NUMERICAL COMPARATIVE ANALYSIS ON THE CLASSICAL VASICEK MODEL FOR DETERMING THE ZERO COUPON BOND'S PRICE

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ABSTRACT: *In finance theory different models aiming to prognose the price of a given bond are considered. The finance modeling constructs an abstract introduction of a model that can describe the real world financial condition. This is a mathematical model that can be used to introduce a simplified version of the extension of the financial assets or a portfolio of business project or another investment. Usually the finance modeling is concerned with the exercising of the assets' prices or cooperative finances of a quantative nature. In the present paper we consider the classical Vasicek model without a risk factor and the classical Vasicek model with a risk factor. We give three numerical experiments for determing the zero coupon bond's price. The first one takes into consideration that the market price of risk is not changing. The other experiments are made for different values of the market price of risk.*

KEYWORDS: *finance modeling, interest rates, bond's price, zero coupon, stochastic differential equations, partial differential equations*

1 Introduction

In finance Vasicek's model is a mathematical model which is describing the random movement of the interest rates. The most universal model is the single-factor short-term model which is describing the random movement of the interest rate influenced by one source of a market risk. Oldrich Vasicek, see Vasicek (1977), [7] introduced this model as a stochastic investment model. In the classical Vasicek model the movement of the interest rate is given by a stochastic differential equation. i.e it is defined by a diffusion process. The assumption that is used in the model is that the bond market is a non-arbitrary market and that there exists a risk-free portfolio with a quantity W. The arbitrary argument is similar to that used in the Black-Scholes model for determing the put and call option's pricing. The main purpose in the classical Vasicek model is to determine the zero coupon bond's price at time t with maturity T. As this price can changes in time quite accidently it can be interpreted as a stochastic process for which the Ito's lemma can be applied.

In this paper we consider two financial models. The first one is the classical Vasicek model without a risk factor and the second one is the classical Vasicek model with a risk factor. For these two models we consider the zero coupon bond's price which is determined by using the apparatus of the stochastic differential equations and the partial differential equations. As there is a connection between the stochastic differential equations and the partial differential equations and it is obvious from the Fayman-Kac's theorem, see Oksendal (1998), [4] it is a good way to determine the final bond's price. We introduce a numerical comparative analysis and give some essential results. The analysis is made thanks to the partial differential equations given in the models with a specific boundary condition.

In section 2 we consider the classical Vasicek model. For this model we pay a great attention on the market price of risk and the value of the zero coupon bond. A numerical analysis and some comparative characteristics are made in Section 3. Some concluding remarks are given in Section 4.

2 A classical Vasicek model

The main application of the classical Vasicek model, see Nazil M. N. (2009), [3] in finance is to determine the zero coupon bond's price using the fact that the interest rate is following a stochastic differential equation. Usually this model is used for a valuation of interest-rate derivatives and it is also adapted to the credit markets. It is one of the earliest and the simplest term structure models based on increasingly realistic assumptions about the random movement of the interest rates.

Let r_0, , $\alpha > 0$, $\sigma > 0$ and μ are real constants. In the classical Vasicek model the interest rate is given by:

(1) $dr(t) = \alpha(\mu - r)dt + \sigma dW_t$ with an initial condition $r(0) = r_0$,

where

r - the instantaneous short rate interest

α - the speed of mean reversion

μ - the long-run expected value for the interest rate r

σ - the instantaneous standard deviation of the interest rate r

W_t - a standard Wiener process with mean 0 and standard deviation 1

The stochastic process used by Vasicek is known as the Ornstein-Uhlenbesk process. The first term in the stochastic process proposed by Vasicek pulls the short rate *r* back towards μ. So μ can be thought of as the long-run level of the short rate. When the short rate *r* is above μ, the first term trends to pull *r* downward since α is assumed to be positive. When the short rate *r* is below μ then *r* trends to drift upward. The impact of the mean reversion is to create realistic interest rate cycles with the level of α determing the length and the violence of rises and falls in interest rates.

The expression $\alpha(\mu - r)$ in the classical Vasicek model is called a drift. It is the expected moment change in the interest rate in time *t*. The constant α is called a rate of drift adjustment and the constant μ a level of drifting. The definition a level of drifting comes from the fact that the drift $\alpha(\mu - r)$ depends on that if $r > \mu$ or $r < \mu$. If the inequality $r > \mu$ holds then the drift is negative as the constant α is always a non-negative constant. In this situation the trend of the interest rate is moving down to the equilibrium. Conversely if the inequality $r > \mu$ holds then the drift is positive the trend of the interest rate is moving up to the equilibrium.

