Analysis of the mutual inductive and capacitive connections and tolerances of memristor’s parameter’s of a memristor memory matrix

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Abstract — In this paper the mutual inductive and capacitive connections and tolerances of memristor’s parameter’s of a memristor memory matrix is investigated. An equivalent circuit of three neighboring memristors of a memory matrix with complicated mutual capacitive and inductive connection is presented. Then the memristor’s own parasitic capacitance and inductance are described. The mutual capacitances between the rims of the memory matrix are calculated. Two possible values of the coefficient of magnetic coupling between the memristors are used. A SIMULINK model of the circuit of three neighboring memristors is developed. The simulation is made at pulse mode with 3 GHz actuating voltages. The main result is that the parasitic parameters do not strongly affect the memristor voltage drops at frequencies up to 3 GHz. However the tolerances of the memristor parameters have stronger effect on the circuit characteristics. Finally, some concluding remarks associated with the magnetic and capacitive influence between the memristors of a memristor memory matrix are given.

Keywords—titanium-dioxide memristor; parasitic parameters; mutual inductance; memristor characteristics

I. INTRODUCTION

One of the most important properties of the memristor is to memorize the amount of charge which has passed through it [2, 3, 14]. Many analyses and simulations on this nonlinear element have been made in last few years [4, 7, 8]. The main properties and the operation of Williams’s memristor have been presented [5, 9, 12]. In the literature a few types of memristors as polymeric memristor, nitride memristors and others have been described [1, 6, 13]. Some physical relationships between the basic electrical quantities of the memristor have been shown in the literature [8, 10, 11]. The titanium-dioxide memristor matrix as an integrated circuit has mutual capacitances and inductances between the memristor cells. These parameters are dependent of the memristor chip size and the position of the cells [6, 7, 8, 10]. In the papers published no data on this topic have been found. The main purpose of this paper is to propose adequate substituting circuit with three memristors connected each other with mutual inductance and capacitance and a suitable SIMULINK model for further analysis.

In Section II an equivalent substituting circuit of the three neighboring memristors is proposed. The SIMULINK model of the memristor as a building block of the next circuit model is presented in Section III. The SIMULINK model of the whole circuit investigated and the simulation results realized are given in Section IV. The concluding remarks are presented in Section V.

II. EQUIVALENT SUBSTITUTING CIRCUIT OF THREE NEIGHBOURING MEMRISTORS CONNECTED WITH MUTUAL CAPACITIVE LINKS OF A MEMRISTOR MEMORY MATRIX

The equivalent scheme of the three nearly placed on a matrix memristors with mutual inductive and capacitive connections is presented in Fig. 1. The capacitors $C_1$, $C_2$ and $C_3$ have value of $3.10^{-16}$ F and present memristor’s own parasitic capacitances. The parameters $L_1$, $L_2$ and $L_3$ are the parasitic inductances of the memristors and have values of $9.4 \, nH$. The coefficients of mutual inductances $M_{12}$, $M_{13}$, $M_{23}$ are almost equal to each of inductances because of the embracement of the magnetic flux induced by the neighboring rims from the wires. Their values are $8.5 \, nH$.  

![Fig. 1. Equivalent circuit of three neighboring memristors of a memory matrix with mutual capacitive connection](image-url)
The mutual capacitances $C_{12}$, $C_{21}$, $C_{23}$, $C_{32}$ are calculated as a capacitance of a plane capacitor. They are due to the electric influence between the nearest parallel platinum rims of the memristor memory matrix. Their values are 0.007 pF.

III. SYNTHESIS OF A SIMULINK MODEL OF A MEMRISTOR CELL IN THE MEMORY MATRIX

Each of the memristors in the circuit investigated is presented with linear ionic drift model. The relationship between the memristor current $i$ and the voltage drop across it $u$ is shown using Eq. (1):

$$i(t) = \frac{u(t)}{R_{\text{OFF}}} \sqrt{\left(1 - \frac{q(t)}{Q} \right)^2 - \frac{2\eta}{Q} \frac{u(t)}{R_{\text{OFF}}}}$$

The quantity $q(t_0)$ is the initial amount of charge memorized in the memristor cell. The resistance of the memristor in opened state is denoted with $R_{\text{OFF}}$. The coefficient $\eta$ represents the polarization of the memristor with respect of the initial sign of the voltage applied across the element. When the memristor is forward biased the positive potential is applied to the anode of the memristor and the coefficient $\eta$ is equal to 1. When the element is reverse biased the positive potential is applied to the cathode of the memristor and then the coefficient $\eta$ is equal to -1. The maximal amount of charge that could be memorized in the memristor cell is denoted with $Q_D$:

$$Q_D = \frac{D^2}{\mu \, R_{\text{ON}}}$$

The quantity $D$ represents the length of the memristor, $\mu$ is the mobility of the charge carriers and $R_{\text{ON}}$ is the resistance of the memristor in closed state. The SIMULINK model of the memristor cell, based on Eq. (1) is shown in Fig. 2. The coefficient $\eta$ is determined automatically in the beginning of the simulation. For this purpose a short positive impulse is multiplied by the sign of the initial memristor voltage drop. After integration and determining the sign of the signal it is equalized to the coefficient $\eta$. The initial charge of the memristor is 5% of the maximal charge of the memristor that could be accumulated in it.

