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PREVENTIVE MAINTENANCE FOR INDUSTRIAL APPLICATION

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Abstract: Preventive maintenance procedures are scheduled actions carried out to either minimize the chance of a failure or to improve the reliability of the system. In many cases, the failure rate is increasing with time, so preventive maintenance restores the system to a state with a lower failure rate. The question is how often preventive maintenance should be scheduled. This paper is presents an algorithm to determine the probability density functions of time-to-failure and recovery time and the reliability and maintainability functions of electrical equipment. In order to schedule preventive maintenance the following preventive maintenance models are discussed: a constant interval replacement/repair model and a replacement/repair at predetermined age model. Depending on the type of electrical equipment – its complexity, its function in the production process, a cost minimization criterion or downtime minimization criterion is used. *Copyright 2001 IFAC*

Keywords: preventive maintenance, reliability, maintainability, distribution, failure, lifetime

1. INTRODUCTION

All man-made systems including electrical equipment have a limited reliability in the sense that their performance deteriorates with age and usage and they finally fail when they are unable to perform their required function under specified operating conditions.

The approaches to maintaining unreliable systems have changed significantly over the last 50 years (Niebel, 1994; Blischke and Murthy 2000). Up to about 1940, maintenance was considered as unavoidable costs and the entire maintenance that was carried out usually was corrective maintenance. Whenever a system failure took place, a specialized maintenance workforce was needed to make the system operational. Maintenance was neither incorporated into the design of the system, nor was the impact of maintenance on system and business performance duly recognized. The development of operation research and their applications during the Second World War to its subsequent use in industry led to the widespread utilization of preventive maintenance. Since 1950 operations research models for maintenance have appeared. It deals with the effect of different maintenance policies and optimal determination of policy parameters. The developed models focussed on the operational level by looking at either the cost of maintenance or some operational performance measure of the system. New defense acquisitions by the United States government required a life cycle costing approach, with maintenance cost being a significant component, and the close linkage between reliability (R) and maintainability (M) was recognized. As a result, the term "R&M" became more widely used in non-civilian systems. This concept was also adopted by manufacturers and operators of civilian aircrafts through the methodology of reliability-centered maintenance in the United States. Concurrently, the Japanese evolved a concept of total productive maintenance in the context of manufacturing.

Two major developments in the mathematical theory of reliability took place in the early 1950's:

- In 1951, W. Weibull of the Royal Institute of Technology, Stockholm, published a statistical distribution to represent the breaking strength of materials (Weibull, 1951).
- In 1952, D. J. Davis presented failure data and the results of several goodness-of-fit tests for competing failure probability distribution. This work provided the support for the assumption of exponential failure distribution, which is widely used today to represent the failure behavior of various engineering items (Davis, 1952).

Every electrical equipment has an inherent reliability that is determined by design and manufacturing decisions. When this electrical equipment is put into use, it deteriorates, thus reducing its reliability. System performance is acceptable as long as the actual reliability is at or above the desired level. Once the actual reliability falls below this level, corrective actions are required in order to restore the reliability of the system to at least an acceptable level.

Maintenance can be defined as actions to control the deterioration process leading to failure of a system and restore the system to its operational state through corrective actions after a failure. Maintenance is of importance to manufacturers as well, since the ease and ability to carry out maintenance actions depends on the inherent properties of system design.

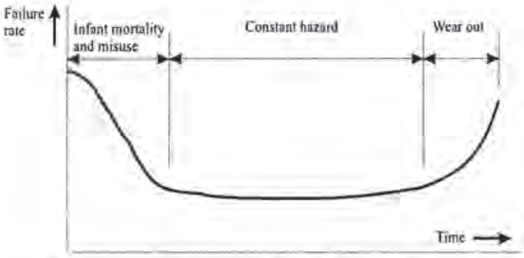

Fig. 1 Theoretical relationship between the magnitude of failure and time

Equipment failure may be classified by four cases: infant mortality failures, chance failures, abuse or misuse failures and wear out failures. Not all equipment will have these four causes of failure. A theoretical relation between the magnitude of failure and time for each of these four causes of failure is shown in fig.1 (Niebel, 1994). Wear out failures begins at the end of the useful life period of the electrical equipment. These failures are no longer random and their causes include aging, friction, wrong overhaul practices, poor maintenance and corrosion. Wear out can usually be postponed by proper preventive maintenance.

