

ADVANCED CONTROL NONLINEAR STATE SPACE MODELS OF PERMANENT MAGNET SYNCHRONOUS MOTORS

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Abstract: State space nonlinear mathematical models for permanent magnet synchronous motors are derived in the paper using the power invariant dqo rotating transformation. The nonlinear state space models proposed are strictly input-output orientated and linear with respect to the model parameters. The current and flux linkage dqo state space models are much simpler than the original model, their signals are non-periodic and they have better numerical properties for simulation. This allows convenient application of advanced control theory differential geometric design approaches including multi-input multi-output feedback linearization and nonlinear adaptive systems design. The nonlinear state space models derived are simulated to verify their equivalence.

Key words: Nonlinear systems, Nonlinear transformations, Permanent magnet synchronous motor

INTRODUCTION

The permanent magnet synchronous motors (PMSM) are part of the permanent magnet motors class. The PMSM have sinusoidal back electro-motive force and require sinusoidal stator currents to produce constant torque [10, 11]. PMSM have no rotor circuit, instead they have strong rare-earth permanent magnets in the rotor that generate very powerful permanent excitation field. These magnets consume almost no power, so the copper losses in the rotor are negligible unlike the wound synchronous motors. As a result PMSM are very efficient energetically while generating comparatively high torque. The absence of mechanical commutators and brushes, leads to lower friction and higher durability. These motors have maximal ratio between generated torque and motor inertia or total mass, conditioned by the high power density of PMSM compared to induction motors or wound rotor synchronous motors. All these PMSM characteristics put them into the top place of the high efficiency category. PMSM are preferred motors for high performance applications in robotics, industrial and aerospace implementations. Most frequently the three-phase PMSM are used in practice.

The transformation of three-phase variables into two-phase coordinates, known also as Park-Clarke transformation, allows the two-phase mathematical model [9, 7, 6, 3] to be utilized for simulation and analysis. The two-phase dq model is widely used for analysis and design with various types of control methods [2, 7, 1, 4, 5, 8, 6].

The mathematical models of PMSM can be derived in vector-matrix form which is the most general and compact notation offering the opportunity to apply systematic approaches and generalized matrix transformations.

The paper presents the derivation of three-phase dqo vector-matrix mathematical models for PMSM, mapping of the three-phase original vector variables in three-phase dqo coordinates by the power invariant dqo transformation. The two dqo state space models derived are suitable for nonlinear adaptive observers and closed-loop systems design methods. The generalized matrix transformations used are power invariant. They are an accumulation of both the stationary phase and rotating coordinate transformations.

PMSM VECTOR-MATRIX DYNAMIC MODEL IN ORIGINAL abc COORDINATES

The derivation of this model is based on the assumptions that

- ◆ The induced by the harmonic fields of the stator currents in the rotor and the iron losses are neglected.
- ◆ The stator windings are balanced with sinusoidal magnetomotive forces (MMF).
- ◆ The saturation and the parameters changes are neglected.

The rotor magnets are modelled as flux linkage sources. The vector-matrix dynamic equation in original coordinates is

$$\mathbf{u}_{abc} = \mathbf{R}_{abc} \mathbf{i}_{abc} + \frac{d\mathbf{L}_{abc} \mathbf{i}_{abc}}{dt} + \frac{d\boldsymbol{\Psi}_{pmabc}}{dt}, \quad (1)$$

where the stator voltages, currents, the permanent magnets flux linkages, the resistance, and the inductance matrices are

$$\mathbf{u}_{abc} = [u_a, u_b, u_c]^T, \quad \mathbf{i}_{abc} = [i_a, i_b, i_c]^T, \\ \boldsymbol{\Psi}_{pmabc} = \begin{bmatrix} \Psi_{pma} \\ \Psi_{pmb} \\ \Psi_{pmc} \end{bmatrix} = \Psi_{pm} \begin{bmatrix} \sin(\theta_e) \\ \sin(\theta_e - \varphi) \\ \sin(\theta_e + \varphi) \end{bmatrix},$$

