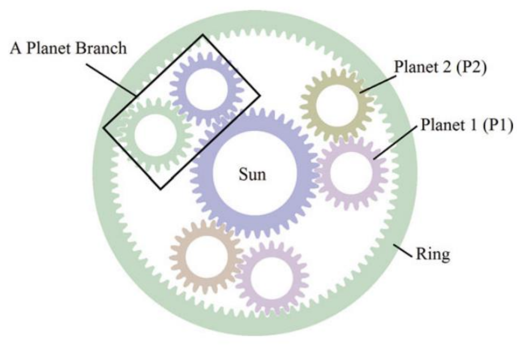

Fig. 4. Schematic representation of dual pin planetary gear train [16]

Can also be seen as "double pinion planetary", "double planet planetary" or 2K-H +. Again, different authors use different designations. 2K-H type D [3], $\overline{AAI}$ [2], [9] 2K-H positive by some German authors [4], [6]. While some articles [12] describe this as a compound planetary, its construction is more related to the simple gear trains with just one carrier.

It has two central gears sun(Z_1) and ring gear(Z_4), but 2 rows of planet gears Z_2, Z_3 mounted on one carrier(H). In the condition of a fixed carrier, this makes the sun gear and ring gears rotate in the same direction. So for the basic speed ratio we have:

$$i_0 = \frac{\omega_1}{\omega_4} = \frac{Z_4}{Z_1} > 0 \tag{5}$$

A. Assembly condition

$$\frac{Z_4 - Z_1}{k} = f \in \text{int} > 0 \tag{6}$$

$$\delta = \frac{f360^0}{Z_4 - Z_1} \tag{7}$$

Where by δ is denoted the spacing angle between the planet pairs (*k*).

B. Neighbor condition

This condition is more complicated in the case of dual pin planetary.

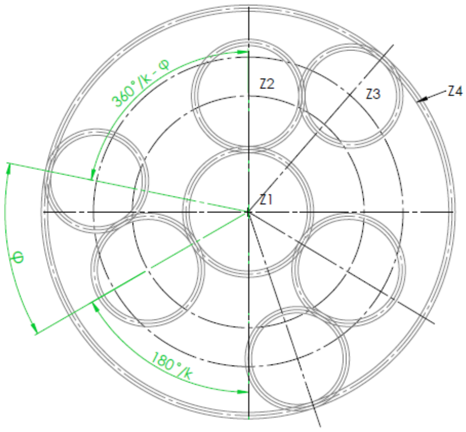

Fig. 5. Angle relations in dual pin planetary

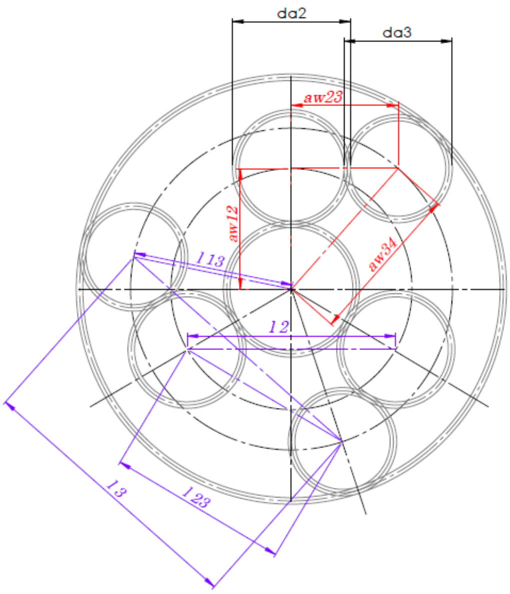

Fig. 6. Center distances in dual pin planetary

Relating to Fig. 5, Fig. 6 all of the following criteria should be met:

$$d_{a2} < l_2 = 2a_{w12} \sin(180°/k)$$

$$d_{a3} < l_3 = 2a_{w34} \sin(180°/k)$$

$$0,5(d_{a2} + d_{a3}) < l_{23} = \\ = [a_{w12}^2 + a_{w34}^2 - 2\, a_{w12}\, a_{w34} \cos(360°/k - \varphi)] \tag{8}$$

$$0,5(d_{a1} + d_{a3}) < l_{13} = a_{w34}$$

$$0,5(d_{a4} + d_{a2}) < l_{24} = a_{w12}$$

C. Coaxiality condition

It can be described as a sum of vectors [2]

$$a_{w12} + a_{w23} = a_{w34} \tag{9}$$

The conditions described above, can also be expressed using the number of teeth as described in article [13].