Preventive maintenance actions are intended to control the deterioration in reliability and to ensure that the actual reliability is at or above the desired level. Preventive maintenance also includes inspections and regular maintenance activities, such as lubrication, cleaning of lines, and changing of filters, planned in order to avoid unforeseen failure of equipment and help to assure that equipment is operating in a satisfactory way. However, in the majority of electrical equipment, it is important that certain maintenance operations take place in advance of a failure to avoid breakdowns and to extend equipment lifetime.

Any industry today that does not have an active preventive maintenance program is undoubtedly inefficient. The indications of a poorly conceived program or the lack of a program are:

- low machine and equipment utilization because of unscheduled work stops due to breakdowns,
- long waiting or idling time of operators because of the machine or equipment downtime,
- high scrap and rejects due to unreliability of the equipment,
- increase in the cost of repairs of equipment because of neglecting regular lubrication, inspection and replacement of worn parts,
- decrease of the expected productive life of capital equipment because of a lack of adequate maintenance.

Then the basic objectives of preventive maintenance include:

- to minimize the number of breakdowns of critical equipment,
- to reduce the loss of production that occurs when equipment failures take place,
- to increase the productive lifetime of all capital equipment,
- to acquire meaningful data related to the history of all capital equipment so that sound decisions as repair, overhaul or replacement can be made to maximize the return on capital investment,
- to obtain a better planning and scheduling of required maintenance work,
- to promote better safety and health of the work force.

As mentioned above, the wear out failures can be prevented with preventive maintenance only. The failure rate is increasing with time, so preventive maintenance restores the system to a state with a lower failure rate. The question is how often preventive maintenance should be scheduled so that to have the maximum utilization of the electrical equipment and maximum availability of the electrical equipment. A high frequency for preventive maintenance increases the total cost of maintenance and reduces the cost due to downtime of the system. A low frequency for preventive maintenance decreases the cost of maintenance but increases the cost due to the downtime of the system. Hence, the fundamental task is finding the optimum preventive maintenance time.

2. MODELS FOR PREVENTIVE MAINTENANCE

Appropriate models for preventive maintenance are constant interval replacement/repair and replacement/repair at predetermined age.

There are many situations where the availability of the electrical equipment is more important than the cost of repair or maintenance. Indeed, the consequences of the downtime of equipment may exceed any measurable cost. In such cases, it is more appropriate to minimize the downtime per unit time than to minimize the total cost per unit time. For this reason, depending on the type of electrical equipment - its complexity, its function in the production processes - could are used the following two optimization criteria: cost minimization criterion or downtime minimization criterion. It is important to note that the following assumptions are made:

- the total cost associated with a failure replacement/repair is greater that the costs associated with a preventive maintenance action,
- the function of the system's failure rate is monotonically increasing with time,
- minor repairs do not change the failure rate of the system.

2.1. Model for Constant Interval Replacement/Repair (MCIRR)

MCIRR using cost minimization criterion. Using this model, two types of actions are performed:

- preventive replacement/repair that occurs at fixed interval of time, and
- failure replacement/repair where parts are replaced/repaired upon failure.

The total cost is the sum of the expected cost of failure replacements/renewals and the cost of the preventive replacement/repair. During the time interval $(0,t_P]$, one preventive replacement/repair is performed at a cost of c_P and $M(t_P)$ failure replacements/repairs at a cost of c_f each. Hence, the expected length of the interval is t_P and it may be obtained by minimizing the total expected replacement/repair cost per time unit given by following expression

$$c(t_P)=\frac{c_P+c_f M(t_p)}{t_P} \tag{1}$$

MCIRR using downtime minimization criterion. The objective is to minimize the total downtime per unit of time – that is, to minimize the unavailability of the equipment. In this case, replacements/repairs are performed at predetermined times regardless of the age of the equipment being replaced/repaired. In addition, replacements/repairs are performed upon failure of equipment. Hence, we have to minimize the following expression

$$D(t_P)=\frac{M(t_P)T_f+T_P}{T_P+t_P} \tag{2}$$

where

$M(t_P)$ expected number of failures in the interval $(0,t_P]$,

T_f time to perform failure replacement/repair,

T_P time to perform preventive replacement/repair.

