Analysis of temperature influence on titanium-dioxide memristor characteristics at pulse mode

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***Abstract* — The main goal of this paper is to investigate the influence of the temperature on the memristor properties and on the inner diffusion rate. Based on experimental data a relationship between ionic mobility and temperature is extrapolated. Then the dependencies between resistances of the memristor in open and closed states and the temperature are given in analytical and graphical form. Based on a SIMULINK algorithm the temperature dependence of maximal quantity of charge memorized is given,. The influence of the temperature on memristor characteristics are presented using MATLAB models in pulse mode. The diffusion processes in Williams's memristor are analyzed. In the end some concluding remarks about the complex influence of temperature on Williams's memristor are presented.**

Keywords—titanium-dioxide memristor;charge carriers mobility; temperature dependence; impulse mode; diffusion.

I. INTRODUCTION

The memristor has been predicted in 1971 by Professor Leon Chua [1]. The first physical prototype of the memristor was invented in 2008 by Stanley Williams of Hewlett-Packard [2, 3]. That memristor consists of two sub-layers of titanium dioxide, sandwiched between two platinum electrodes. The basic unique property of this nonlinear element is to memorize the full amount of charge which has passed through it [4, 5]. Many research results and simulations on the memristor investigation have been made in the last few years [6, 7]. The main properties and the principle of operation of Williams's memristor have been described. Some physical relationships between the basic electrical quantities of the memristor are presented. The resistances of the memristor in opened and closed states, respectively R_{OFF} and R_{ON} are very important parameters for its operation. The charge carriers' mobility μ_v is also one of the basic parameters of the element. These parameters are dependent on the memristor temperature but in the papers published no data on this topic have been found. The main purpose of this paper is to propose adequate models of these temperature dependencies of the titanium-dioxide memristor.

In section II the relationship between charge carriers mobility and the temperature of the memristor is described. The temperature dependences of resistances of the titanium-dioxide memristor in open and closed states are depicted in Section III. The main formulae and SIMULINK model and the results taken of a simple memristor circuit with temperature influence accounting are given in Section IV. In Section V the diffusion processes in the memristor are analyzed. Final conclusions and remarks about the temperature influence on memristor parameters and characteristics are presented in Section VI.

II. DEPENDENCE BETWEEN CHARGE CARRIERS' MOBILITY AND TEMPERATURE

In [8] an experimental diagram of the relationship between oxygen vacancies mobility and temperature of pure titanium dioxide is given. It is interesting that in this type of material the derivative of the experimental curve is positive. The oxygen vacancies mobility at room temperature has a value of $\mu_v = 1.10^{-14}$ ($m^2/V.s$). Using these experimental data an extrapolation polynomial of 20^{-th} power is created in MATLAB environment [9]. It is given with Eq. (1):

$$\mu_v = 1.10^{54}\left(-0,0005T^{-18} + 0,2394T^{-17}\right) \qquad (1)$$

The graph of this polynomial is presented in Fig. 1. It is clear that the charge mobility increases rapidly with increasing the memristor temperature.

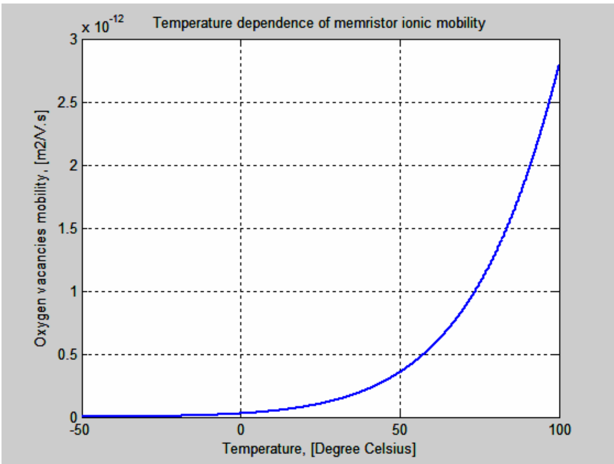

Fig. 1. Dependence between charge carriers' mobility and Celsius temperature

III. DEPENDENCE BETWEEN RESISTANCES OF THE MEMRISTOR IN OPEN AND CLOSED STATES AND THE CELSIUS TEMPERATURE

An experimental relationship between the specific conductivity of titanium-dioxide σ and the absolute temperature T is presented in [10]. The quantities on the axes are in logarithmic scale. The dependence between them is approximately presented with Eq. (2):