Let $P(r,t,T)$ be the price value of a zero coupon bond at time *t* and maturity *T*. The price depends from the interest rate movement and also the market value of the price reflects to the market expectation of the interest rate in the future. By Ito's lemma we obtain the following movement in the price of a zero coupon bond:

(2) $$dP(r,t,T) = P_r dr + \frac{1}{2} P_{rr} (dr)^2 + P_t dt,$$

where $P_r = \dfrac{\partial P(r,t,T)}{\partial r}$, $P_t = \dfrac{\partial P(r,t,T)}{\partial t}$ and $P_{rr} = \dfrac{\partial^2 P(r,t,T)}{\partial r^2}$.

Taking into account that $dr(t) = \alpha(\mu - r)dt + \sigma dW_t$ and substituting in (2) we obtain the following equation for the zero coupon bond's price:

(3) $$dP = P_r[\alpha(\mu - r)dt + \sigma dW_t] + \frac{1}{2}P_{rr}\sigma^2 dt + P_t dt$$

The calculations are obtained thanks to the following differential table:

X	dW_t	dt
dW_t	dt	0
dt	0	0

i.e $(dW_t)^2 = dt,\ dW_t.dt = 0$ and $dt.dt = 0$, see Stoyanov (1978) [6].

In the further exposition of the model is extremely important to find a formula for the zero coupon bond's price $P(r,t,T)$. The realization of this task is achieved by the assumption of that the bond market is non-arbitrary. Let on this market an investor forms a portfolio with a quantity *W* which includes a unit quantity of a zero coupon bond 1 with maturity T_1 and a quantity *w* of a zero coupon bond 2 with a maturity T_2. Then the portfolio's value is given by:

(4) $$W = P_1 + w.P_2,$$

where P_1 is the zero coupon bond's price of the first portfolio's bond and P_2 is the zero coupon bond's price of the second portfolio's bond. Then the change of the portfolio's value in time is given by:

(5) $$dW = dP_1 + w.dP_2,$$

We write the Ito's formula separately for the differentials dP_1, dP_2 and obtain the following partial differential equations:

$$dP_1 = P_{1_r} dr + \frac{1}{2}P_{1_{rr}}(dr)^2 + P_{1_t} dt = P_{1_r}[\alpha(\mu - r)dt + \sigma dW_t] + \frac{1}{2}P_{1_{rr}}\sigma^2 dt + P_{1_t} dt$$
$$dP_2 = P_{2_r} dr + \frac{1}{2}P_{2_{rr}}(dr)^2 + P_{2_t} dt = P_{2_r}\left[\alpha(\mu - r)dt + \sigma dW_t\right] + \frac{1}{2}P_{2_{rr}}\sigma^2 dt + P_{2_t} dt$$

We substitute the obtained values of dP_1 and dP_2 in the stochastic differential equation (5) and after that we obtain the following equation:

(6) $$dW = \left[\frac{1}{2}P_{1_{rr}}\sigma^2 + P_{1_r}\alpha(\mu - r) + P_{1_t} + wP_{2_t}\right]dt + w\left[\frac{1}{2}P_{2_{rr}}\sigma^2 + P_{2_r}\alpha(\mu - r)\right]dt + \left(P_{1_r} + wP_{2_r}\right)\sigma dW_t$$

If the coefficient in front of the differential dW_t is a zero i.e. $P_{1_r} + w.P_{2_r} = 0$ or $w = -\frac{P_{1_r}}{P_{2_r}}$, then we say that on the market is forming a riskless portfolio. Once the risk has been removed the change of the portfolio's value in time is given by:

(7) $$dW = \left(rP_1 + rwP_2\right)dt$$

Equating the right sides of the equations (6) and (7) and substituting by $w = -\frac{P_{1_r}}{P_{2_r}}$ we obtain the following equation:

(8) $$rP_1 - \frac{P_{1_r}}{P_{2_r}}P_2 r = \frac{1}{2}P_{1_{rr}}\sigma^2 + P_{1_r}\alpha(\mu - r) + P_{1_t} - \frac{P_{1_r}}{P_{2_r}}.P_{2_t} - \frac{1}{2}\sigma^2\frac{P_{1_r}}{P_{2_r}}.P_{2_{rr}} - \frac{P_{1_r}}{P_{2_r}}.P_{2_r}\alpha(\mu - r)$$

After simplifying and dividing by $P_{1_r}.\sigma$ we obtain the following equality:

(9) $$\frac{\frac{1}{2}\sigma^2 P_{1_{rr}} + \alpha(\mu - r)P_{1_r} + P_{1_t} - rP_1}{\sigma P_{1_r}} = \frac{\frac{1}{2}\sigma^2 P_{2_{rr}} + \alpha(\mu - r)P_{2_r} + P_{2_t} - rP_2}{\sigma P_{2_r}} = -\lambda$$

i.e. the market price of risk is given by:

$$-\lambda = \frac{\frac{1}{2}\sigma^2 P_{rr} + \alpha(\mu - r)P_r + P_t - rP}{\sigma P_r}$$

The market price of risk λ is assumed to be constant. From the last equation we obtain the partial differential equation of the zero coupon bond's price for the classical Vasicek model of the form:

(10) $\frac{1}{2}\sigma^2 P_{rr} + [\alpha(\mu - r) + \lambda\sigma]P_r + P_t - rP = 0$ with boundary condition

(11) $P(r,T,T) = 1$

i.e the zero coupon bond's price at maturity equals its principal amount, 1.

The price value of a zero coupon bond in the classical Vasicek model is analytically found, see Donald R. van Deventer et. al (1997), [1] and is given by the following formula:

(12) $$P(r,t,T) = P(r,\tau) = e^{-rF(\tau) - G(\tau)},$$

where $F(t,T)=F(\tau)=\frac{1}{\alpha}\left(1-e^{-\alpha\tau}\right)$ and $G(t,T)=G(\tau)=\left[\mu+\frac{\lambda\sigma}{\alpha}-\frac{\sigma^2}{2\alpha^2}\right]\left[\tau-F(\tau)\right]+\frac{\sigma^2}{4\alpha}F^2(\tau)$ are functions of the remaining time to maturity $T-t=\tau$.

The stochastic process proposed by Vasicek, see Donald R. van Deventer et al. (1997), [1] allows to calculate the expected value and the variance of the short rate at any time in the future s from the perspective of the current time t. Denoting the short rate at time t by $r(t)$, the expected value of the short rate at future time s is as follows:

$$(13) \qquad E_t\left[r(s)\right]=\mu+\left[r(t)-\mu\right]e^{-\alpha(s-t)}$$

The standard deviation of the potential values of the interest rate r around this mean value is:

$$(14) \qquad \text{Standard deviation}_t\left[r(s)\right]=\sqrt{\frac{\sigma^2}{2\alpha}\left[1-e^{-2\alpha(s-t)}\right]}$$

Of course there are some financial problems that are interested also in the constant $\sigma^2/2\alpha$ which is called a long term variance. The meaning of this constant is that the future expected values for the interest rate r may be regrouped in a long-term period of time with this value of the variance. Because the interest rate $r(s)$ at future time s is normally distributed there is a positive probability that $r(s)$ can be negative. This is inconsistent with a no-arbitrage economy in the special sense that consumers hold an option to hold cash instead of investing at negative interest rates. The magnitude of this theoretical problem with the Vasicek model depends on the level of the interest rates and the chosen parameters.

3 Numerical analysis on the classical Vasicek model and some comparative characteristics

In section 2 we consider the classical Vasicek model and the analytical obtaining of the zero coupon bond's price. The equation (10) given in the model with the boundary condition (11) is a parabolic partial differential equation and we are interested in the market price of risk λ. As we have the condition that the market is a non-arbitrary and the portfolio is riskless then we decide to take $\lambda=0$. By this decision we eliminate the market price of risk and obtain a model without a risk factor. The parabolic partial differential equation for this model is given by:

$$(15) \qquad \frac{1}{2}\sigma^2P_{rr}+\alpha(\mu-r)P_r+P_t-rP=0 \text{ with boundary condition (11)}$$

This is the first model used in our numerical analysis. We compare it with the classical Vasicek model for given different values of the constant λ. When $\lambda\neq0$ we can also use another probabilistic numerical method for valuating options, see Sobhani and Milev (2018), [5]. When λ is a function of time, see Gzyl et. al (2017), [2] then the interest rate and the volatility parameters depend on time and such model reflects more realistic on the random movement of the asset prices in financial market.

Starting from the parabolic partial differential equations (10) and (15) and using the software Mathematica 11 we give a numerical solution for the zero coupon bond's prices for the two considered models. The experiments are as follows:

3.1 First numerical experiment

This numerical experiment uses the data given in the table below. It is seen that for each year the market price of risk λ is a constant. Also the interest rate r is taking different values for the period of 1 year. The other parameters are fixed. Based on these data we obtain the zero coupon bond's prices without a risk and with a risk factor given in Figure 1 and Figure 2.