$$\mathbf{R}_{abc} = \begin{bmatrix} R & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & R \end{bmatrix}, \quad \mathbf{L}_{abc} = \begin{bmatrix} l_m + l_{sl} & -\frac{l_m}{2} & -\frac{l_m}{2} \\ -\frac{l_m}{2} & l_m + l_{sl} & -\frac{l_m}{2} \\ -\frac{l_m}{2} & -\frac{l_m}{2} & l_m + l_{sl} \end{bmatrix}.$$

The magnetizing and the stator leakage inductances are denoted as l_m , l_{sl} and R , Ψ_{pm} , and θ_e are the stator resistances, the magnets flux linkages magnitude, the electrical rotor angle and $\varphi = 2\pi/3$ is a phase shift. Solving the dynamic equation (1) with respect to the currents yields the currents vector-matrix dynamics in the form

$$\frac{d\mathbf{i}_{abc}}{dt} = \mathbf{L}_{abc}^{-1} \mathbf{u}_{abc} - \mathbf{L}_{abc}^{-1} (\mathbf{R}_{abc} + \frac{d\mathbf{L}_{abc}}{dt}) \mathbf{i}_{abc} - \mathbf{L}_{abc}^{-1} \frac{d\boldsymbol{\Psi}_{pmabc}}{dt}. \quad (2)$$

The electromagnetic torque in original coordinates is

$$T_{eabc} = n_p \Psi_{pm} (i_a \cos(\theta_e) + i_b \cos(\theta_e - \varphi) + i_c \cos(\theta_e + \varphi)), \quad (3)$$

where n_p is the number of the pole pairs. The equation for the motor mechanical dynamics is

$$\frac{d\omega}{dt} = \frac{T_e - T_L}{J}, \quad (4)$$

where J is the motor inertia and T_L is the load torque. The electrical rotor speed and angle are connected to their mechanical analogues by $\theta_e = n_p \theta$, $\omega_e = n_p \omega$. The PMSM componentwise dynamic model in original abc coordinates based on equations (4), (2), and $T_e = T_{eabc}$ according to (3) reads

$$\frac{d\theta}{dt} = \omega$$

$$\frac{d\omega}{dt} = [n_p \Psi_{pm} (i_a \cos(\theta_e) + i_b \cos(\theta_e - \varphi) + i_c \cos(\theta_e + \varphi)) - T_L] / J$$

$$\frac{di_a}{dt} = -\frac{R}{L} i_a + \frac{1}{L} u_a - \frac{\Psi_{pm}}{L} \omega_e \cos(\theta_e)$$

$$\frac{di_b}{dt} = -\frac{R}{L} i_b + \frac{1}{L} u_b - \frac{\Psi_{pm}}{L} \omega_e \cos(\theta_e - \varphi)$$

$$\frac{di_c}{dt} = -\frac{R}{L} i_c + \frac{1}{L} u_c - \frac{\Psi_{pm}}{L} \omega_e \cos(\theta_e + \varphi)$$

After choosing the state and control vectors

$$\mathbf{x} = [x_1, x_2, x_3, x_4, x_5]^T = [\theta, \omega, i_a, i_b, i_c]^T,$$

$$\mathbf{u} = [u_1, u_2, u_3, T_L]^T = [u_a, u_b, u_c, T_L]^T,$$

the currents state space model of the PMSM in original abc coordinates is obtained

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= c_4(x_3 \cos(c_5 x_1) + x_4 \cos(c_5 x_1 - c_7) + x_5 \cos(c_5 x_1 + c_7)) - c_6 u_4 \\ \dot{x}_3 &= c_1 u_1 - c_2 x_3 - c_3 x_2 \cos(c_5 x_1) \\ \dot{x}_4 &= c_1 u_2 - c_2 x_4 - c_3 x_2 \cos(c_5 x_1 - c_7) \\ \dot{x}_5 &= c_1 u_3 - c_2 x_5 - c_3 x_2 \cos(c_5 x_1 + c_7) \end{aligned} \quad (5)$$

with parameters $c_1 = 1/L$, $c_2 = R/L$, $c_3 = n_p \Psi_{pm} / L$, $c_4 = n_p \Psi_{pm} / J$, $c_5 = n_p$, $c_6 = 1/J$, $c_7 = \varphi$.