While this type of design doesn't have the ability to create large ratios, compared to other type of planetary arrangements, there are particular qualities that make it useful in certain applications. Being a positive drive ***i₀* >0**, the sun gear and the ring gear rotate in the same direction. However, the sun gear and the carrier rotate in opposite directions thanks to the second row of planets that shift the motion of the carrier. This also affects the ratio of the gear train compared to the 2k-H negative (fig.2, fig.3). In the condition of a fixed ring gear the torque and speed ratio is as follows:

$$T_{\text{H}} = \frac{Z_4}{Z_1} - 1 = i_0 - 1, \quad \omega_{\text{H}} = \frac{\omega_1}{\omega_4} - 1 = i_0 - 1 \tag{10}$$

Subtracting "1" from the ratio gives interesting opportunities:

- Gear train with ratios closer to 1 can be designed which is rather difficult with the 2k-H negative.
- When a gear train with $i_0 = 2$ is designed, according to formula (10) the ratio of the carrier is $i_0 - 1 = 2 - 1 = 1$. This makes the ratio between the sun gear and the carrier equal

This gives the opportunity to create an equal power split(differential) or power combinatory (fig 6) transmission.

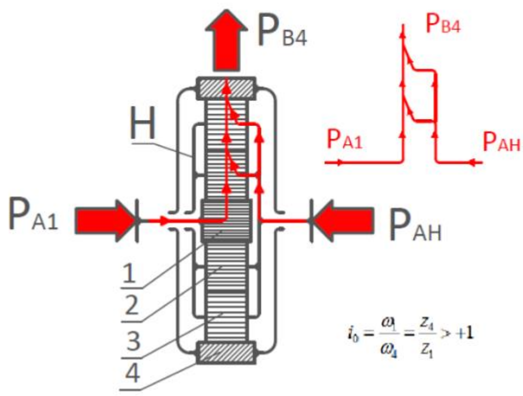

Fig. 7. Power flow in combinatory transmission [14]

Such an arrangement can be seen in symmetrical automotive differentials [2], [6], [14]. Being very axially compact and thanks to the internal balance of forces a very lightweight and small casing can be designed, resulting in reduced overall weight and size. Hence, it's use in high performance automotive and motorsport applications [15], as well as some designs of automatic gearboxes and transfer cases of vehicles [6], [7].

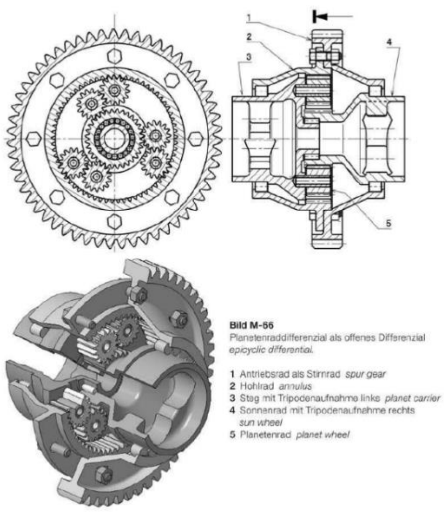

Fig. 8. High performance motorsport differential [15]

However, adding a second row of planet gears inevitably affects the dynamic loads in the gear train [16], [17] as well as the load sharing ratio of the planets [11], [18]. Number and spacing of the planets also affect load bearing characteristic [19], [20].

3. Full planet engagement planetary gear train

There is a surprisingly little evidence for this kind of arrangement in the literature. Honda technical journal 2014 has an article that describes the use of such planetary mechanism for a prototype of a differential for their 2009 season F1 car [21]. In this article they explain the idea of using an arrangement called "full pinion engagement" in which all of the planets mesh with each other(Fig.5*)*. Authors claim reduced weight and radial dimensions of this arrangement compared to the dual pin planetary – $\overline{AAI}$ **,** 2k – H positive. Both of these features contributing to the performance of the car.

