**Robust DRC-Systems**

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**Abstract** - In this work are proposed, researched and analyzed structures, methods and algorithms for designing of fractional DRC-, DRC.Clegg-, DRC.Clegg.Inverse- robust systems. For their development, inverse solutions of the synthesis problem were used using rational fractional operators from the theory of generalized fractional calculus. Their potential advantages are proven. Results of perturbation analysis for numerical example are presented.

**Index Terms** - inverses models, fractional DRC-, DRC.Clegg-, DRC.Clegg.Inverse-systems, ML-Clegg-differentiation filters, perturbation quality analysis.

I. INTRODUCTION

The structure, methods and algorithms for synthesis of fractional DRC- and DRC.Clegg-control systems (DRC-Disturbance Rejection Control) are known [1 - 4, 8 - 19]. The structure and methods for synthesis of control systems with inverse model of the control plant [6, 7, 15 - 18, 20 - 23] are known, as well. Unlike to the classical ones (Fig. 1), DRC-systems are characterized (Fig. 2) by the presence of an estimator-observer with a nominal model \( G^* \) of the control plant and a fractional order rejecting filter \( F_\beta \), but in DRC.Clegg-systems (Fig. 3) is used an estimator-observer with Clegg-differentiator of fractional order. The denotations used are: control plant \( G(\zeta) \) with a priori known or set nominal model \( G^* \) and with disturbed at the upper limit model \( G^\beta \); basic controller \( R^* \) of integer or fractional order having fixed parameters and structure; set point \( y^* \); control variable \( y \); error \( y \); load \( v \); control \( u \); noise \( f \); reparametrical and structural disturbance \( \zeta \); analog memory key \( M.C., P \), -reset factor; \( \kappa \), -reset-coefficient; \( \varepsilon \), -reset delay. The present article aims to analyze the possibilities for configuring fractional DRC.Clegg-control systems with an inverse model of the control plant, setting itself the tasks to propose: structure of DRC.Clegg.Inverse-systems; methods and algorithms for their synthesis; results of perturbation comparative analysis of the considered systems based on a numerical example.

II. PREPARE CONFIGURATION OF ROBUST DRC.Clegg.INVERSE- SYSTEMS WITH INVERSE MODEL OF THE CONTROL PLANT

A structure of fractional DRC.Clegg.Inverse-systems (Fig.4) proposed, which unlike DRC.Clegg-systems (Fig.3) contains an inverse model (rational function) \( G^\beta = (G^*)^{-1} \) of the nominal model \( G^* \) of the control plant \( G(\zeta) \) and an estimator-observer \( \Phi^{\beta}_{app} \) with fractional Clegg-differentiator \( \Phi^{\beta}_{app} \) and \( \Phi^{\beta}_{app} \).

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The use of \( \Phi^{\beta}_{app} \) determines the proposed DRC.Clegg.Inverse-systems in the class of \( ML \) Reset -systems with a linear analog memory key \( M.C., P \).

III. ANALYTICAL METHODS AND ALGORITHMS FOR SYNTHESIS OF ROBUST DRC.Clegg.INVERSE SYSTEMS USING OPERATORS FROM THE GENERALIZED FRACTIONAL CALCULUS

For control systems with inverse model including DRC.Clegg.Inverse systems (Fig. 4) in the absence of parametrical and structural disturbances (\( \zeta = 0 \) ), the dependences (1), (2) are valid. The design of both \( G^\beta \) and \( \Phi^{\beta}_{app} \) of the robust DRC.Clegg.Inverse system (Fig.4) is proposed to be based on the method [14 - 18] of "double-transferred frequency-limited approximation". The analytical synthesis of a rational inverse \( G^\beta \) filter (3) of order \( \gamma \) (frequency-limited for \( \forall \omega \in [\omega_m, \omega_c] \) approximating the inverse model \( G^\beta \) of the control plant \( G^* \) uses algorithm (4), (5) [14 - 18] for criterion (2) where \( \omega_m \) is the cut frequency of \( G^* \). \( \omega_c \) is the unit frequency of \( G^\gamma \), and (6) - (9) are requirements to the parameters of (3).