ROTATING dqo MATRIX TRANSFORMATIONS

The forward and inverse dqo matrix transformations in rotating dqo coordinates are used for the state space models derivation in the form

$$\mathbf{T}_{dqo}(\theta_k) = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\theta_k) & \cos(\theta_k - \varphi) & \cos(\theta_k + \varphi) \\ -\sin(\theta_k) & -\sin(\theta_k - \varphi) & -\sin(\theta_k + \varphi) \\ \sqrt{2}/2 & \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}, \quad (6a)$$

$$\mathbf{T}_{abc}(\theta_k) = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\theta_k) & -\sin(\theta_k) & \sqrt{2}/2 \\ \cos(\theta_k - \varphi) & -\sin(\theta_k - \varphi) & \sqrt{2}/2 \\ \cos(\theta_k + \varphi) & -\sin(\theta_k + \varphi) & \sqrt{2}/2 \end{bmatrix} \quad (6b)$$

These transformations are related as $\mathbf{T}_{dqo} = \mathbf{T}_{abc}^{-1}$. The forward transformation \mathbf{T}_{dqo} maps the original stator and rotor abc signals in rotating dqo coordinates. The inverse transformation carries out the respective inverse signal mappings from dqo to original abc coordinates

$$\mathbf{u}_{abc} = \mathbf{T}_{abc} \mathbf{u}_{dqo}, \quad \mathbf{i}_{abc} = \mathbf{T}_{abc} \mathbf{i}_{dqo}, \quad \Psi_{pmabc} = \mathbf{T}_{abc} \Psi_{pmdqo}, \quad (7)$$

where

$$\mathbf{u}_{dqo} = [u_d, u_q, u_o]^T, \quad \mathbf{i}_{dqo} = [i_d, i_q, i_o]^T,$$

$$\Psi_{pmdqo} = [\Psi_{pmd}, \Psi_{pmq}, \Psi_{pmo}]^T$$

are the voltage, current, and flux linkage vectors in dqo coordinates. The matrix transformations in rotating dqo coordinates are power invariant because $\mathbf{T}_{abc}^T = \mathbf{T}_{abc}^{-1}$ and

$\mathbf{T}_{dqo}^T = \mathbf{T}_{dqo}^{-1}$, i.e. the transformation matrices are unitary.

PMSM dqo STATE SPACE MODELS

The transformation of the original currents vector-matrix model (2) in rotating dqo coordinates is achieved by applying the substitutions (7), solving for $d\mathbf{i}_{dqo}/dt$ and considering that

$$\begin{aligned} \mathbf{L}_{dqo} &= \mathbf{T}_{dqo} \mathbf{L}_{abc} \mathbf{T}_{abc} = \text{const} \quad \text{and} \quad \mathbf{R}_{dqo} = \mathbf{T}_{dqo} \mathbf{R}_{abc} \mathbf{T}_{abc} \quad \text{giving} \\ \frac{d\mathbf{i}_{dqo}}{dt} &= \mathbf{L}_{dqo}^{-1} [\mathbf{u}_{dqo} - (\mathbf{R}_{dqo} + \mathbf{T}_{dqo} \frac{d\mathbf{T}_{abc}}{dt} \mathbf{L}_{dqo}) \mathbf{i}_{dqo} \\ &\quad - \mathbf{T}_{dqo} \frac{d\mathbf{T}_{abc}}{dt} \Psi_{pmdqo} - \frac{d\Psi_{pmdqo}}{dt}], \end{aligned} \quad (8)$$