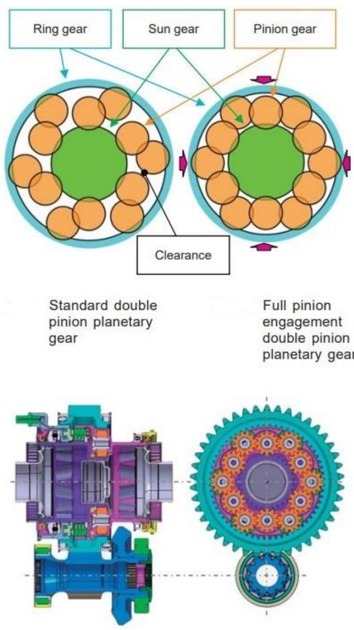

Fig. 9. Comparison model of a double pin planetary gear train and full pinion engagement planetary [21]

There are patents [22] that have some similarity with such an arrangement, as well as some developments for military aviation that use the idea of a closed loop meshing [23]. Yet there is little evidence and research on the use and characteristics of such gear train. In can easily be seen that this arrangement can be considered a derivative of the $\overline{AAI}$. However, while being kinematically similar, the assembly and neighbor conditions differ. A CAD simulation was done as a test.

A. Assembly condition

Applying formulas (6) and (7) a successful engagement between the sun gear and planet gears and planet gears and ring gears is established. However, this condition is not enough to prevent interference between the planets themselves (Fig. 10). Further simulation shows that not only the number of teeth, but also the angle between the center distances has to be of specific value.

B. Neighbor condition

While most of the neighbor condition rules in other planetary mechanism ensure that the addendum diameters of the planet gears are not touching, quite the opposite is required here. The pitch diameters of all the planet gears have to be tangent for successful meshing. Even more their addendum diameters have to be the right size so that they fit inside the space between the sun gear and ring gear and also meet all the assembly requirements.

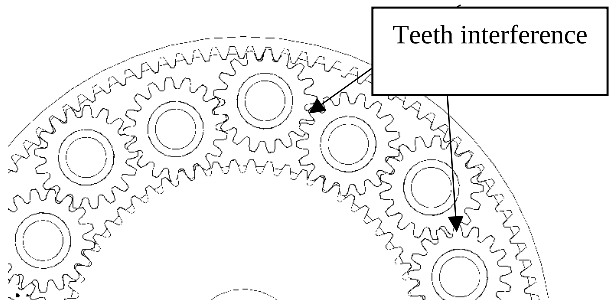

Fig. 10. Teeth interference

Again, a simple drawing simulation shows that these conditions are very hard to be met without profile shifting of the gears and adjusting their center distances. Using an integer number for pitch diameters of one of the gears, generates fraction numbers for the diameters of the others in order to have tangent circles – Fig. 11.

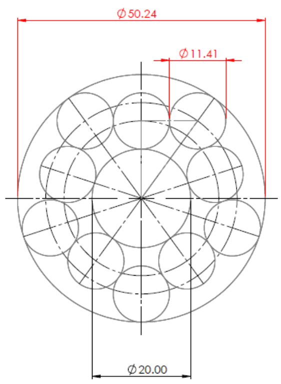

Fig. 11. Center distance relations in full planet engagement gear train

4. Torque split transmissions and constrained gear systems

One possible solution to the problems described in the assembly condition for the full planet engagement planetary can be found in the synthesis of so-called torque split transmissions [23]. Constrained gear systems used in some braiding machines [24], and portal axles of off-road vehicles [25]. In these designs, gear meshing forms a closed loop similar to the full planet engagement planetary gear train. The assembly condition is given as follows:

$$\frac{z_1\theta_1}{180^{\circ}}+\frac{z_2(180^{\circ}+\theta_1+\theta_2)}{180^{\circ}}+\frac{z_3\theta_2}{180^{\circ}}=\text{integer} \quad (11)$$