2.2. Model for Preventive Replacement/Repair at Predetermined Age (MPRRPA)

MPRRPA using cost minimization criterion. The disadvantage of the MCIRR is that the units or components are replaced/repaired at failures and at a constant interval of time since the last preventive replacement/repair. This may result in performing preventive replacements/repairs on units shortly after failure replacements/repairs.

For MPRRPA the units are replaced/repaired upon failure or at an age t_P, whichever occurs first. Then, the optimum value of the length of the preventive replacement/repair cycle is obtained by determining t_P that minimizes the total cost per unit time given with

$$c(t_P)=\frac{c_P R(t_P)+c_f[1-R(t_P)]}{t_P R(t_P)+\int_{-\infty}^{t_P} t f(t)dt} \tag{3}$$

where

$R(t_P)=\int_{t_P}^{\infty} f(t)dt$ is the probability that the equipment survives during the planned replacement/repair age and

$1-R(t_P)=F(t)=\int_{0}^{t_P} f(t)dt$ is the probability of equipment failure before t_P,

c_P the cost of preventive replacement/repair,

c_f the cost of failure replacement/repair.

The equation (3) can be rewritten as

$$c(t_P)=\frac{c_P\int_{t_P}^{\infty} f(t)dt+c_f\int_{0}^{t_P} f(t)dt}{t_P\int_{t_P}^{\infty} f(t)dt+\int_{-\infty}^{t_P} t f(t)dt} \tag{4}$$

MPRRPA using downtime minimization criterion. Using this model, preventive replacements/repairs are performed upon equipment failure or when the equipment reaches an age of t_P. The objective is to determine the optimal preventive replacement age t_P that minimizes the downtime per unit time

$$D(t_P)=\frac{T_P R(t_P)+T_f[1-R(t_P)]}{(t_P+T_P)R(t_P)+\left[\int_{-\infty}^{t_P} tf(t)dt+T_f\right][1-R(t_P)]} \tag{5}$$

3. TIME-TO-FAILURE DISTRIBUTION, RELIABILITY AND MAINTAINABILITY FUNCTIONS OF ELECTRICAL EQUIPMENT

The Time-To-Failure (TTF) for electrical equipment is a random variable, and can be represented by a probability distribution. A few basic failure distributions are useful for modeling TTF: exponential distribution, Weibull distribution, normal distribution, lognormal distribution, Gamma distribution, Erlangian distribution or Rayleigh distribution (Blischke and Murthy 2000).

To use some of the indicated above preventive maintenance models, have to know the TTF distribution, its probability density function f(t), parameters and its reliability function R(t). Fig.2 presents an algorithm for determining the probability density functions of TTF and Time-To-Recovery (TTR), the reliability and the maintainability functions of electrical equipment.