where the transformed inductance and resistance matrices have the form

$$\mathbf{L}_{dqo} = \begin{bmatrix} L_d & 0 & 0 \\ 0 & L_q & 0 \\ 0 & 0 & L_o \end{bmatrix}, \quad \mathbf{R}_{dqo} = \begin{bmatrix} R & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & R \end{bmatrix},$$

with $L_d = L_q = 3l_m/2 + l_{sl}$, $L_o = l_{sl}$. The stator dqo homopolar dynamic equation reads

$$\frac{di_o}{dt} = \frac{1}{L_o} u_o - \frac{R}{L_o} i_o.$$

The synchronous motors considered and the input voltages are symmetric, hence the homopolar dqo voltage $u_o = 0$. Then the dqo homopolar dynamic equation will be asymptotically stable. Therefore, it can be neglected and will be further omitted in the state space models. The transformed electromagnetic torque (3) is given by the expression

$$T_{edqoi} = n_p (\sqrt{3}/2 \Psi_{pm} i_d + (L_d - L_q) i_d i_q) \quad (9)$$

The currents state space model in rotating dqo coordinates is based on equations (4), (8), $T_e = T_{edqoi}$ according to (9).

Choosing the state and control vectors

$$\mathbf{x} = [x_1, x_2, x_3, x_4, x_5]^T = [\theta, \omega, i_d, i_q]^T,$$

$$\mathbf{u} = [u_1, u_2, u_3, u_4]^T = [u_d, u_q, \omega_k, T_L]^T,$$

results in the dynamics

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -c_9 u_4 + c_{10} x_3 + c_8 x_3 x_4 \end{aligned} \quad (10)$$

$$\dot{x}_3 = c_1 u_1 - c_7 u_3 - c_2 x_3 + c_3 u_3 x_4$$

$$\dot{x}_4 = c_4 u_2 - c_6 u_3 x_3 - c_5 x_4$$

with $c_1 = 1/L_d$, $c_2 = R/L_d$, $c_3 = L_q/L_d$, $c_4 = 1/L_q$,

$$c_5 = \frac{R}{L_q}, \quad c_6 = \frac{L_d}{L_q}, \quad c_7 = \frac{\sqrt{3}/2 \Psi_{pm}}{L_d}, \quad c_8 = \frac{n_p (L_d - L_q)}{J},$$

$$c_9 = 1/J, \quad c_{10} = \sqrt{3}/2 n_p \Psi_{pm} / J.$$

The connection between the currents and the flux linkages in dqo coordinates is expressed by the relation

$$\mathbf{i}_{dqo} = \mathbf{L}_{dqo}^{-1} (\Psi_{dqo} - \Psi_{pmdqo}). \quad (11)$$

The electromagnetic torque in dqo coordinates in terms of flux linkages can be obtained by substituting (11) in the expression for the electromagnetic torque (9) in terms of currents

$$T_{edqov} = n_p \left(\frac{\sqrt{3}/2 \Psi_{pm}}{L_q} \Psi_d + n_p \left(\frac{1}{L_q} - \frac{1}{L_d} \right) \Psi_d \Psi_q \right) \quad (12)$$

The flux linkages state space model in rotating dqo coordinates is derived by substituting (11) in (8), (12) in (4), and choosing the state space and control vectors

$$\mathbf{x} = [x_1, x_2, x_3, x_4, x_5]^T = [\theta, \omega, \Psi_d, \Psi_q]^T,$$

$$\mathbf{u} = [u_1, u_2, u_3, u_4]^T = [u_d, u_q, \omega_k, T_L]^T,$$

which yields the following system

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -c_6 u_4 + c_5 x_3 + c_4 x_3 x_4$$

$$\dot{x}_3 = u_1 - c_1 x_3 + u_3 x_4$$

$$\dot{x}_4 = u_2 - c_2 - u_3 x_3 - c_3 x_4$$

with model coefficients $c_1 = R/L_d$, $c_2 = \sqrt{3/2} R \psi_{pm} / L_q$,

$$c_3 = \frac{R}{L_q}, c_4 = \frac{n_p}{L_q J} - \frac{n_p}{L_d J}, c_5 = \frac{\sqrt{3/2} n_p \psi_{pm}}{L_q}, c_6 = \frac{1}{J}.$$

