



Original articles

I-Delaporte process and applications

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Received 27 November 2014; received in revised form 6 October 2015; accepted 15 December 2015

Available online 6 January 2016

Abstract

In this paper we introduce a mixed Pólya–Aeppli process with shifted gamma mixing distribution and call it an Inflated-parameter Delaporte process (I-Delaporte process). We derive the probability mass function, moments and some basic properties. Then we define the process as a pure birth process and derive differential equations for the probabilities. As application, we consider a risk model in which the claim counting process is the defined I-Delaporte process. For the defined risk model we derive the joint distribution of the time to ruin and the deficit at ruin as well as the ruin probability. We discuss in detail the particular case of exponentially distributed claims.

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Keywords: Mixed distributions; Pure birth process; Delaporte process; Ruin probability

1. Introduction

The Delaporte process was introduced by Grandell (1997), [5] as a mixed Poisson process, where the parameter λ has a shifted gamma distribution with density

$$g(\lambda) = \frac{\beta^r}{\Gamma(r)} (\lambda - \alpha)^{r-1} e^{-\beta(\lambda-\alpha)}, \quad \beta > 0, \lambda > \alpha. \quad (1)$$

The parameter α can be interpreted as a basic risk, see [5]. The resulting process is a sum of Poisson process and a process with negative binomial distribution, independent of the Poisson process. In a similar way, in this paper we introduce a generalization of the Delaporte process as a mixed Pólya–Aeppli process. The corresponding generalization of the Delaporte distribution was defined in Minkova (2012), [10] and is called I-Delaporte distribution. The process defined in this paper is a sum of two independent compound processes: Pólya–Aeppli process and compound Negative binomial process.

In many cases in the risk model the counting process is the Poisson process. The corresponding risk model is called a classical risk model. There are many directions in which the classical risk model is generalized in order to become

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a reasonably realistic description. One direction of generalization is by compounding the Poisson process, see for example Minkova (2004), [8], where the Pólya–Aeppli process was defined. Another direction of generalization is by mixing. In Minkova (2011), [9] a mixed Pólya–Aeppli process with Gamma mixing distribution was introduced and the resulting process, called I-Pólya process was applied in analyzing the risk model. Recently, in many papers, the authors introduce some dependence in the model including copula also, see for example Albrecher et al., (2011) [1], Boudreault et al., [3] and Cossette (2008), [4]. In this paper we consider a mixed Pólya–Aeppli process with shifted Gamma mixing distribution. Then we apply the resulting process as a counting process in the risk model.

In Section 2 we give the definition of the I-Delaporte process with probability mass function, moments and interpretation. In Section 3 the process is defined as a pure birth process. As application, in Section 4 we consider a risk model with I-Delaporte counting process, ruin probability and particular case of exponentially distributed claims.

2. Definition

Suppose that for a given $\lambda > 0$, the number of items in the time interval $[0, t]$ has a Pólya–Aeppli distribution, i.e.,

$$P(N(t) = k | \lambda) = \begin{cases} e^{-\lambda t}, & k = 0, \\ e^{-\lambda t} \sum_{i=1}^k \binom{k-1}{i-1} \frac{[\lambda(1-\rho)t]^i}{i!} \rho^{k-i}, & k = 1, 2, \dots, \end{cases} \tag{2}$$

where λ and $\rho \in [0, 1)$ are parameters. We use the notation $N(t) \sim PA(\lambda t, \rho)$. The mean number of arrivals and the variance are given by $E(N(t) | \lambda) = \frac{\lambda}{1-\rho}t$ and $\text{Var}(N(t) | \lambda) = \frac{\lambda(1+\rho)}{(1-\rho)^2}$. Then, for the Fisher index of dispersion we obtain $FI(N(t)) = \frac{\text{Var}(N(t))}{E(N(t))} = \frac{1+\rho}{1-\rho}$, i.e., Pólya–Aeppli process is over-dispersed with respect to the Poisson process. The probability generating function (PGF) is given by

$$\psi_{N(t)|\lambda}(s) = e^{-\lambda t(1-\psi_1(s))},$$

where

$$\psi_1(s) = Es^X = \frac{(1-\rho)s}{1-\rho s} \tag{3}$$

is the probability generating function of the geometric distribution, $Ge_1(1-\rho)$.