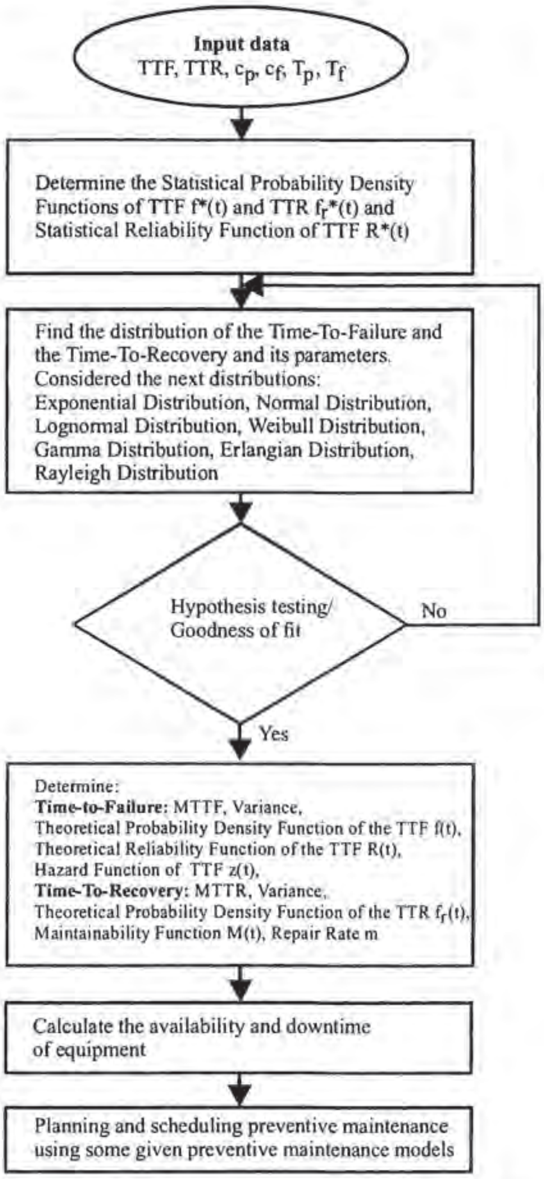

Fig. 2 Algorithm to determine the reliability and the maintainability functions of the electrical equipment

To estimate the parameters of TTF distribution and TTR distribution we need the complete information of mean TTF, mean TTR, and its variances.

The Mean Time-To-Failure MTTF is

$$MTTF = \frac{\sum_{i=1}^{n} TTF_i}{n} \tag{6}$$

where n is the number of failures.

The variance of TTF is

$$s^2 = \frac{\sum_{i=1}^{n} (TTF_i - MTTF)^2}{n-1} \tag{7}$$

For estimating the statistical failure density function the following expression is applied

$$f^*(t)_i = \frac{\Delta N_i}{n \Delta t} \quad \text{for i=1,2,\dots,m} \tag{8}$$

where

$m = 5 \lg n$ number of intervals for $6 \le m \le 20$,

$\Delta t = \frac{\max TTF}{m}$ size of time interval,

max TTF maximum TTF,

ΔN_i number of failures in each time interval.

The statistical reliability function is

$$R^*(t)_i = 1 - \frac{\sum_{1}^{i} \Delta N_i}{n} \tag{9}$$

To find the maintainability function of electrical equipment we have to know the distribution of the TTR and its probability density function $f_r(t)$. Then, the Mean Time-To-Recovery MTTR is

$$MTTR = \frac{\sum_{i=1}^{n} TTR_i}{n} \tag{10}$$

The variance of the recovery time is

$$s_r^2 = \frac{\sum_{i=1}^{n} (TTR_i - MTTR)^2}{n-1} \tag{11}$$

For estimating the statistical density function of the TTR the following expression is made use of

$$f_r^*(t)_i = \frac{\Delta N_{ri}}{n \Delta t_r} \quad \text{for i=1,2,\dots,m} \tag{12}$$

where:

$m = 5 \lg n$ number of intervals,

$\Delta t_r = \frac{\max TTR}{m}$ size of time interval,

max TTR maximum recovery time,

ΔN_{ri} number of renewals in each time interval.

Therefore, the maintainability is defined as

$$M(t) = P(T < t) = \int_0^t f_r(x) dx \tag{13}$$

where P(T<t) is the probability of completing the repairs in a time is less then T and $f_r(x)$ is the repair time probability density function.

The classic chi-square test due to Karl Pearson may be applied to test the fit data to any specific distribution. The test statistic is