IV. SYNTHESIS OF A SIMULINK MODEL OF THE WHOLE MEMRISTOR CIRCUIT INVESTIGATED AND PRESENTATION OF THE SIMULATION RESULTS AT IMPULSE MODE

The SIMULINK model of the circuit investigated is presented in Fig. 3. This circuit will be used also for analysis of the equivalent scheme when the three memristors have manufacture tolerances of their basic parameters. The voltage sources produce rectangular pulse signals with frequency of 3 GHz and magnitude of 1 V. The model of the circuit presented in Fig. 1 is made with respect to the structure of the circuit and using the method for investigating of electrical circuits with the branch currents. The simulation is made using Bogachi – Shampine integration algorithm for solving the differential equations of the complicated nonlinear electrical memristor circuit.
In what follows the main results of the simulation made in MATLAB environment are presented. The time diagrams of source voltage $e_1$ and the memristor voltage drop over the first memristor $u_{m1}$ are shown in Fig. 4. The diagrams of the second and third memristor voltage drops have similar form. It is obvious that there are transient processes with pseudo-periodic character. The ricochets have very high values but there are no losses of information.

![Time diagrams of the source voltage $e_1$ and the memristor voltage drop $u_{m1}$](image4)

**Fig. 4.** Time diagrams of the source voltage $e_1(t)$ and the memristor voltage drop $u_{m1}(t)$

The time diagram of memristor current $i_{m3}$ is presented in Fig. 5. It has graphical form similar to the shape of the memristor voltage drop $u_{m1}$.

![Time diagram of the memristor current $i_{m3}$](image5)

**Fig. 5.** Time diagram of the memristor current $i_{m3}$

The time diagram of the source current $i_{e1}$ is presented in Fig. 6. It is interesting that the magnitude of the current pulses is 10 times bigger than the magnitude of the memristor current $i_{m1}$. That is due to the presence of capacitive currents flowing between the neighboring rims of the memristors. The time diagrams of these capacitive currents are shown in Fig. 7.

![Time diagrams of mutual capacitive currents $i_{c12}$, $i_{c23}$](image7)

**Fig. 7.** Time diagrams of the mutual capacitive currents

The time diagrams of memristor current at different resistances $R_{ON}$ at opened state $R_{OFF}$ are presented in Fig. 8. The two currents investigated almost coincide. That means that the alteration of the memristor resistance in closed state in very wide range (about 50 %) do not affect the current. But the resistance $R_{ON}$ is one of the quantities that determine the whole amount of charge that could be memorized in the memristor cell. When the quantity $Q_d$ decreases the memristor needs less time and current intensity to pass over from opened to closed state and vice versa.

![Time diagrams of memristor current at different $R_{ON}$](image8)

**Fig. 8.** Time diagrams of the memristor currents at different $R_{ON}$

The time diagrams of memristor current at different resistances at opened state $R_{OFF}$ are presented in Fig. 9. It is obvious that the changes in this parameters with 50 % strongly affect the magnitude of the memristor current pulses. When the value of $R_{ON}$ remains constant but $R_{OFF}$ increases the ratio between $R_{OFF}$ and $R_{ON}$ increases too. Then the current pulses have less distortions but the switching properties of the element are decreased.
The time diagram of whole source power $s_j$ is presented in Fig. 10. It is clear that there is a little permanent component in the power due to the active power consumed by the memristor. In some time intervals there is power pulses due to the action of the reactive parasitic elements – mainly due to the mutual capacitances and inductances.

The time diagram of the power consumed by the memristor $M_j$ is shown in Fig. 11. In the duration of the plateaus of the pulses the power has constant value but when there is transient process the power increases rapidly.

V. CONCLUSIONS

From the results presented above it is obvious that the parasitic own parameters and the mutual inductances and capacitances slightly affect the parameters and the characteristics of Williams’s memristor. The influence of the parasitic mutual parameters is mainly on the fronts of the pulses. The transient process has pseudo-periodic character and the first ricochet has very high magnitude. At frequencies up to 3 GHz there are no losses of information and the pulses of logical zero and logical one have its nominal values. The changes in parameters of the memristors have stronger effect on the characteristics of the circuit investigated. The changes of $R_{OFF}$ very slightly affect the current pulses. The altering of parameter $R_{OFF}$ has effect on changing of the magnitude of current pulses and the switching properties of the memristors. The result of presented investigations is that the changes of the ionic mobility in very high interval do not affect the characteristics of the memristor circuit at high frequencies.

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