	α	μ	σ	λ	r	t (days)	T (year)	**Price**
Vasicek model without a risk factor	**0.10**	**0.01**	**0.01**		**0.010**	**0**	**1**	**0.1296**
	0.10	**0.01**	**0.01**		**0.015**	**73**	**1**	**0.1843**
	0.10	**0.01**	**0.01**		**0.020**	**146**	**1**	**0.2618**
	0.10	**0.01**	**0.01**		**0.025**	**219**	**1**	**0.3719**
	0.10	**0.01**	**0.01**		**0.030**	**292**	**1**	**0.5288**
	0.10	**0.01**	**0.01**		**0.019**	**365**	**1**	**1.00**
Vasichek model with a risk factor	**0.10**	**0.01**	**0.01**	**0.10**	**0.010**	**0**	**1**	**0.0050**
	0.10	**0.01**	**0.01**	**0.10**	**0.015**	**73**	**1**	**0.0140**
	0.10	**0.01**	**0.01**	**0.10**	**0.020**	**146**	**1**	**0.0390**
	0.10	**0.01**	**0.01**	**0.10**	**0.025**	**219**	**1**	**0.1073**
	0.10	**0.01**	**0.01**	**0.10**	**0.030**	**292**	**1**	**0.2945**
	0.10	**0.01**	**0.01**	**0.10**	**0.019**	**365**	**1**	**1.00**

Table 1
Specific values determing the bond's price
in Vasicek models

For fixed values of the parameters α, μ, σ and λ and changing the value of r and t we obtain the following figures.

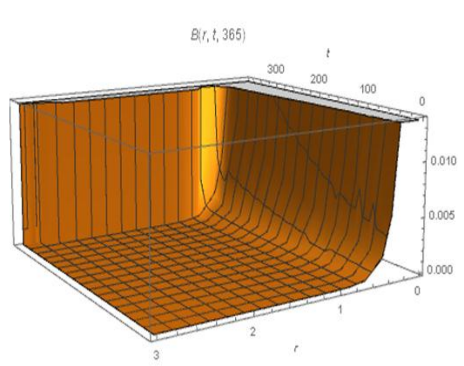

Figure 1
The zero coupon bond's price
for the Vasicek model without a risk factor
$\alpha = 0.10,\ \mu = 0.01,\ \sigma = 0.01$

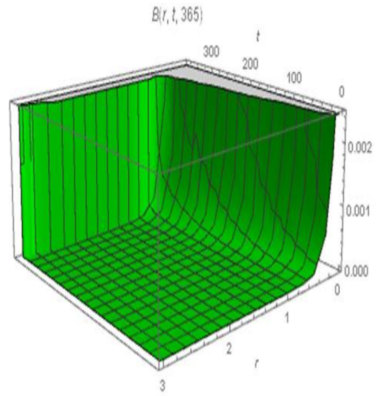

Figure 2
The zero coupon bond's price
for the Vasicek model with a risk factor
$\alpha = 0.10,\ \mu = 0.01,\ \sigma = 0.01,\ \lambda = 0.10$

At first we cannot find an essential difference between Figure 1 and Figure 2. When we compare them we obtain the following graphic:

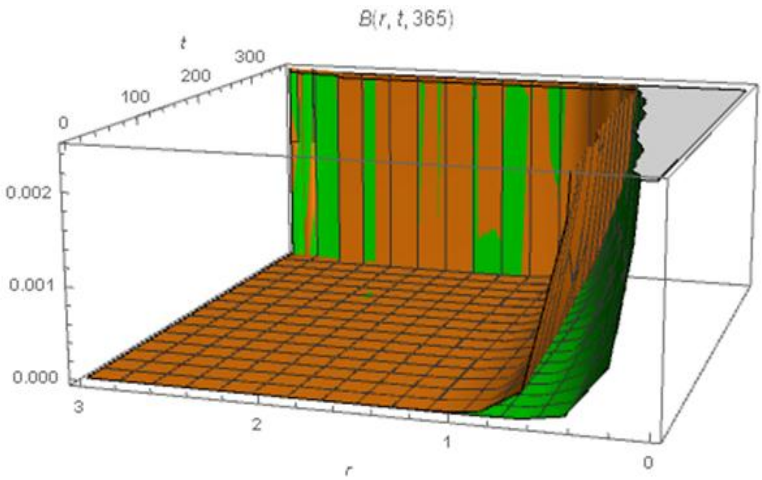

Figure 3
A comparative graphic characteristic showing
the difference in zero coupon bond's prices between
the model with risk and the model without a risk for
given parameters $\alpha = 0.10,\ \mu = 0.01,\ \sigma = 0.01,\ \lambda = 0.10$

The graphic and the table below show the difference in the comparative characteristic.