Suppose that the parameter λ has a shifted gamma distribution, given in (1). Then, the unconditional probability mass function (PMF) is given by

$$P(N(t) = m) = \int_{\alpha}^{\infty} P(N_t = m | \lambda) \frac{\beta^r}{\Gamma(r)} (\lambda - \alpha)^{r-1} e^{-\beta(\lambda-\alpha)} d\lambda. \tag{4}$$

Calculating the integral in (4) gives the following probability mass function

$$P(N(t) = m) = \begin{cases} e^{-\alpha t} \left(\frac{\beta}{\beta+t}\right)^r, & m = 0 \\ e^{-\alpha t} \left(\frac{\beta}{\beta+t}\right)^r \left[r(1-\rho) \frac{t}{\beta+t} + \alpha(1-\rho)t \right], & m = 1 \\ e^{-\alpha t} \left(\frac{\beta}{\beta+t}\right)^r \left[\sum_{i=1}^m \binom{m-1}{i-1} \binom{r+i-1}{i} \left((1-\rho) \frac{t}{\beta+t} \right)^i \rho^{m-i} \right. \\ \quad \left. + \sum_{i=1}^m \binom{m-1}{i-1} \frac{[\alpha(1-\rho)t]^i}{i!} \rho^{m-i} + \sum_{i=1}^{m-1} \sum_{j=1}^i \binom{i-1}{j-1} \frac{[\alpha(1-\rho)t]^j}{j!} \rho^{i-j} \right. \\ \quad \left. \times \sum_{k=1}^{m-i} \binom{m-i-1}{k-1} \binom{r+k-1}{k} \left((1-\rho) \frac{t}{\beta+t} \right)^k \rho^{m-i-k} \right], & m = 2, 3, \dots \end{cases}$$

From the PMF it follows that the process $N(t)$ is a sum of two independent processes, $N(t) = N_1(t) + N_2(t)$, where $N_2(t) \sim PA(\alpha t, \rho)$, and $N_1(t)$ has a PMF

$$P(N_1(t) = m) = \begin{cases} \left(\frac{\beta}{\beta+t}\right)^r, & m = 0 \\ \left(\frac{\beta}{\beta+t}\right)^r \sum_{i=1}^m \binom{m-1}{i-1} \binom{r+i-1}{i} \left[(1-\rho)\frac{t}{\beta+t}\right]^i \rho^{m-i}, & m = 1, 2, \dots \end{cases} \tag{5}$$

The process in (5) has an Inflated-parameter negative binomial distribution with parameters $\frac{\beta}{\beta+t}$, ρ and r , see Minkova (2002), [7]. We say shortly I-Negative binomial and use the notation $N_1(t) \sim INB(\frac{\beta}{\beta+t}, \rho, r)$. For the process $N_1(t)$, called also I-Pólya process, and application in risk models see Minkova (2011), [9]. The PGF of $N_1(t)$ is given by

$$\psi_{N_1(t)}(s) = \left[\frac{\pi}{1 - (1 - \pi)\psi_1(s)} \right]^r,$$

where $\pi = \frac{\beta}{\beta+t}$ and $\psi_1(s)$ is given by (3). The mean and the variance of $N_1(t)$ are given by

$$E(N_1(t)) = \frac{rt}{\beta(1-\rho)} \quad \text{and} \quad \text{Var}(N_1(t)) = \frac{r[t + \beta(1 + \rho)]t}{\beta^2(1-\rho)^2}.$$

For the Fisher index of dispersion we obtain

$$FI(N_1(t)) = \frac{1 + \rho}{1 - \rho} + \frac{t}{\beta(1 - \rho)},$$

i.e. the I-negative binomial process is over-dispersed related to Pólya–Aeppli process.