$$\lg \sigma = 5{,}72 \lg T - 9 \tag{2}$$

After processing of (2) and expression of specific conductivity with the resistance R_{OFF} we have the next formula (3):

$$R_{OFF} = 2{,}06.10^{18} T^{-5{,}72} \quad [\Omega] \tag{3}$$

The resistance of the memristor in closed state has more steep temperature relationship, expressed with Eq. (4):

$$R_{ON} = 6{,}33.10^{16} T^{-6} \quad [\Omega] \tag{4}$$

In Fig. 2 the relationship between R_{OFF} and the Celsius temperature t^0 is presented. It is obvious that the resistance R_{OFF} decreases lightly with increasing the temperature.

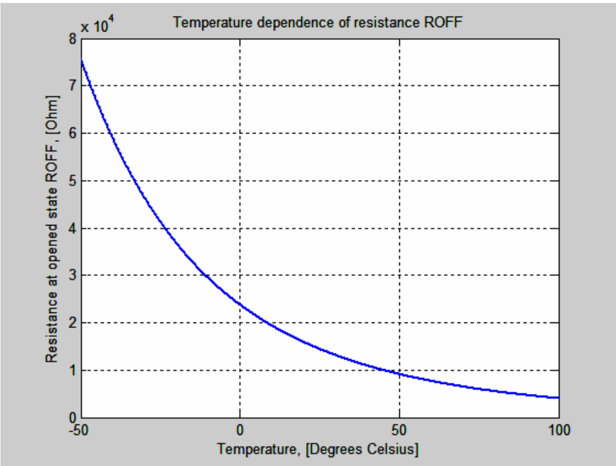

Fig. 2. Dependence between memristors' resistance in opened state R_{OFF} and the Celsius temperature t^0

In Fig. 3 the relationship between R_{ON} and the Celsius temperature t^0 is shown. The resistance in closed state decreases with increasing the temperature. The character of these relationships is due to the thermal generation of charge carriers in this type semiconductor material. The changing of these resistances is non-desirable at a working mode.

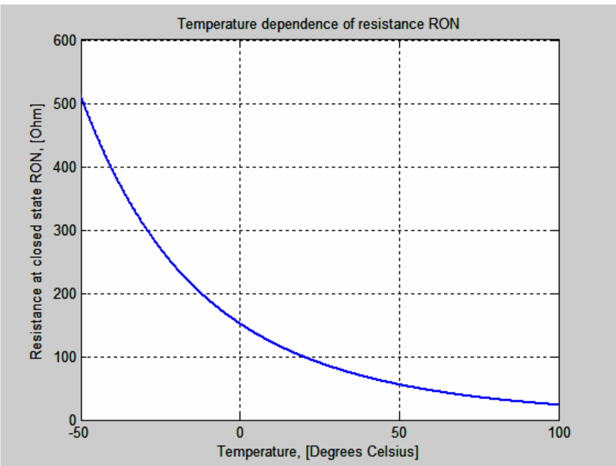

Fig. 3. Dependence between memristors' resistance in closed state R_{ON} and the Celsius temperature t^0

IV. Synthesis and Analysis of a SIMULINK Model of a Simple Memristor Circuit at Impulse Mode

The basic relationship between memristor current *i(t)* and voltage across it *u(t)* is presented with (5) [2]:

$$i(t) = \frac{u(t)}{R_{OFF}\sqrt{\left(1 - \frac{q(t_0)}{Q_D}\right)^2 - \frac{2\eta}{Q_D R_{OFF}} \int u(t)dt}} \tag{5}$$

where $q(t_0)$ is the initial charge in the doped region, and Q_D is the quantity of charges which the memristor could memorize in its whole volume. This quantity Q_D is presented with (6) [2]:

$$Q_D = \frac{D^2}{\mu_v R_{ON}} \tag{6}$$

where the constant D is the length of the whole memristor. The temperature dependence of the maximal charge memorized in the memristor Q_d is presented in Fig. 4.