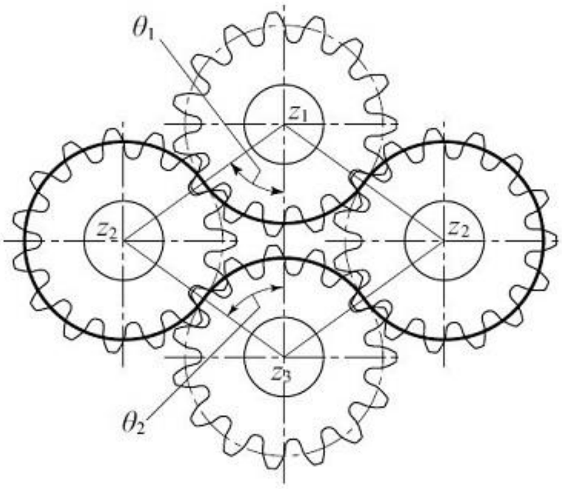

Fig. 12. Assembly condition in constrained gear system Copyright© 2021 Kohara Gear Industry [26]

A more detailed information on the assembly conditions of a split torque gear system can be found in [27].

Again, as with planetary gear trains and other power branching transmissions there is an unequal load sharing between the members affected by many parameters [24], [28], [29], [30], [31].

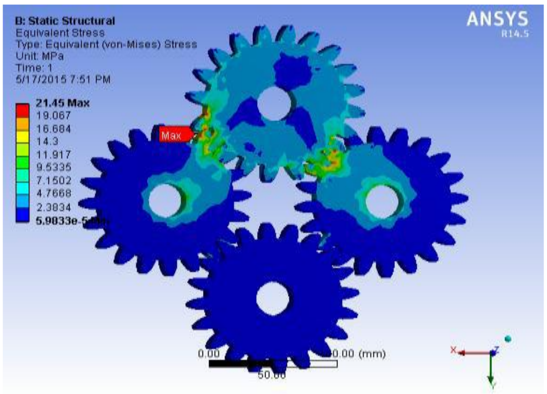

Fig. 13. FEA analysis of the load sharing in constrained gear system of portal axle drive [29]

III. Reverse Engineering of the Honda Differential

A prototype based on the Honda differential was developed by the Bulgarian FSAE team of the Technical University of Sofia for season 2018. During the development the planet arrangement and number of teeth of the model described in [21] was used as a starting point. The gear module and general design was chosen according to the specific case – to be used in a small motorcycle engine car with roller chain final drive. During the development 2 sets of gear were manufactured, with slightly different teeth correction and tolerances (Fig. 14).

Fig. 14. Manufactured prototype gearing for a full planet engagement gear train

During the test assembly the following was observed:

- This type of arrangement is very sensitive to geometrical error especially in the planet gears. With radial runout, and deviation from cylindricity being the worst affecting factors. One of the gear sets had poor geometrical tolerances after heat treatment and resulted in very rough meshing. The second gear set was left untreated and instead of changing the backlash or addendum truncation, a slightly smaller value of the profile shift coefficient was used. Both of these resulted in a significantly better meshing.
- A floating mounting of the planet gears has a positive effect on the gear train.

The prototype model (Fig. 15) worked kinematically but was not integrated in the car and was never tested during intended operation.

Fig. 15. Exploded showcase assembly of the full planet engagement differential developed for TU Sofia Racing team

IV. Potential Use

Other than the use described in the cited article [21] as a high-performance automotive differential this arrangement can rarely be seen. However, taking into account the characteristics of other types of planetary and power branching transmissions a theoretical analysis can be made. In the case of the 2k-H setup and not only, the torque split is proportional to the number of planets. Theoretically speaking the higher the number of planets, the higher the specific load bearing capacity of the transmission. Meaning that more performance can be extracted from a given weight and size. Yet the number of planets is limited by the neighbor and assembly conditions. Full planet engagement allows for highest potential load split with regards to size and available space in a 2K-H setup. With this in mind, it seems that these kinds of transmissions have potential use in highly loaded low speed applications, as well as the already mentioned high performance automotive use as a differentials and combinatory transmissions and other applications where high specific load bearing capacity is needed and the size envelope is limited.

