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Introducing bachelors, studying for math teachers, with computer-aided graphical solving and exploration of equations

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Graphs give *intuitive and complete overall picture* of the behavior of functions: increasing, decreasing, convexity, asymptotes, intersection points, etc. Therefore graphs are used in many branches: economy, politics, meteorology, engineering, biology, etc. The use of graphical methods and graphical solving of equations with a computer is strategic continuation of graphical approach. Another benefit from graphical solving (with computer) is the wide range solvable equations. It is hard or impossible to solve by standard methods equations like $x^2+\sqrt{7}x+\sqrt{3}=0$, $x^3+1{,}83x+0{,}86-e=0$, $143x^3-149x^2+11933x+811=0$ and $1{,}01x^3-x^2+\sqrt{7}x+\sqrt{3}=0$, where coefficients aren't small enough whole numbers. Other computer-solvable equations are "mixed type" ones: $x^2+x=\ln x$, $x=\sin x$, $\ln\left(x^2-x\right)=\tan x$, etc. Mixed here means polynomial-logarithmic, logarithmic-trigonometric, etc. All these equations *cannot* be solved graphically by paper and pen. A colleague with PhD in physics said that equations obtained in his practice are *very rarely* solvable by traditional school means and the most - by computer. Hence it's perspective to introduce to the future math teachers computer aided graphical solving, problems related to it and didactic technologies to introduce it to school students.

When we solve graphically equations of the type $f(x)=g(x)$ or $F(x)=0$, there might occur several problems, considered in the paper. Future teachers have to know about these difficulties and we've introduced them with. Most important and frequent among these problems are:

1) The graph of $F(x)$ may not appear on the screen or not all the

equation's roots may appear on it. First-case-example in our lecture was the equation $x^2 - 4x - 192 = 0$. It seemed like the computer did not respond. Our second-case-examples were: $x = 5\ln x$ (**Fig. 1**) and $x^3 - 18x^2 + 17x + 240 = 0$ (**Fig. 2**). The solution in these cases is to "zoom out" until enough from the graph appear on the screen. It seemed (**Fig. 1**) there are no other solutions, as (according to the best student): "The one graph goes upwards, and the other - downwards." The concavity of logarithm curve helped us to prognose eventual second root and no more ones.

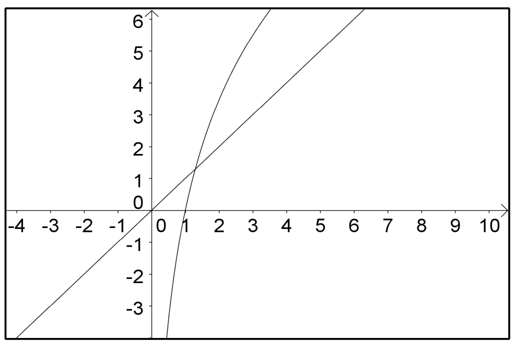

Fig. 1. $x = 5\ln x$.

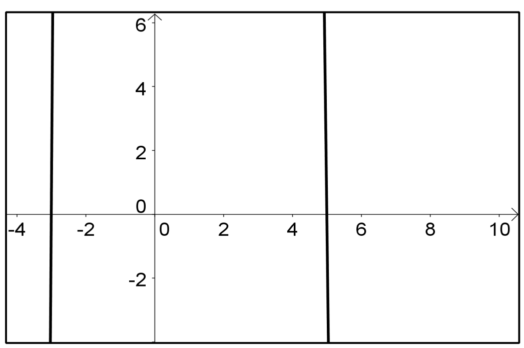

Fig. 2. $x^3 - 18x^2 + 17x + 240 = 0$.

2) The graph of $F(x)$ may not be relevant, if the function $F(x)$ is not "standard". Replacing the equation $F(x) = 0$ with convenient equivalent one in the form $f(x) = g(x)$ usually removes this problem. Example: $\tan x - x^2 = 0$ (**Fig. 3**).

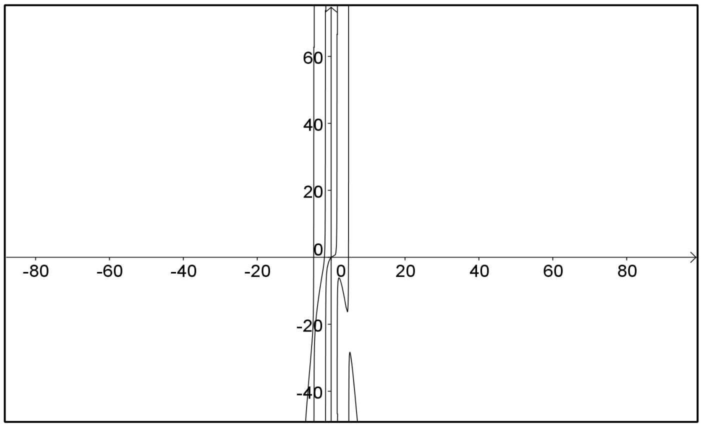

Fig. 3. $\tan x - x^2 = 0$.

Even powerful SCA like MAPLE 11 err the graph of $y = \tan x - x^2$, as shown in the paper. The equation $\tan x - x^2 = 0$ is solvable in its equivalent form $\tan x = x^2$. Such examples show that the computer is not omnipotent. They are convenient for teachers to know about and show them to their students.

Getting ready graphs from the computer, the students can do different explanations, interpretations, suggestions, etc. I.e. they're not passive. Further, the dynamics geometry software GeoGebra, used by us, is convenient environment for experimentation. Exploration of parametric equations was another activity during our lesson. It is creative activity. The equation was $7x^k + 2x^3 - x + \sqrt{3x^2 + x} = 7$ in our case. I reminded the students what means a term to be on negative/rational power.

After our experiment colleague-docent gave us positive overall estimation. The (university) students were interested during it and gave us positive estimation after it too.