This curve is monotonically decreasing line. The quantity Q_d varies in very large interval with varying of the Celsius temperature. This phenomenon is non-desirable because of changing the logical levels of the signals.

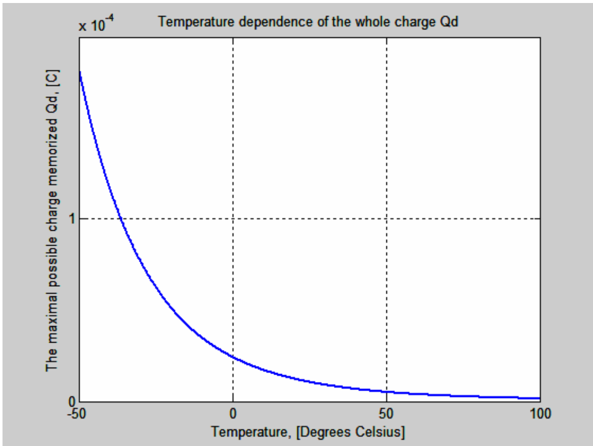

Fig. 4. Dependence between memristors' full possible charge memorized Q_d and the Celsius temperature t^0

The electrical scheme of the circuit investigated is shown in Fig. 5.

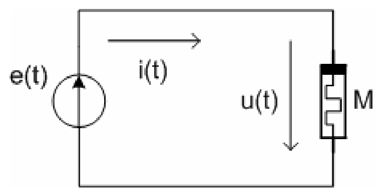

Fig. 5. Electrical scheme of the circuit investigated

Based on the formulae presented above and on the circuit presented in Fig. 5 a SIMULINK model of memristor circuit with one memristor and with a pulse voltage source is created. It is shown in Fig. 6.