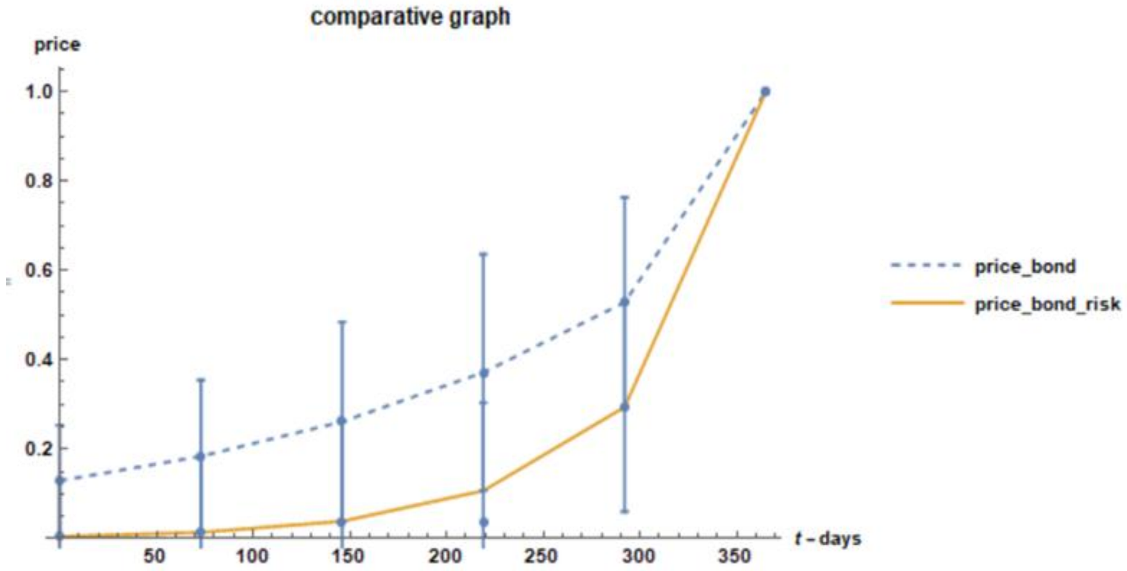

Figure 4
A comparative graph with given deviations for the first experiment

t days	0	73	146	219	292	365=1 year
error	0.1246	0.1703	0.2228	0.390	0.2343	0

Table 2
The deviation of the value of the bond's price for different periods of time

Conclusion:

The lines in Figure 4 represent the change in the value of the zero coupon bond's price over a period of one year. It can be seen that they are the contour of Figure 3. Based on the two tables and the graphics we can conclude that with a risk factor the zero coupon bond's price is decreasing in time.

3.2 Second numerical experiment

This numerical experiment uses the data given in the table below. It is seen that for each year the market price of risk λ is increasing. Also the interest rate r is taking different values in the time period of 5 years. The other parameters are fixed. Based on these data we obtain the zero coupon bond's prices without a risk and with a risk factor given in Figure 5 and Figure 6.

	α	μ	σ	λ	r	t (year)	T (year)	**Price**
Vasicek model without a risk factor	**0.20**	**0.10**	**0.05**		**0.010**	**0**	**5**	**0.8275**
	0.20	**0.10**	**0.05**		**0.015**	**1**	**5**	**0.8601**
	0.20	**0.10**	**0.05**		**0.020**	**2**	**5**	**0.8938**
	0.20	**0.10**	**0.05**		**0.025**	**3**	**5**	**0.9287**
	0.20	**0.10**	**0.05**		**0.030**	**4**	**5**	**0.9644**
	0.20	**0.10**	**0.05**		**0.019**	**5**	**5**	**1.00**
Vasichek model with a risk factor	**0.20**	**0.10**	**0.05**	**0.05**	**0.010**	**0**	**5**	**0.8087**
	0.20	**0.10**	**0.05**	**0.10**	**0.015**	**1**	**5**	**0.8337**
	0.20	**0.10**	**0.05**	**0.15**	**0.020**	**2**	**5**	**0.8693**
	0.20	**0.10**	**0.05**	**0.20**	**0.025**	**3**	**5**	**0.9125**
	0.10	**0.10**	**0.05**	**0.25**	**0.030**	**4**	**5**	**0.9588**
	0.10	**0.10**	**0.05**	**0.30**	**0.019**	**5**	**5**	**1.00**

Table 3
Another values determing the bond's price
in Vasicek models

The Table 3 is constructed based on the increasing values of the parameters α, μ, σ and λ in comparison with Table 1. Again the values of r and t are changing and the other parameters are fixed. This leads to the following figures.