Then, the PGF of $N(t)$ is then given by

$$\psi_{N(t)}(s) = \left[\frac{\pi}{1 - (1 - \pi)\psi_1(s)} \right]^r e^{-\alpha t(1-\psi_1(s))}. \tag{6}$$

Remark 1. In the case of $\rho = 0$, the distribution of $N(t)$ simplifies to the Delaporte distribution, Grandell (1997), [5]. In this case it is a composition of independent negative binomial and Poisson distributions. In a similar way, the non-central negative binomial distribution (NNBD), introduced by Ong and Lee (1979), [11], is a composition of independent negative binomial and Pólya–Aeppli distribution.

This motivated us to say that the process $N(t)$ with PGF (6), has an Inflated-parameter Delaporte distribution or I-Delaporte distribution, and to give the following definition.

Definition 1. The process $N(t)$, defined by the PGF (6), is called an Inflated-parameter Delaporte process (I-Delaporte process).

Interpretation: Let us denote by $N_0(t)$ the Delaporte process, given in Grandell (1997), [5]. Then, the process $N(t)$ can be represented as a compound Delaporte process, i.e., $N(t) = X_1 + \dots + X_{N_0(t)}$, where X_i , $i = 1, 2, \dots$ are independent, geometrically distributed with success probability $1 - \rho$, and PGF given in (3), independent of $N_0(t)$.

Moments: Let us rewrite the PGF in (6) as

$$\psi_{N(t)}(s) = \left[\frac{\pi(1-\rho s)}{1 - (1 - \pi(1-\rho))s} \right]^r e^{-\alpha t \left(1 - \frac{(1-\rho)s}{1-\rho s}\right)}. \tag{7}$$

By differentiation in (7) we obtain the mean and the variance of the I-Delaporte process

$$EN(t) = \left(\alpha + \frac{r}{\beta}\right) \frac{t}{1-\rho}$$

and

$$\text{Var}(N(t)) = \left[\alpha(1 + \rho) + \frac{r((1 + \rho)\beta + t)}{\beta^2} \right] \frac{t}{(1 - \rho)^2}.$$

For the Fisher index we obtain

$$FI(N(t)) = \frac{1 + \rho}{1 - \rho} + \frac{rt}{(1 - \rho)\beta^2 \left(\alpha + \frac{r}{\beta} \right)},$$

i.e., I-Delaporte process is over-dispersed related to Pólya–Aeppli process.

3. I-Delaporte process as a pure birth process

In this section we define the I-Delaporte process as a pure birth process. Suppose that $N(t)$, $t \geq 0$ is the number of times a certain event occurs in time interval $(0, t]$. The transition probabilities of the counting process $N(t)$, for every $m = 0, 1, \dots$ are specified by the following postulates:

$$P(N(t+h) = n \mid N(t) = m) = \begin{cases} 1 - \left(\alpha + \frac{r}{\beta+t} \right) h + o(h), & n = m, \\ (1 - \rho) \left[\alpha \rho^{k-1} + \frac{r}{\beta} \left(\frac{\beta}{\beta+t} \right)^2 \left(1 - (1 - \rho) \frac{\beta}{\beta+t} \right)^{k-1} \right] h + o(h), & n = m+k, k = 1, 2, \dots, \end{cases}$$

where $o(h) \rightarrow 0$ as $h \rightarrow 0$. Denote the probabilities of $N(t)$ by $P_m(t) = P(N(t) = m)$, $m = 0, 1, 2, \dots$. Then the above postulates yield the following Kolmogorov forward equations:

$$\begin{aligned} P_0'(t) &= - \left(\alpha + \frac{r}{\beta+t} \right) P_0(t), \\ P_m'(t) &= - \left(\alpha + \frac{r}{\beta+t} \right) P_m(t) \\ &\quad + (1 - \rho) \sum_{k=1}^m \left[\alpha \rho^{k-1} + \frac{r}{\beta} \left(\frac{\beta}{\beta+t} \right)^2 \left(1 - (1 - \rho) \frac{\beta}{\beta+t} \right)^{k-1} \right] P_{m-k}(t), \quad m = 1, 2, \dots \end{aligned} \quad (8)$$

with initial conditions

$$P_0(0) = 1 \quad \text{and} \quad P_m(0) = 0, \quad m = 1, 2, \dots \quad (9)$$