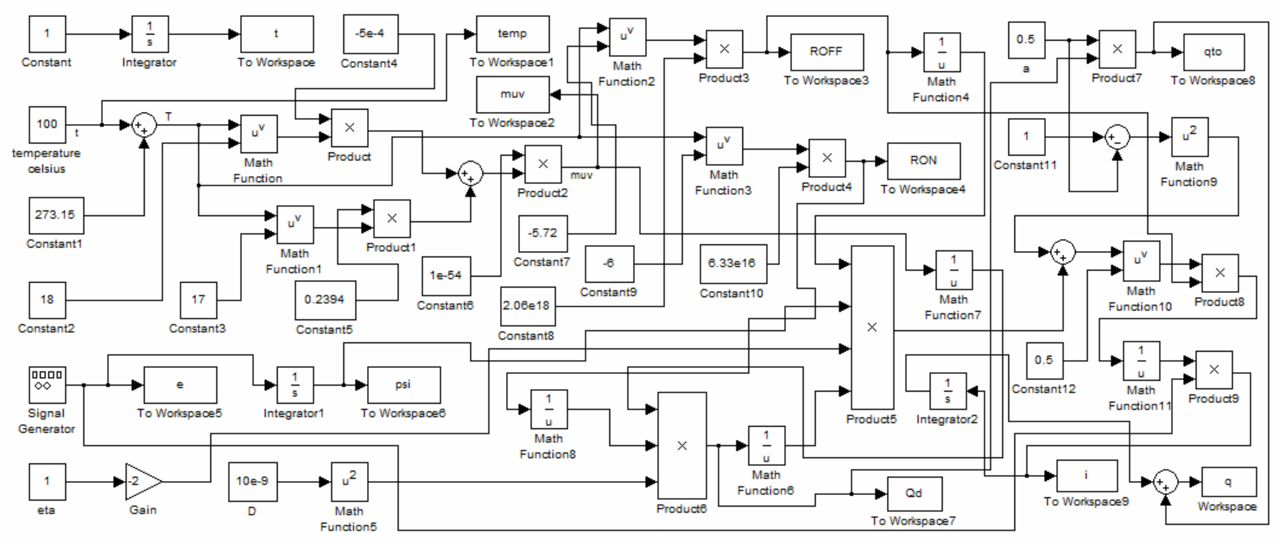

Fig. 6. SIMULINK model of the circuit investigated

The time diagrams of memristor current and charge accumulated in the memristor are presented at different temperatures. The time diagram of source voltage is presented in Fig. 7.

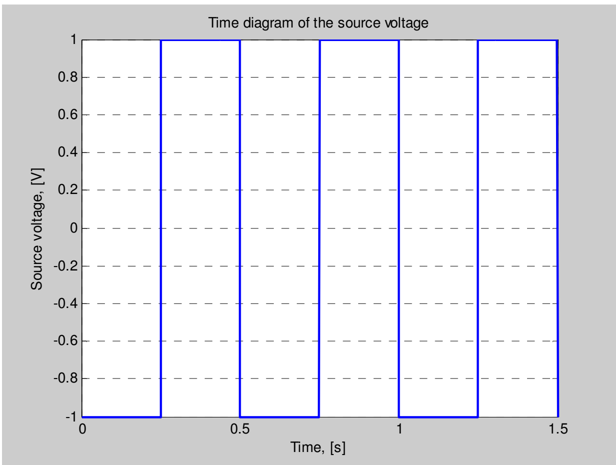

Fig. 7. Time diagram of the source voltage

The time diagrams of memristor current at different temperatures are presented in Fig. 8 and Fig. 9. It is clear that when the ambient temperature increases, the current intensity increases too. But this effect is more obvious at low temperatures.

In the temperature range between 25 and 75 degrees Celsius this effect has very slight influence on the increasing of the current magnitude.

The influence of the temperature on the charge accumulated in the memristor is presented in Fig. 10 and Fig. 11. It is clear from the characteristics in graphical form that with increasing the temperature the charge accumulated in the memristor decreases.

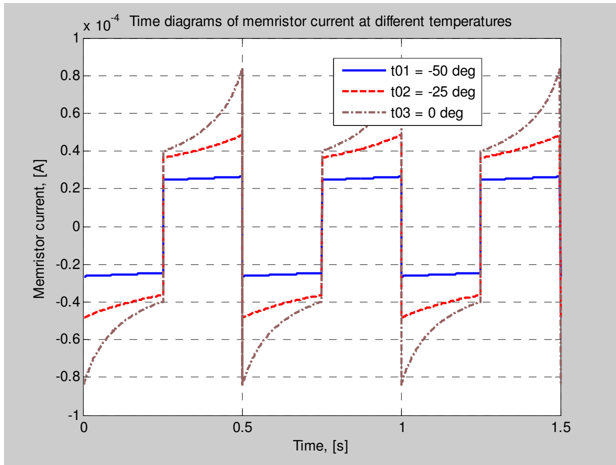

Fig. 8. Time diagram of memristor current at temperatures -50, -25 and 0 degrees Celsius