The control input $u_3 = \omega_k$ in the above two state space models is the angular speed of the dqo coordinate frame, which is $\omega_k = \omega_e$, hence $\theta_k = \theta_e$. The rotating matrix transformations used assume the following three phase stator voltage sequence

$$\mathbf{u}_{abc} = u_m [\sin(\omega t), \sin(\omega t - 2\pi/3), \sin(\omega t + 2\pi/3)]^T.$$

The transformed stator voltages in dqo coordinates are

$$\mathbf{u}_{dqo} = \begin{bmatrix} u_d \\ u_q \\ u_o \end{bmatrix} = \mathbf{T}_{dqo} \mathbf{u}_{abc} = \sqrt{\frac{3}{2}} \begin{bmatrix} u_m \sin[\omega t - \theta_k(t)] \\ -u_m \cos[\omega t - \theta_k(t)] \\ 0 \end{bmatrix}. \quad (13)$$

These voltages are non-periodic signals because of the time evolution of the argument $[\omega t - \theta_k(t)]$, which converges to a steady state value and becomes constant. This determines the non-periodicity of all signals in dqo coordinates. It cannot be shown symbolically because the dynamic equations of the state space models are nonlinear, they involve the variables in a substantially non-algebraic way and cannot be solved for $\theta_k(t) = \theta_e(t)$. The advantage of the dqo transformation is that it transforms the three-phase original PMSM model with periodic signals into a much simpler two-phase model with non-periodic signals. This leads to simplification of the control design and allows the model integration with a bigger step without losing numerical stability.

SIMULATION AND SYSTEM TIME RESPONSES

The three state space models in original abc and canonical dqo coordinates are simulated for the purpose of verification. The motor parameters are $R = 0.5 \Omega$, $L_m = 1 \text{ mH}$, $L_{sl} = 0.1 \text{ mH}$, $L = 1.5 L_m + L_{sl}$, $L_d = L_q = L_o = L$, $\psi_{pm} = 0.069 \text{ Vs}$, $n_p = 2$, $J = 17e-6 \text{ kgm}^2$, $T_L = 0.3 \text{ Nm}$ and the initial conditions for the original and current state space models are zero. The initial conditions for the flux linkage model are determined according to the relation (11). Figures 1 and 2 show the control input voltages and the stator currents of the original abc state space model. Figure 3 depicts the evolution of the