From the equations of (8) we get the following differential equation for the PGF of $N(t)$

$$\frac{\partial \psi_{N(t)}(s)}{\partial t} = - \left[\alpha + \frac{r}{\beta + t(1 - \psi_1(s))} \right] (1 - \psi_1(s)) \psi_{N(t)}(s). \quad (10)$$

The solution of (10) with the initial condition $\psi_{N(t)}(1) = 1$ is given by (6). This leads to the second definition.

Definition 2. The stochastic process, defined by the differential equations (8) with initial conditions (9) is called I-Delaporte process.

4. Application to risk models

As application of the I-Delaporte process, we consider the standard risk model $\{X(t), t \geq 0\}$, defined on the complete probability space (Ω, \mathcal{F}, P) and given by

$$X(t) = ct - \sum_{i=1}^{N(t)} Z_i, \quad \left(\sum_1^0 = 0 \right). \quad (11)$$

Here c is a positive real constant representing the risk premium rate. The sequence $\{Z_i\}_{i=1}^\infty$ of non-negative i.i.d. random variables is independent of the counting process $N(t)$, $t \geq 0$, and represents the claim sizes to the insurance company. The claim sizes $\{Z_i\}_{i=1}^\infty$ are distributed as the random variable Z with distribution function F , $F(0) = 0$ and mean μ .

We consider the risk model (11), where $N(t)$ is an I-Delaporte process and we call this process I-Delaporte risk model. The interpretation of the counting process is the following. Suppose that the successive claims are of two types, such that the first type of claims are counted by the INB process and the counting process of the second type of claims is the Pólya–Aeppli process. Our interest is in counting all the claims in total. Then, the number of claims has an I-Delaporte distribution.

Let $\tau = \inf\{t : X(t) < -u\}$ with the convention of $\inf \emptyset = \infty$ be the time to ruin of an insurance company having initial capital $u \geq 0$. We denote by $\Psi(u) = P(\tau < \infty)$ the ruin probability and by $\Phi(u) = 1 - \Psi(u)$ the non-ruin probability. The relative safety loading θ for the I-Delaporte risk model in (11), is given by

$$\theta = \frac{EX(t)}{E \sum_{i=1}^{N(t)} Z_i} = \frac{c(1 - \rho)}{\mu \left(\alpha + \frac{r}{\beta} \right)} - 1.$$

If the safety loading $\theta < 0$, then $\Psi(u) = 1$ for all u , see Asmussen and Albrecher (2010), Proposition 1.1, [2]. If $\theta > 0$, then $\Psi(u) < 1$ for all sufficiently large u . In the case of positive safety loading $\theta > 0$, the premium income per unit time c should satisfy the following inequality

$$c > \frac{\mu}{1 - \rho} \left(\alpha + \frac{r}{\beta} \right).$$

The main in the application is to analyze for this model the joint probability distribution $G(u, y)$ of the time to ruin τ and the deficit at the time of ruin $D = |u + X(\tau)|$. The function $G(u, y)$ is given by

$$G(u, y) = P(\tau < \infty, D \leq y), \quad y \geq 0, \tag{12}$$

see Klugman et al. (2004), [6].