The analytical synthesis of an estimator-observer \( \Phi^{\beta}_{app} \) in DRC.Clegg.Inverse (Fig.4) consists of the design of a frequency-limited for \( \forall \omega \in [\omega_m, \omega_c] \) fractional differentiator \( \Phi^{\beta} \) (10) of row \( \beta \) (11), approximating the inverse function of \( \Phi^{\beta} \) [14 - 18] on algorithm (11), (12), under the criterion "minimum deviation from the nominal trajectory" (17), where \( \omega_{m^\beta} \) is the cut frequency of \( \Phi^{\beta} \). \( \omega_{c^\beta} \) is the unit frequency of \( \Phi^{\beta} \), and (13) - (16) are requirements to the parameters of (10).

IV. NUMERICAL EXAMPLE

The proposed methods and algorithms (3) - (17) are used in the design of DRC.Clegg.Inverse systems (Fig. 4) for control of an exemplary plant presented (18) - (20) by previously known nominal model \( G^* \) of \( G(\zeta) \) and disturbed at the upper limit model \( G^\beta \) of \( G(\zeta) \). The systems (Fig. 1 - Fig. 4), configured parametrically based on the solutions (21) - (26) in the numerical example, are modeled and simulated in parallel. The fulfillment of the requirements of criterion (2) in the synthesis of \( G^\beta_{app} \) is proved by the frequency characteristics \( G^\beta \), \( G^* \), \( (G^* G^\gamma) \) on Fig.5, Fig.6. The fulfillment of the requirements of criterion (17) in the synthesis of \( \Phi^{\beta}_{app} \) is proved on Fig. 7, Fig. 8 using comparative estimations of and deviation \( \varepsilon_{\beta_{app}} \) and \( \varepsilon_{\beta_{app}} \) using the criteria (17) and (18).

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\begin{align*}
G^*(p) &= k^*(T^*p+1)^{-1}
end{align*}
\]
\[ G^*(p) = k^* \left( T^* p + 1 \right)^{e^{-\tau^*}} = 80.723 \left( 4 p + 1 \right)^{e^{-0.2 p + 100 p^2}} \cdot \left( k^* = 80.7235, \tau^* = 20 \right. \];

\[ 0.7235 \leq k^* \leq 77.07235; \; 3, \text{sec} \leq \tau^* \leq 20, \text{sec} \];

\[ R^*(p) = k^* \left( T^* p + 1 \right)^{e^{-\tau^*}}; \; \left( k^* = 0.50; \; T^* = 3 \text{sec} \right); \; R^* \ll_{k^*, \tau^*} \; G^* \];

\[ G^{D^*} \left( j \omega \right) = \frac{1 + \left( \omega \right)^{e^{-\tau^*}}}{1 + \left( \omega \right)^{e^{-\tau^*}}}; \]
\[ i^{\text{reg}}(\tau) < i^{\text{tool}}(\tau) < i^{*}(\tau); \]
\[ g\mathcal{M}^{\text{reg}}(\zeta) >> g\mathcal{M}^*(\zeta) >> g\mathcal{M}^{\text{tool}}(\zeta); \]
\[ \varphi\mathcal{M}^{\text{reg}}(\zeta) >> \varphi\mathcal{M}^*(\zeta) >> \varphi\mathcal{M}^{\text{tool}}(\zeta). \]

(27)
(28)
(29)

V. NUMERICAL EXAMPLE PERFORMANCE ANALYSIS

For the quality indicators determining and aiming perturbation comparative analysis, the models of the synthesized systems (Fig. 2, Fig. 3, Fig. 4) are simulated in parallel for the range of parametrical fluctuations (20). The results illustrated on Fig. 9 - Fig. 12 allow to determine the ratios (27) - (29) in the quantitative indicators of quality – time of regulation and margins of stability.

VI. CONCLUSION

The new and original in the present article are:

- structure of robust \texttt{DRC.Clegg.Inverse} systems (Fig. 4) with fractional inverse model of the control plant and estimator-observer with fractional \texttt{Clegg}-differentiator;
- method and algorithm for analytical synthesis of \texttt{DRC.Clegg.Inverse} control systems;
- proven and determined superiority of the quality indicators of robust \texttt{DRC.Clegg.Inverse} systems in comparison with the indicators of \texttt{DRC.Clegg} of DRC systems in parametrically disturbed mode.

REFERENCES


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