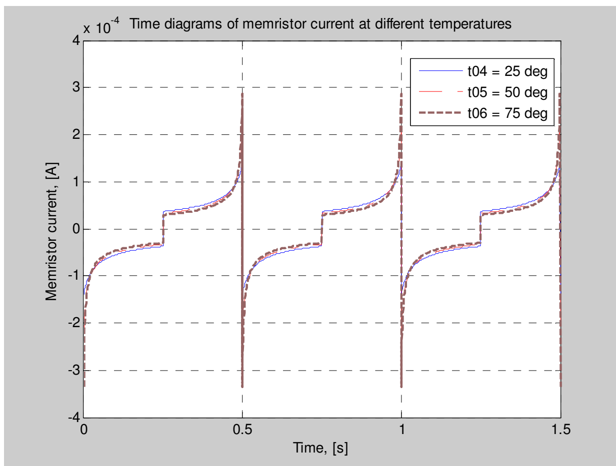

Fig. 9. Time diagram of memristor current at temperatures 25, 50 and 75 degrees Celsius

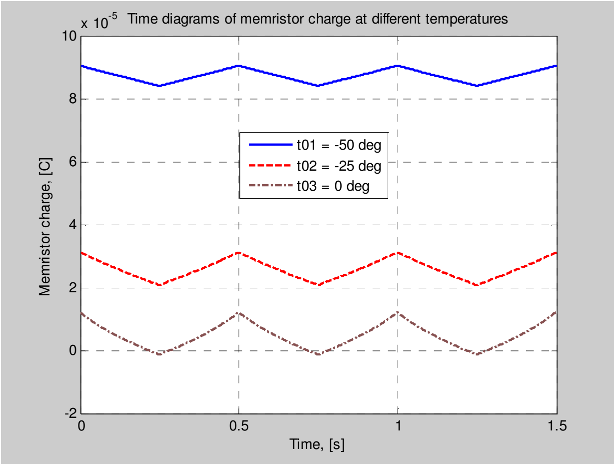

Fig. 10. Time diagram of memristor charge q at temperatures -50, -25 and 0 degrees Celsius, at $q_{t0} / Q_d = 0{,}5$

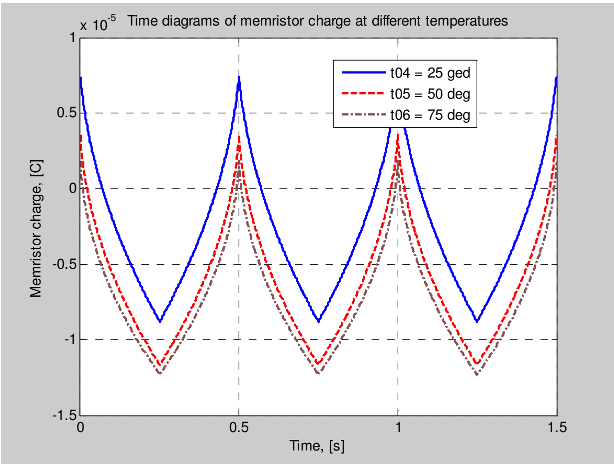

Fig. 11. Time diagram of memristor charge q at temperatures 25, 50 and 75 degrees Celsius, at $q_{t0} / Q_d = 0{,}7$

The current-voltage characteristics of the memristor at different temperatures are given in Fig. 12.

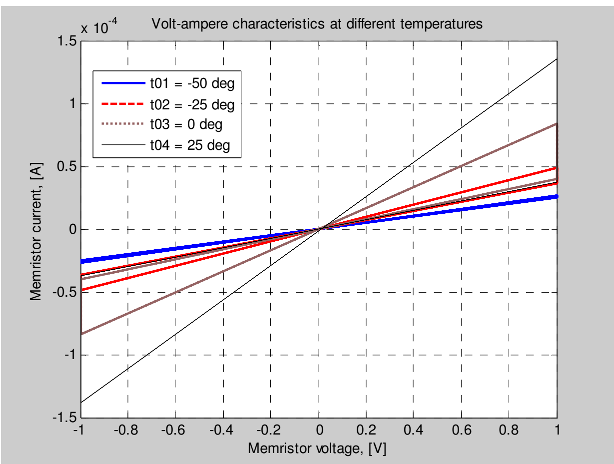

Fig. 12. Volt-ampere characteristics of the memristor at several different Celsius temperatures

The power consumed by the memristor cell at several different temperatures is given in Fig. 13 and Fig. 14.