V. CONCLUSION

The current literature review resulted in very little information about the full planet engagement planetary gear train. While similarities and relations with other types of gear mechanisms can be found, a guide for synthesis and characteristics of such gear train is lacking or incomplete.

With regards to the design itself the following theoretical conclusions can be made:

- Using the full planet engagement design yields the highest potential power split with respect to neighbor and assembly conditions
- In comparison with the $\overline{AAI}$ there is a further power split and mesh load reduction between the second row of planets because each one is meshing with two adjacent. Similar to the torque split transmissions

Nonetheless there are certain concerns that may affect these advantages:

- The effect of the mutual engagement of the planets on the load sharing ratio is unknown.
- CAD simulations and attempted virtual synthesis show that the possible gear ratios of such designs is strictly limited.
- As proven with the reverse engineered differential project, this arrangement is sensitive to manufacturing errors and requires a carefully designed tolerance
- The effect of high speeds, high heat expansions and high dynamic loads in general is unknown.

One important area that is not covered in this article is the load bearing characteristics of such a gear train. Of particular interest would be the effect of the full planet engagement on the load sharing ratio. This leaves room for studies, as well as empirical experiments on the topic of load bearing characteristics.

REFERENCES

[1] VDI-Richtlinie 2157, Planetengetriebe – Begriffe, Symbole, Berechnungsgrundlagen, 2012.

[2] K. Arnaudov, D. Karaivanov, Planetary Gear trains, Sofia, Bulgaria, Publishing house of BAS “Prof. Marin Drinov”, 2017 (in Bulgarian).

[3] Kudryavtsev, Y. Kirdyashev. Planetary Gear Trains, Leningrad, Mashinostroenie, 1977 (in Russian).

[4] H. W. Muller, Epicyclic Drive Trains. Detroit, MI: Wayne State University Press, 1982.

[5] D. T. Jelaska, Gears and gear drives, John Wiley & Sons Ltd, 2012 (English), https://doi.org/10.1002/9781118392393

[6] J. Looman, Zahnradgetriebe – Grundlagen, Konstruktion, Andwen-dung in Fahrzeugen, Auflage, Berlin: Springer-Verlag, 1996

[7] L. Jinxia, Y. Luda, Z. Qingliang, L. Qingye, Synthesis of multi-row and multi-speed planetary gear mechanism for automatic transmission, Mechanisms and machine theory, vol. 128, Pp. 616-627, 2018, https://doi.org/10.1016/j.mechmachtheory.2018.07.007

[8] D. Yu, KHV Planetary gearing, MCP Corporation. Skokie, Il 1987

[9] V. A. Tkachenko. "Design of multi-planet planetary gear trains". Kharkov: Kharkov National University, 1961.

[10] V. A. Tkachenko. “Planetary Mechanisms”, Kharkiv: Kharkiv Avia-tion Institute Publ., 2003 (in Russian).

[11] K. Arnaudov, A. Dobreva, D. Karaivanov, V. Ivanov, Internal involute gearing, Sofia, Publishing house of Technical University of Sofia 2019 (in Bulgarian)

[12] W. Liu, L. Junqing, L. Yanlong, L. Yanfang, X. Xiangyang, D. Peng, “Load Sharing Behavior of Double-Pinion Planetary Gear Sets Considering Manufacturing Errors”, 2019 IOP Conf. Ser.: Mater. Sci. Eng. 677 052068, https://doi.org/10.1088/1757-899X/677/5/052068

[13] Z. Naiwei , W. Yuting , S. Chang and D. Zhou, “Single Planetary Mechanism Teeth Matching Conditions”, College of Mechanical Engineering, Jiamusi University, Jiamusi Heilongjiang China (ICAMIA 2015)

[14] K. Arnaudov, S. Petrov, E. Hristov, “Work of the planetary gear trains as differentials and their capabilities”, MATEC Web of Conferences 287, 04001 (2019), https://doi.org/10.1051/matecconf/201928704001

[15] M. Trzesniowski, Rennwagentechnik Grundlagen, Konstruktion, Komponenten, Systeme, Vieweg+ Teubner Verlag 2012, https://doi.org/10.1007/978-3-8348-2209-3