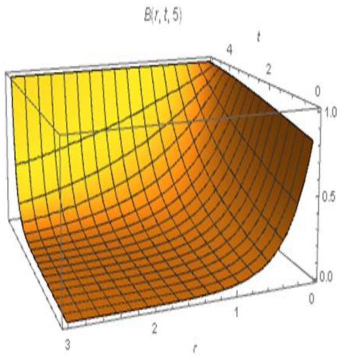

Figure 5
The zero coupon bond's price
for the Vasicek model without a risk factor
with given parameters
$\alpha = 0.20,\ \mu = 0.10,\ \sigma = 0.05$

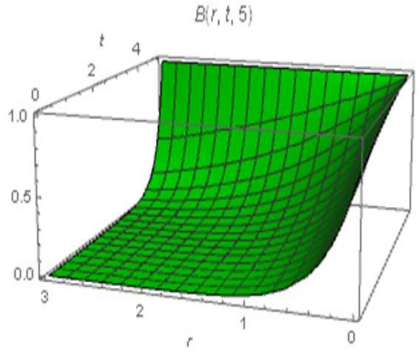

Figure 6
The zero coupon bond's price
for the Vasicek model with a risk factor
with given parameters
$\alpha = 0.20,\ \sigma = 0.05,\ \lambda = 0.20,\ \mu = 0.10$

At first we cannot find an essential difference between Figure 5 and Figure 6. When we compare them we obtain the following graphic:

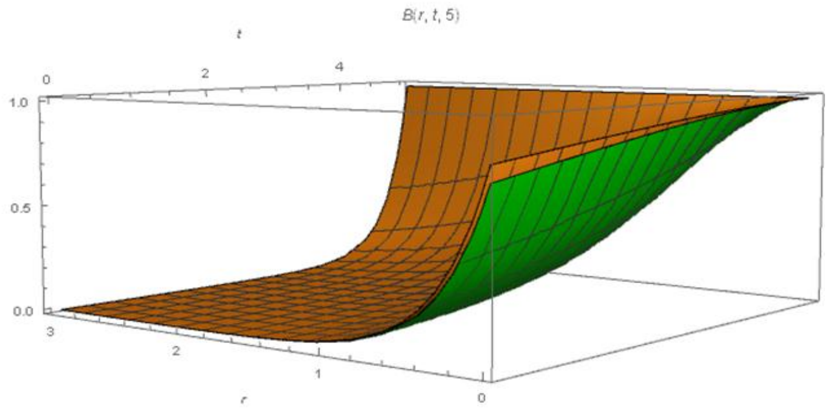

Figure 7
A comparative graphic characteristic showing
the difference in zero coupon bond's prices between
the model with risk and the model without a risk for
given parameters $\alpha = 0.20,\ \mu = 0.10,\ \sigma = 0.05,\ \lambda = 0.20$

The graphic and the table below show the difference in the comparative characteristic.