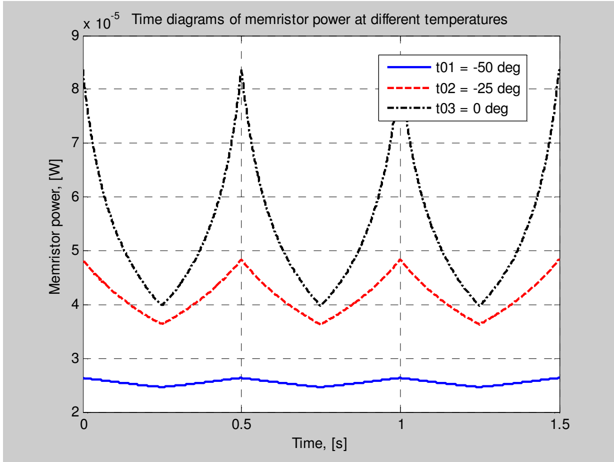

Fig. 13. Time diagrams of the memristor power consumed at temperatures-50, -25 and 0 degrees Celsius

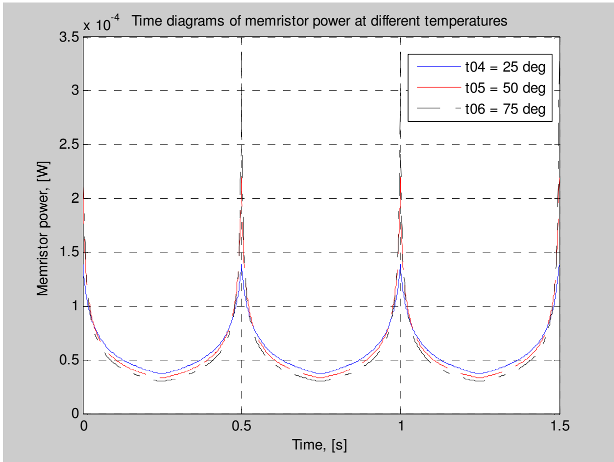

Fig. 14. Time diagrams of the memristor power consumed at temperatures 25, 50 and 75 degrees Celsius

V. INVESTIGATION OF THE INNER DIFFUSION PROCESSES IN THE WILLIAMS'S MEMRISTOR

The structure scheme of a titanium-dioxide memristor is presented in Fig. 15. In the structure of the Williams's memristor a concentration gradient exists near to the boundary between the doped and the undoped regions [11].

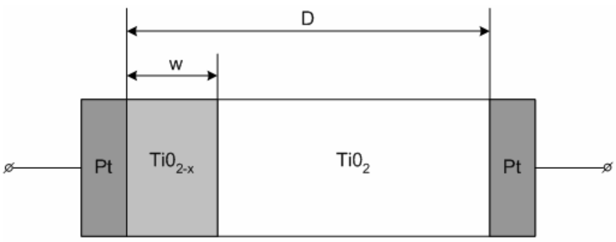

Fig. 15. Structure scheme of a titanium-dioxide memristor

This concentration gradient of the oxygen vacancies causes diffusion from the doped layer to the undoped region which is equivalent to flowing of a diffusion current. The density of this diffusion current J *[A/m²]* is presented by the first Fick's law [12, 13]:

$$J_{V_{o^{2-}}} = -qD\frac{\partial N}{\partial x} \tag{7}$$

where $q = 2.e = 3,2.10^{-19}$ *[C]* is the charge of an oxygen vacancy, D *[m²/s]* is the diffusion coefficient, N *[m⁻³]* is the volumetric concentration of oxygen vacancies in the memristor, and x is the coordinate in which the diffusion realizes. The diffusion coefficient of the oxygen vacancies is presented with Eq. (8) [12]:

$$D = \frac{\mu_v k_B T}{q} \tag{8}$$

where $k_B = 1,3787.10^{-23}$ *[J/K]* is the Boltzmann constant. The dependence between the coefficient D and the temperature is drawn up in MATLAB environment using (1) and Eq. (8) and it is presented in Fig. 16. It is clear that the coefficient of diffusion D increases rapidly with increasing the temperature.