$$\chi^2 = \sum_{i=1}^{m} \frac{(v_i - v_{iT})^2}{v_{iT}} \qquad (14)$$

In the goodness-of-fit application of this test, is assumed a sample of size n, with each observation falling into one of m possible classes. v_i is the observed frequency in class i; v_{iT} is the frequency that would be expected if the specified distribution were the correct one. In calculating χ^2, it is necessary to assume that expected frequencies are not too small; if they are, this may greatly distort the result. A rule of thumb is that if $v_{iT} < 1$ for some i, combine that class with either the previous or the succeeding class, repeating the process until $v_{iT} \geq 1$ for all i. If $sign(\chi^2 - \chi_T^2) < 0$ the hypothesis of distribution is accepted and if $sign(\chi^2 - \chi_T^2) \geq 0$ the hypothesis of distribution is not accepted. Here $\chi_T^2 = f(m - r - 1, \alpha)$ and it is found from special table of χ^2 (Blischke and Murthy 2000). In the former case, it is usually appropriate to test at a low level of significance, say α=0.01, the idea being that we do not want to reject the theoretical distribution unless there is strong evidence against it. In the second case, it may be more appropriate to test at a much higher level of significance, say α=0.1 or 0.2, in order to narrow the list of candidates somewhat. As always, the choice is to a great extent subjective and depends on the particular application. This χ^2 test has the advantages of being easy to apply and being applicable even when parameters are unknown.

As an example, a metal-cutting machine with digital program control is considered. Its TTF and TTR are given in APPENDIX.

MTTF, MTTR and its variances are obtained by using expressions (6), (10), (7) and (11)

MTTF=6250.3 min and *MTTR=798.9 min*
σ^2*=6503.1 min* and σ^2_r*=1380.7 min.*

Using MATLAB checks the type of TTF distribution and TTR distribution. To obtain the statistical density function of TTF and statistical reliability function have to been found: maxTTF=26455min, the intervals number m=8, the size of time interval $\Delta t = \dfrac{\max TTF}{m} = 3307$ min.

In the Table 1 are given the number of TTF for each interval ΔN_i, statistical density function and reliability function for each interval calculating by expressions (8) and (9) and its plots are shown in fig.3 and fig.4, respectively. Fig.4 shows the theoretical reliability function too.

Table 1 Statistical density and reliability functions

Interval number	Time interval [min]	ΔN_i	$f^*(t).10^{-4}$	$R^*(t)$
1	[0 ; 3307]	14	1.176	1.000
2	(3307 ; 6614]	10	0.840	0.6110
3	(6614 ; 9921]	3	0.252	0.3330
4	(9921 ; 13228]	2	0.168	0.2500
5	(13228 ; 16535]	5	0.420	0.1940
6	(16535 ; 19842]	1	0.084	0.0556
7	(19842 ; 23149]	0	0.000	0.0278
8	(23149 ; 26455]	1	0.084	0.0278

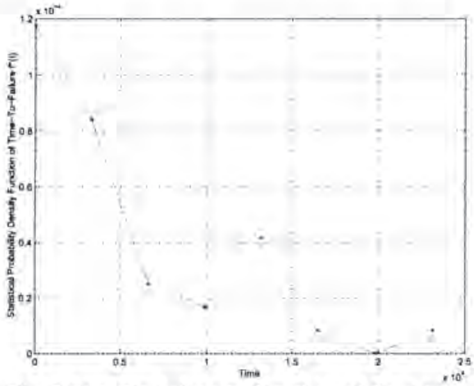

Fig. 3 Statistical density function of time-to-failure f*(t)

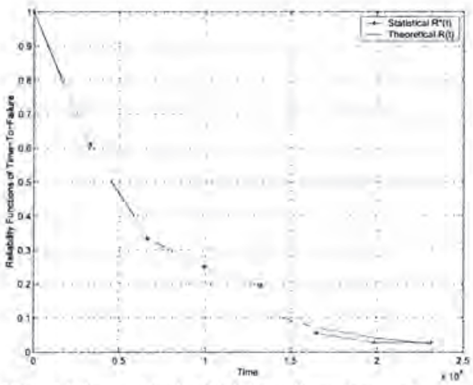

Fig. 4 Theoretical reliability function and statistical reliability function of TTF

Hence, the goodness-of-fit is obtained when TTF is modeled with an exponential distribution with parameter $\lambda = 1.5999.10^{-4}$, because chi-square test is $\chi^2 = 2.3532$, $\chi^2_{T(\alpha=0.05; m-r-1=6)} = 12.5916$ and $sign(\chi^2 - \chi_T^2) < 0$. Then the density function of TTF is $f(t) = \lambda e^{-\lambda t} = 0.00015999\ e^{-0.00015999\ t}$ and the reliability function of TTF is $R(t) = e^{-\lambda t} = e^{-0.00015999\ t}$.