It is clear that

$$\lim_{y \rightarrow \infty} G(u, y) = \Psi(u). \tag{13}$$

Using the postulates, we get

$$G(u, y) = \left[1 - \left(\alpha + \frac{r}{\beta + t} \right) h \right] G(u + ch, y) + (1 - \rho) \sum_{k=1}^\infty \left[\alpha \rho^{k-1} + \frac{r}{\beta} \left(\frac{\beta}{\beta + t} \right)^2 \left(1 - (1 - \rho) \frac{\beta}{\beta + t} \right)^{k-1} \right] h \times \left[\int_0^{u+ch} G(u + ch - x, y) dF^{*k}(x) + \left(F^{*k}(u + ch + y) - F^{*k}(u + ch) \right) \right],$$

where $F^{*k}(x)$, $i = 1, 2, \dots$ is the distribution function of $Z_1 + Z_2 + \dots + Z_k$. Rearranging the terms leads to

$$\begin{aligned} \frac{G(u + ch, y) - G(u, y)}{ch} &= \left(\alpha + \frac{r}{\beta + t} \right) \frac{1}{c} G(u + ch, y) \\ &- \frac{1 - \rho}{c} \sum_{k=1}^\infty \left[\alpha \rho^{k-1} + \frac{r}{\beta} \left(\frac{\beta}{\beta + t} \right)^2 \left(1 - (1 - \rho) \frac{\beta}{\beta + t} \right)^{k-1} \right] \\ &\times \left[\int_0^{u+ch} G(u + ch - x, y) dF^{*k}(x) + F^{*k}(u + ch + y) - F^{*k}(u + ch) \right]. \end{aligned} \tag{14}$$

Let us denote by

$$H(x) = \frac{1 - \rho}{\alpha + \frac{r}{\beta + t}} \sum_{k=1}^\infty \left[\alpha \rho^{k-1} + \frac{r}{\beta} \left(\frac{\beta}{\beta + t} \right)^2 \left(1 - (1 - \rho) \frac{\beta}{\beta + t} \right)^{k-1} \right] F^{*k}(x), \tag{15}$$

the probability distribution function of the aggregated claims. It follows from (15), that $H(0) = 0$ and $H(\infty) = 1$, i.e., $H(x)$ is a proper distribution function. By letting $h \rightarrow 0$ in (14), we obtain the following differential equation

$$\frac{\partial G(u, y)}{\partial u} = \left(\alpha + \frac{r}{\beta + t} \right) \frac{1}{c} \left[G(u, y) - \int_0^u G(u - x, y) dH(x) - [H(u + y) - H(u)] \right]. \quad (16)$$

In the following theorem we obtain the initial condition.

Theorem 1. *The function $G(0, y)$ is given by*

$$G(0, y) = \left(\alpha + \frac{r}{\beta + t} \right) \frac{1}{c} \int_0^y [1 - H(u)] du. \quad (17)$$

Proof. Integrating (16) from 0 to ∞ with $G(\infty, y) = 0$ leads to

$$\begin{aligned} -G(0, y) &= \left(\alpha + \frac{r}{\beta + t} \right) \frac{1}{c} \\ &\times \left[\int_0^\infty G(u, y) du - \int_0^\infty \int_0^u G(u - x, y) dH(x) du - \int_0^\infty (H(u + y) - H(u)) du \right]. \end{aligned}$$

The change of variables in the double integral and simple calculations yield to

$$G(0, y) = \left(\alpha + \frac{r}{\beta + t} \right) \frac{1}{c} \int_0^\infty [H(u + y) - H(u)] du$$

and then (17). \square

4.1. Ruin probability

Theorem 2. *For $u \geq 0$, the ruin probability $\Psi(u)$ satisfies the equation*

$$\frac{\partial \Psi(u)}{\partial u} = \left(\alpha + \frac{r}{\beta + t} \right) \frac{1}{c} \left[\Psi(u) - \int_0^u \Psi(u - x) dH(x) - [1 - H(u)] \right]. \quad (18)$$