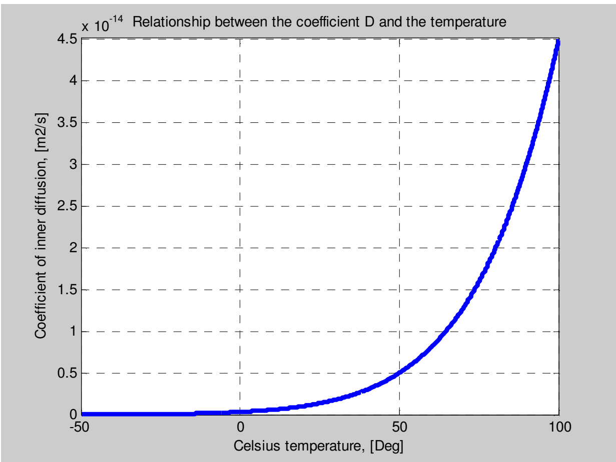

Fig. 16. Relationship between the coefficient of inner diffusion D and the Celsius temperature

The volume of a memristor cell may be calculated as a volume of rectangular parallelepiped with length $l = 10$ *nm*, width $a = 50$ *nm* and height $b = 50$ *nm*:

$$V_{total} = lab = 10.10^{-9}.(50.10^{-9})^2 = 2,5.10^{-23}\,[\mathrm{m}^3] \tag{9}$$

The volume of the doped region is:

$$V_{doped} = l_d ab = 0,1.V_{total} = 2,5.10^{-24}\,[\mathrm{m}^3] \tag{10}$$

The density of titanium-dioxide is $\rho = 4230$ [*kg/m³*]. Its molar weight is $M(TiO_2) = 79,88$ [*g/mol*] [13, 14]. The weight of a molecule of TiO_2 is:

$$m_{TiO_2} = \frac{M(TiO_2)}{N_A} = \frac{79,88}{6,02.10^{23}} = 1,3269.10^{-22}\,[\mathrm{g/molec}] \tag{11}$$

where N_A is the Avogadro number [14].

The number of the molecules of TiO_2 in $1\ m^3$ is:

$$N_{TiO_2} = \frac{\rho_{TiO_2}}{m_{TiO_2}} = \frac{4230}{1,3269.10^{-25}} = 3,1879.10^{28}\,[\mathrm{m}^{-3}] \tag{12}$$

The concentration of the oxygen vacancies in the doped layer of the memristor is:

$$N_{V(O^{2-})} = xN_{TiO_2} = 0,03.3,18.10^{28} = 9,56.10^{26}\,[\mathrm{m}^{-3}] \tag{13}$$

The number of oxygen vacancies in the doped layer is:

$$n_{V(O^{2-})} = V_{doped}N_{V(O^{2-})} = 2,5.10^{-24}.9,56.10^{26} = 2390 \tag{14}$$

The ionic diameter of an oxygen vacancy has approximate value of d = 0,2 nm [14]. The number of atomic layers in the doped region is:

$$n_{at.layers} = \frac{w}{d} = \frac{1}{0,2} = 5 \tag{15}$$

Then the number of oxygen vacancies nearest to the boundary between the two sub-layers is:

$$n_{V(O^{2-})at.layer} = \frac{n_{V(O^{2-})}}{n_{at.layers}} = \frac{2390}{5} = 478 \tag{16}$$

The surface concentration of oxygen vacancies Q_0 is:

$$Q_0 = \frac{n_{V(O^{2-})at.layer}}{ab} = \frac{478}{(50.10^{-9})^2} = 1,91.10^{17}\,[\mathrm{m}^{-2}] \tag{17}$$