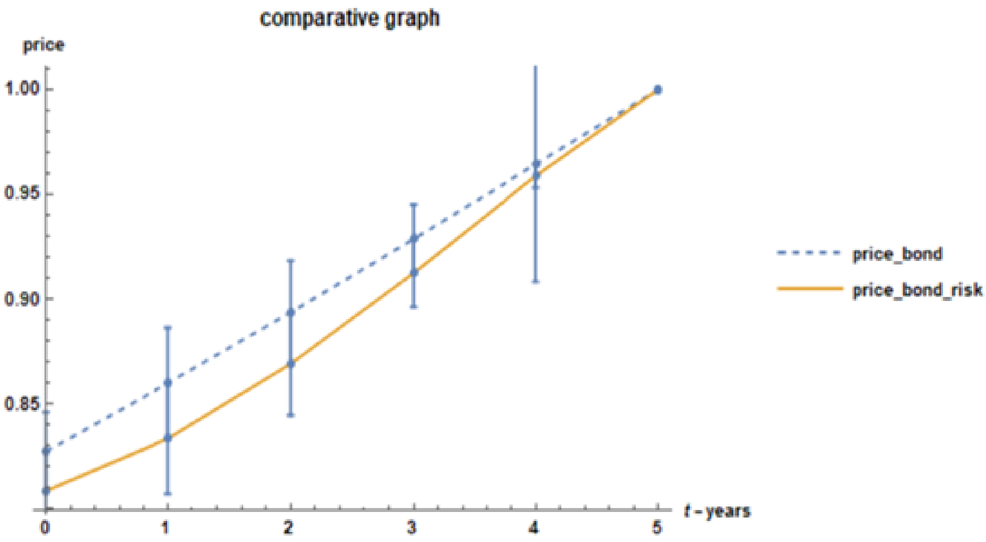

Figure 8
A comparative graph with given deviations for the second experiment

t years	0	1	2	3	4	5
error	0.0188	0.0264	0.0245	0.0162	0.0056	0

Table 4
The deviation of the value of the bond's price for different periods of time

Conclusion:

The lines in Figure 8 represent the change in the value of the zero coupon bond's price over a period of five years. Obviously they are the contour of Figure 7. Based on the two tables and the graphics we can conclude once again that with a risk factor the zero coupon bond's price is slightly decreasing in time.

3.3 Third numerical experiment

This numerical experiment uses the data given in the table below. It is seen that for each year the market price of risk λ is increasing. Also the interest rate r is taking different values in the time period of 10 years. The other parameters are fixed. Based on these data we obtain the zero coupon bond's prices without a risk and with a risk factor given in Figure 7 and Figure 8.

	α	μ	σ	λ	r	t (year)	T (year)	**Price**
Vasicek model without a risk factor	**0.30**	**0.25**	**0.10**		**0.010**	**0**	**10**	**0.2357**
	0.30	**0.25**	**0.10**		**0.015**	**2**	**10**	**0.3366**
	0.30	**0.25**	**0.10**		**0.020**	**4**	**10**	**0.4742**
	0.30	**0.25**	**0.10**		**0.025**	**6**	**10**	**0.6516**
	0.30	**0.25**	**0.10**		**0.030**	**8**	**10**	**0.8517**
	0.30	**0.25**	**0.10**		**0.019**	**10**	**10**	**1.00**
Vasichek model with a risk factor	**0.30**	**0.25**	**0.10**	**0.05**	**0.010**	**0**	**10**	**0.2103**
	0.30	**0.25**	**0.10**	**0.10**	**0.015**	**2**	**10**	**0.2852**
	0.30	**0.25**	**0.10**	**0.15**	**0.020**	**4**	**10**	**0.4038**
	0.30	**0.25**	**0.10**	**0.20**	**0.025**	**6**	**10**	**0.5829**
	0.30	**0.25**	**0.10**	**0.25**	**0.030**	**8**	**10**	**0.8172**
	0.30	**0.25**	**0.10**	**0.30**	**0.019**	**10**	**10**	**1.00**

Table 5
Values determing the bond's price
in the last experiment Vasicek models

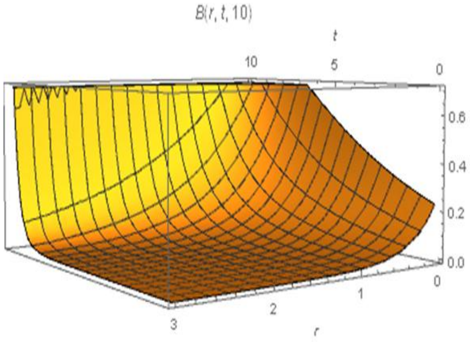

Figure 7
The zero coupon bond's price
for the Vasicek model without a risk factor
with given parameters:
$\alpha = 0.30,\ \mu = 0.25,\ \sigma = 0.10$

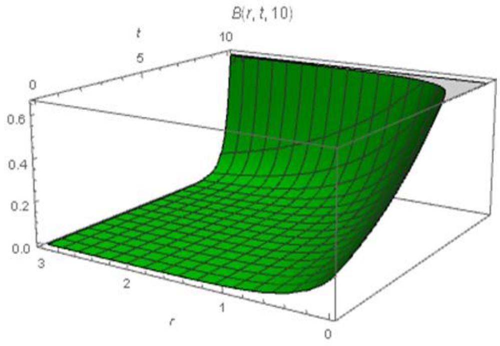

Figure 8
The zero coupon bond's price
for the Vasicek model with a risk factor
with given parameters:
$\alpha = 0.30,\ \mu = 0.25,\ \sigma = 0.10,\ \lambda = 0.20$