[16] T. Wuzhong, W. Jiangming, Y. Hongzhi, X. Enming, L. Shuangqi, "Dynamic Modeling and Simulation of Double-Planetary Gearbox Based on Bond Graph", Mathematical Problems in Engineering, vol. 2021, Article ID 3964808, 14 pages, 2021. https://doi.org/10.1155/2021/3964808

[17] D. C. Fyler, M. Inalpolat, S. H. Lee, and H. K. Lee, “A Design Framework to Improve the Dynamic Characteristics of Double Planet Planetary Gearsets” in Rotating Machinery, Hybrid Test Methods, Vibro-Acoustics & Laser Vibrometry, Volume 8: Proceedings of the 34th IMAC, A Conference and Exposition on Structural Dynamics 2016, https://doi.org/10.1007/978-3-319-30084-9_37

[18] H. S. Kwon, Investigation of the Influences of Thrust Loads Resulted by Helix Angle Directions of Planetary Gear Sets on Bearing Power Losses in Automotive Planetary Gear Trains, Appl. Sci. 2021, 11, 8827, https://doi.org/10.3390/app11198827

[19] J. Y. Chen, "Assessment of Planetary Gear Train with Multiple-Pinion Arrangements", SAE Int. J. Passenger Cars -Mech. Syst. 7(1):2014, https://doi.org/10.4271/2014-01-1763

[20] S. Avinash, “Influence of Planetary Needle Bearings on the Performance of Single and Double Pinion Planetary Systems”, Journal of Mechanical Design JANUARY 2007, Vol. 129, https://doi.org/10.1115/1.2359472

[21] T. Hiyoshi, Y. Suenaga, H. Yokoyama, Y. Katsumasa, “Development of lightweight and compact differential for Formula 1 car”, Honda R&D Technical Review, 2009

[22] B.j. Simon, “Planetenradträgeranordnung für einen Ravigneaux-Radsatz“, DE102005006649A1. 09. 22.2005.

[23] J. G. Kish. Sikorsky, “Aircraft Advanced Rotorcraft Transmission (ART) Program - Final Report”, NASA CR-191079, NASA Lewis Research Center, Cleveland, OH, 1993.

[24] L. Xianqing, Q. Meng, M. Wensuo, “Study on the Load Sharing Capability of Closed-loop Gear Transmission in Braiding Machine”, MATEC web of Conferences 100, 04012 (2017), https://doi.org/10.1051/matecconf/201710004012

[25] S. Aparna, C. Wani, P. Satav, A. D. Diwate, “Design and Analysis of Portal Axle for SUV”, IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE) e-ISSN: 278-1684, p-ISSN: 2320-334X PP. 71-74. 2018

[26] https://khkgears.net/new/gear_knowledge/gear_technical_reference/gear_systems. Accessed 2.15.2022.

[27] Segade-Robleda, J. Vilán-Vilán, M. López-Lago and E. Casarejos-Ruiz, “Split Torque Gearboxes: Requirements, Performance and Applications”, Mechanical Engineering, IntechOpen, London. https://doi.org/10.5772/37258

[28] Y. Lingling, M. Zhuo, B. Jianqiu , Y Sun, “Non-Linear Dynamic Feature Analysis of a Multiple-Stage Closed-Loop Gear Transmission System for 3D Circular Braiding Machine”, Symmetry 2020, 12, 1788, https://doi.org/10.3390/sym12111788

[29] P. D. Devan, K. M. Senthilkumar & K. K. Arun,” Investigation on Static Stress Analysis of Portal Axle Gearbox”, International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 7 (2018) pp. 5244-5250, https://doi.org/10.37622/IJAER/13.7.2018.5244-5250

[30] D. Anand, A. Barangule, S. Chavan, N. Jadhav, B.D. Patil, “Design and Analysis of Portal Axle of Vehicle”, International Journal of Current Engineering and Technology E-ISSN 2277 – 4106, P-ISSN 2347 – 5161. 2017

[31] T. L. Krantz, “A Method to Analyze and Optimize the Load Sharing Split Path Transmissions”, NASA Technical Memorandum 107201, ARL-TR 1066. 1996.