Proof. The result follows from (16) and (13). \square

Remark 2. The nonruin probability $\Phi(u)$ satisfies the equation

$$\frac{\partial \Phi(u)}{\partial u} = \left(\alpha + \frac{r}{\beta + t} \right) \frac{1}{c} \left[\Phi(u) - \int_0^u \Phi(u - x) dH(x) \right]. \quad (19)$$

Theorem 3. *The ruin probability with no initial capital is given by*

$$\Psi(0) = \frac{\mu}{c(1 - \rho)} \left(\alpha + \frac{r}{\beta} \right). \quad (20)$$

Proof. According to (13) and (17) we get

$$\Psi(0) = \lim_{y \rightarrow \infty} G(0, y) = \left(\alpha + \frac{r}{\beta + t} \right) \frac{1}{c} \int_0^\infty [1 - H(u)] du.$$

If X is a random variable with distribution function $H(x)$, then, by the definition of $H(x)$ and $EZ = \mu$ we obtain

$$EX = \frac{\mu \left(\alpha + \frac{r}{\beta} \right)}{(1 - \rho) \left(\alpha + \frac{r}{\beta + t} \right)}.$$

Using the fact that $EX = \int_0^\infty [1 - H(x)] dx$ we obtain (20). \square

4.2. Exponentially distributed claims

Let us consider the case of exponentially distributed claim sizes with mean μ , i.e. $F(x) = 1 - e^{-\frac{x}{\mu}}$, $x \geq 0$, $\mu > 0$. In this case, for the distribution function $H(x)$ we obtain

$$H(x) = 1 - \frac{\alpha}{\alpha + \frac{r}{\beta+t}} e^{-\frac{1-\rho}{\mu}x} - \frac{\frac{r}{\beta+t}}{\alpha + \frac{r}{\beta+t}} e^{-\frac{1-\rho}{\mu} \frac{\beta}{\beta+t}x},$$

i.e. it is a mixer of two exponentially distributed variables. For the function $G(0, y)$ in (17) we obtain

$$G(0, y) = \frac{\mu}{(1-\rho)c} \left[\alpha \left(1 - e^{-\frac{1-\rho}{\mu}y} \right) + \frac{r}{\beta} \left(1 - e^{-\frac{1-\rho}{\mu} \frac{\beta}{\beta+t}y} \right) \right].$$

By differentiation of (18), in the case of exponentially distributed claims, we obtain the following differential equation for the ruin probability.

$$\begin{aligned} \frac{\partial^3 \Psi(u)}{\partial u^3} - \left[\left(\alpha + \frac{r}{\beta+t} \right) \frac{1}{c} - \frac{1-\rho}{\mu} \left(1 + \frac{\beta}{\beta+t} \right) \right] \frac{\partial^2 \Psi(u)}{\partial u^2} \\ - \frac{1-\rho}{\mu} \left[\left(\alpha + \frac{r}{\beta+t} \right) \frac{1}{c} - \frac{\alpha}{c} \frac{t}{\beta+t} - \frac{1-\rho}{\mu} \frac{\beta}{\beta+t} \right] \frac{\partial \Psi(u)}{\partial u} = 0. \end{aligned} \quad (21)$$

The characteristic equation of (21) has a root zero and two negative roots. The solution has the form $\Psi(u) = C_1 + C_2 e^{R_1 u} + C_3 e^{R_2 u}$, where R_1 and R_2 are the nonzero roots of the characteristic equation and C_i , $i = 1, 2, 3$ are constants. According to the condition $\Psi(\infty) = 0$, it follows that $C_1 = 0$ and ruin probability is given by

$$\Psi(u) = C_2 e^{R_1 u} + C_3 e^{R_2 u},$$

where $C_2 + C_3 = \frac{\mu}{c(1-\rho)} \left(\alpha + \frac{r}{\beta} \right)$.

5. Concluding remarks

In this paper we have defined a risk model with I-Delaporte counting process, which gives a possibility to count two types of claims. The model is a generalization of the classical risk model, the Pólya–Aeppli risk model and the I-Pólya risk model.

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