The comparative graphic is:

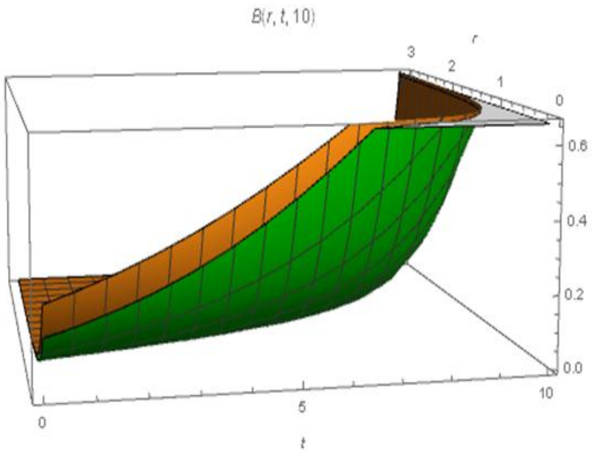

Figure 9
A comparative graphic characteristic showing
the difference in zero coupon bond's prices between
the model with risk and the model without a risk for
given parameters $\alpha = 0.30,\ \mu = 0.25,\ \sigma = 0.10,\ \lambda = 0.20$

The graphic and the table below show the difference in the comparative characteristic.

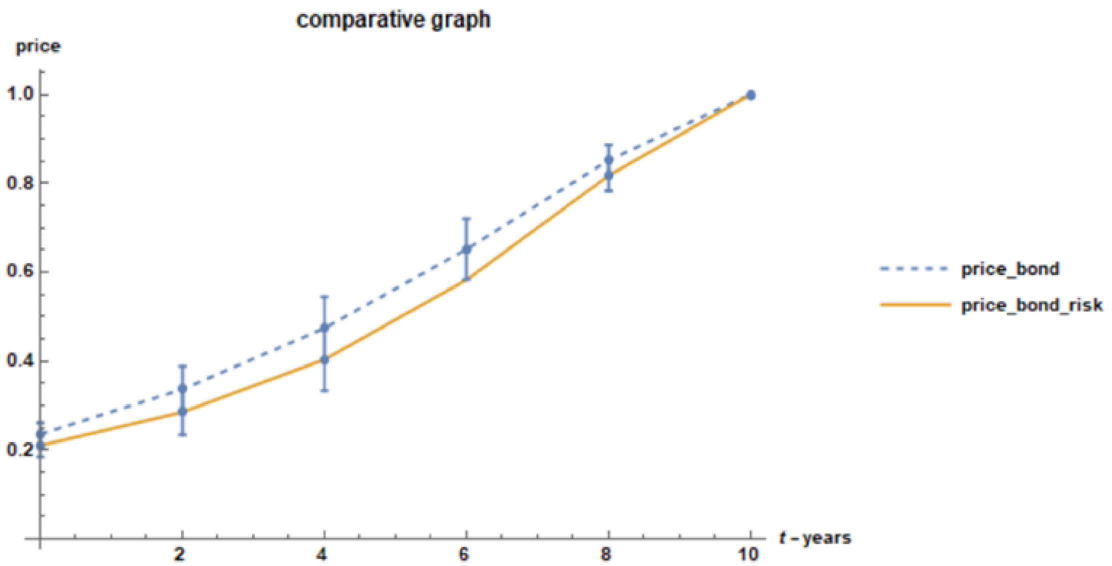

Figure 10
A comparative graph with given deviations for the third experiment

t years	0	2	4	6	8	10
error	0.0254	0.0514	0.0704	0.0687	0.0345	0

Table 6
The deviation of the value of the bond's price for different periods of time for the last experiment

Conclusion:

The lines in Figure 10 represent the change in the value of the zero coupon bond's price over a period of ten years. Based on the two tables and the graphics we can conclude once again that with a risk factor the zero coupon bond's price is strongly decreasing in time.

4 Concluding remarks

In this paper we introduced the zero coupon bond's price in the classical Vasicek model. We considered the influence of the market price of a risk λ over the bond's price and called this model – Vasicek model with a risk factor. Analogously we eliminate the influence of the market price of a risk and called the model - Vasicek model without a risk factor. Taking different values of the risk factor and the other parameters we made a numerical comparative analysis for the changing of the bond's prices in selected periods of time. Finally, we gave three numerical experiments and made the conclusions that the zero coupon bond's price with risk factor is decreasing in time.

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