The second Fick's law [12] for the process of diffusion in the memristor is presented with Eq. (18):

$$\frac{\partial N_{V(O^{2-})}}{\partial t} = D\frac{\partial^2 N_{V(O^{2-})}}{\partial x^2} \tag{18}$$

In the case presented here diffusion from limited source of dopant exists. The initial and the limit conditions are:

$$\begin{aligned} &N(x,t) = N_S, && t = 0, \quad x = 0; \\ &N(x,t) = 0, && t \succ 0, \quad x \to \infty; \\ &\frac{dN}{dx} = 0, && 0 \le t \le \infty, \quad x = 0 \end{aligned}$$

The solution of Eq. (18) at the initial and limit conditions presented above is:

$$N(x,t) = N_s^{'} \exp\left(-\frac{x^2}{4Dt}\right) [m^{-3}] \quad (19)$$

where N_s^I is the instant volumetric concentration near the boundary between the doped and undoped layers:

$$N_s^{'} = \frac{Q_0}{\sqrt{\pi D t}} [m^{-3}] \quad (20)$$

The relationships between concentration of oxygen vacancies N and the length x at three different values of the diffusion time t are presented in Fig. 17. It is obvious that with increasing the diffusion time the concentration of oxygen vacancies decreases. This is due to the penetration of the dopant in the volume of the memristor and its depletion of oxygen vacancies.

With increasing the distance from the dopant source the concentration of oxygen vacancies decreases but after a very long time the concentration of the dopant ions equalizes in the whole volume of the memristor cell.

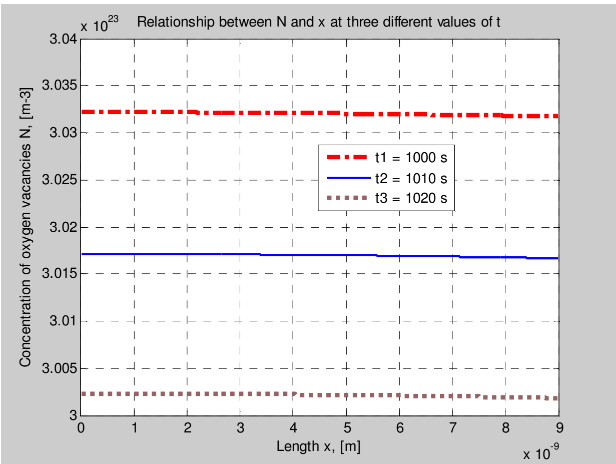

Fig. 17. Relationship between the concentration of oxygen vacancies N and the length of penetration of the charges

VI. CONCLUSIONS

It is clear from the results presented above that with increasing the temperature the charge carriers' mobility increases too but the resistances of the memristor at opened and closed states decrease. As a result at absolute temperature of $T = 400$ K the whole charge of the memristor that it can memorize is about 100 times less than its value at a room temperature. The current through the memristor investigated is higher than the current at room temperature. So the quickness of the memristor increases too and the operational mode changes. But the characteristics of the memristor as an electronic switch are worse at high temperature.

The diffusion processes in the Williams's memristor are parasitic phenomena and they are due to the concentration gradient around the boundary between the doped and undoped regions. The coefficient of diffusion D increases with increasing the ambient temperature. This is an undesired process because the inner diffusion in the memristor causes losses of information. For example, if in the memristor cell has been written logical zero (the resistance of the element is near to R_{OFF}), after a very long time because of the diffusion the dopant penetrates from the doped region to the whole volume of the memristor so that the resistance will have a new value – R_{ON}. This process of loosing of information is quicker at very high temperatures.

In the end it could be said that the increasing of ambient and working temperatures of Williams's memristor has very bad influence on its properties and characteristics. For improvement of memristors's properties in working mode the use of a cooling device is recommended.

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