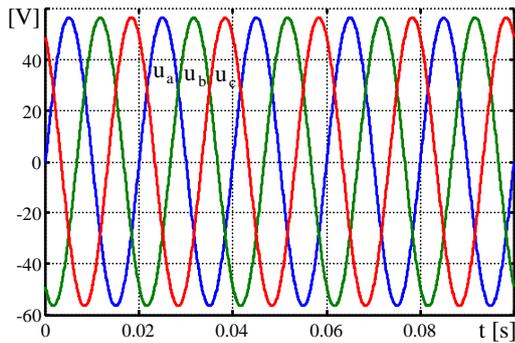


Figure 1: Control voltages in abc coordinates

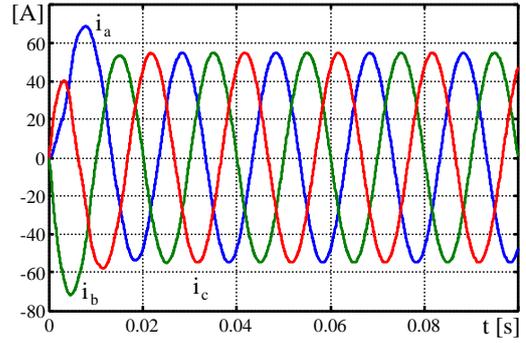


Figure 2: Currents in abc coordinates

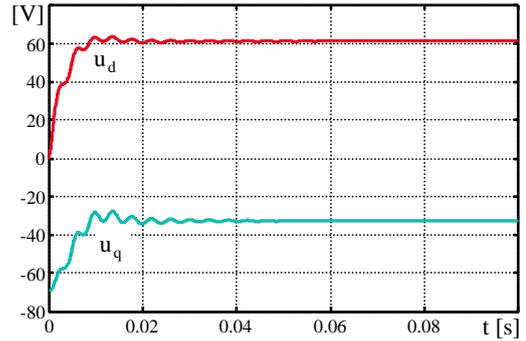


Figure 3: Control voltages in dqo coordinates

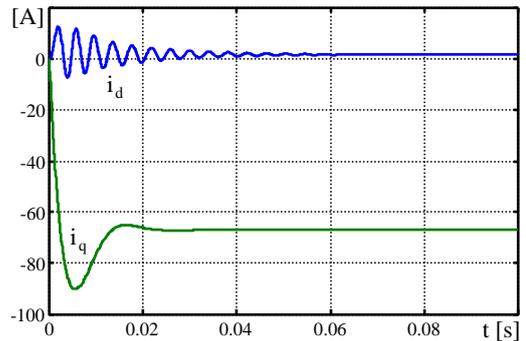


Figure 4: Currents in dqo coordinates

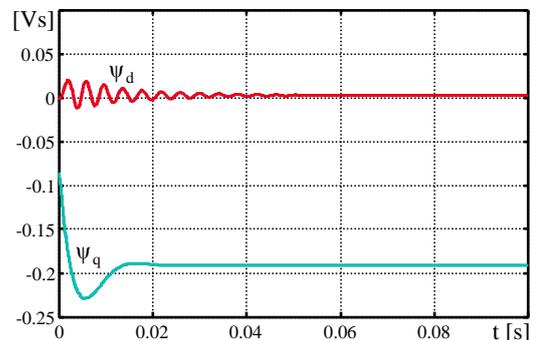


Figure 5: Flux linkages in dqo coordinates

control voltages in dqo coordinates, which are one and the same for the current and the flux linkage state space models. Figures 4 and 5 present the currents and the flux linkages in the canonical coordinates. Obviously these signals are non-periodic. The electromagnetic torques of the original, current and flux linkage models are illustrated on figure 6. They are simulated simultaneously and coincide exactly, which is a

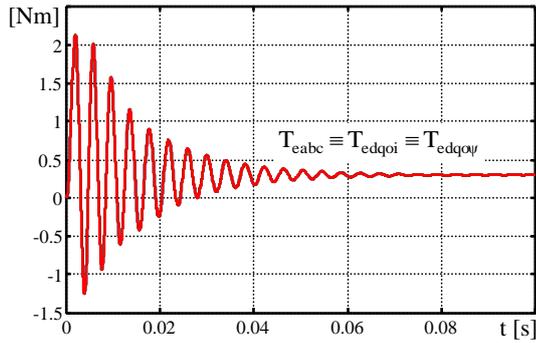


Figure 6: Motor electromagnetic torque

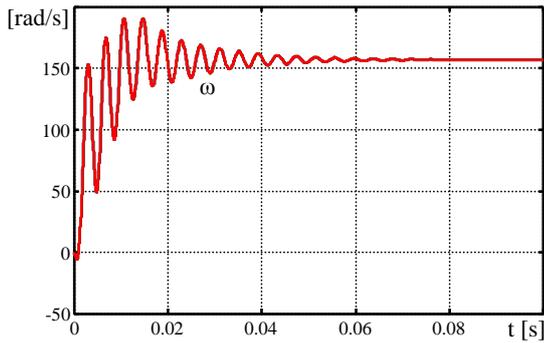


Figure 7: Motor mechanical angular speed

necessary experimental confirmation for the equivalence of these models. The motor mechanical angular speeds from the three models are shown on figure 7. These signals coincide exactly as they are outputs and therefore are equal.