The expected number of replacements (renewals) during the preventive maintenance interval t_p when the TTF has an exponential distribution is $M(t_P) = \lambda t_P = 0.00015999 t_P$

In the same way the distribution of TTR are found. In this case, the goodness-of-fit $sign\left(\chi^2 - \chi^2_{T(\alpha=0.05;m-r-1=5)}\right) = sign(8.0543 - 11.0705) < 0$ is obtained when the TTR is modeled with Weibull distribution with parameters β=0.7 and θ=901.57. Hence, the density function of TTR is

$$f_r(t) = \frac{\beta t^{(\beta-1)}}{\theta^{\beta}} \exp\left[-\left(\frac{t}{\theta}\right)^{\beta}\right]$$

$$f_r(t) = \frac{0.7 t^{(0.7-1)}}{901.57^{0.7}} \exp\left[-\left(\frac{t}{901.57}\right)^{0.7}\right]$$

The maintainability function of the considered machine is shown in fig. 5.

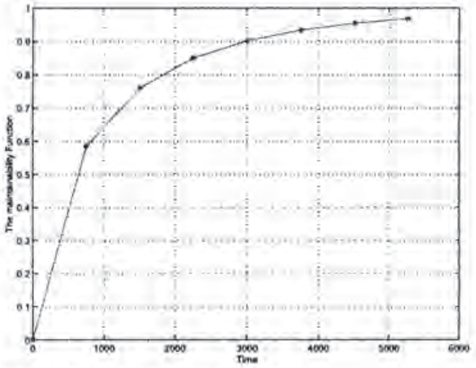

Fig. 5 Maintainability function

The availability and downtime are respectively

$$\text{Availability} = \frac{MTTF}{MTTF + MTTR} = 0.887$$

$$\text{Downtime} = \frac{MTTR}{MTTF + MTTR} = 0.113$$

4. CONCLUSIONS

The most important reason for a preventive maintenance program is the reduction of costs placed in specific perspectives:

- reduced production downtime, resulting in less machine breakdowns,
- better conservation of equipment and increased life expectancy of the equipment,
- reduced overtime costs and more economical use of maintenance workers due to working on a schedule basis instead of crash basis to repair the breakdowns,
- timely, routine repairs circumvent fewer large-scale repairs,
- reduced cost of repairs by reducing secondary failures,
- reduced product rejects, rework, and scrap due to a better overall equipment condition,
- identification of the equipment with excessive maintenance costs, indicating the need for corrective maintenance,
- improvement of safety and quality conditions.

Using the presented algorithm (fig.2) and MATLAB the probability distribution of TTF and TTR are easily obtained. The probability density function of TTF is a most important information of scheduling preventive maintenance. The optimal time for preventive maintenance is calculated using the presented models with cost minimization or downtime minimization. Moreover, the most important appeared to be the failure rate of equipment with time. It may increase, decrease or stay constant. If the equipment's failure rate is decreasing with time, then the equipment is likely to improve with time and any preventive maintenance actions or replacement is considered as a waste of resources. Likewise, if the equipment has a constant failure rate, then we have to estimate whether the preventive maintenance is necessary. For this reason, the costs in relation to preventive maintenance and the costs in relation to corrective maintenance are calculated. If the equipment's failure rate is increasing preventive maintenance is an urgent requirement.

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APPENDIX

N	TTF [min]	Recovery time [min]	N	TTF [min]	Recovery time [min]
1	300	225	19	3 570	270
2	4 635	270	20	6 340	266
3	15 460	170	21	1 119	537
4	26 455	448	22	423	495
5	3 414	400	23	3 340	460
6	1 756	1 180	24	19 165	120
7	9 640	330	25	14 215	340
8	510	455	26	3 940	1 050
9	479	346	27	7 505	165
10	160	130	28	5 800	1 890
11	1 466	1 254	29	810	90
12	10 440	5 220	30	11 810	100
13	2 460	3 960	31	15 635	85
14	6 975	105	32	14 100	180
15	3 730	290	33	16 450	500
16	1 620	180	34	1 110	140
17	780	480	35	800	6 020
18	3 380	288	36	5 220	320