CONCLUSIONS

The paper has presented generalized straight and inverse vector-matrix transformations for the transition of original abc into canonical dqo coordinates and vice versa. These transformations are constructed on the principle of power equivalency for the processes in all coordinates and are power invariant, which is guaranteed by the fact that $\mathbf{T}_{abc}^T = \mathbf{T}_{abc}^{-1}$, $\mathbf{T}_{dqo}^T = \mathbf{T}_{dqo}^{-1}$, i.e. the straight and inverse transformations are unitary.

A vector-matrix dynamical state space model of the PMSM in original abc coordinates in terms of currents is derived. This model is transformed into dqo coordinates with the dqo transformation. Thus, the current dqo state space model is obtained. The second possible model – the flux linkage state space model is derived by the application of the transformation $\mathbf{i}_{dqo}(\boldsymbol{\Psi}_{dqo})$. The original, current and flux

linkage state space models are simulated simultaneously and their equivalence is verified.

The original, current and flux linkage state space models proposed are strictly input-output orientated and linear with respect to their coefficients. On the other hand the canonical dqo state space models are much simpler than the original model and their signals are non-periodic. This allows easier close-loop system design and model simulation with a bigger integration step, conditioned by the improved numerical properties. Finally, the current and the flux dqo state space models proposed permit the application of advanced nonlinear and adaptive control design approaches, analysis of controllability, observability and stability of the open and closed-loop systems based on these models.

REFERENCES

1. Chiasson J., Modeling and High Performance Control of Electric Machines, Wiley-IEEE Press, 2005.
2. Solsona J., M. I. Valla, C. Muravchik, Nonlinear Control of a Permanent Magnet Synchronous Motor with Disturbance Torque Estimation, IEEE Transactions on Energy Conversion, p. 15(2), 2000, 163-168.
3. Mishkov R., Petrov V., Strictly orientated state space models of permanent-magnet synchronous motors for feedback linearization control, Journal of the Technical University Sofia, Branch Plovdiv, Fundamental Sciences and Applications, Vol. 16, book 1, 2011, pp. 335-340
4. Ichikawa S., M. Tomita, S. Doki, S. Okuma, Sensorless control of permanent-magnet synchronous motors using online parameter identification based on system identification theory, IEEE Transactions on Industrial Electronics, Vol. 53, No. 2, 2006, p. 363-372.
5. Kaewjinda W., M. Konghirun, Vector Control Drive of Permanent Magnet Synchronous Motor Using Resolver Sensor, ECTI Transactions on Electrical Eng., Electronics, and Communication, 5(1), 2007, p. 134-138.
6. Krishnan R., Permanent Magnet Synchronous and Brushless DC Motor Drives, Taylor and Francis Group, 2010.
7. Krishnan R., Electric Motor Drives: Modeling, Analysis and Control, Prentice Hall, 2001.
8. Merzoug M. S., H. Benalla, Nonlinear Backstepping Control of Permanent Magnet Synchronous Motor (PMSM), International Journal of System Control, 2010, p. 30-34.
9. Ong Chee-Mun, Dynamic Simulation of Electric Machinery: Using Matlab/Simulink, Prentice Hall, 1998.
10. Pillay P., R. Krishnan, Modeling of Permanent-magnet Motor Drives, IEEE Transactions on Industrial Electronics, 35(4), 1988, p. 537-541.
11. Pillay P., R. Krishnan, Modeling, Simulation, and Analysis of Permanent-magnet Motor Drives. I. The Permanent-magnet Synchronous Motor Drive, IEEE Transactions on Industrial Applications, 25(2), 1989, p. 265-273.