**Canadian Committee for the Theory of Machines and Mechanisms**

**Commission Canadienne pour la Théorie des Machines et des Mécanismes**

*2003 CCToMM Symposium on Mechanisms, Machines, and Mechatronics*

# 2003 CCToMM M$^3$

*Symposium 2003 sur les mécanismes, les machines et la mécatronique de la CCToMM*

May 30, 2003, the Canadian Space Agency, Saint-Hubert (Montréal), Québec, Canada

Le 30 mai 2003, l'Agence spatiale canadienne, Saint-Hubert (Montréal), Québec, Canada

## FOREWORD

Welcome to the 2003 edition of the Second CCToMM *Symposium on Mechanisms, Machines, and Mechatronics*, or $M^3$. We hold $M^3$ on odd-numbered years. In even-numbered years, we hold our Symposium within the CSME Forum.

We have 29 papers in the symposium proceedings; each of these papers was duly reviewed by two referees. The final version of the papers incorporates the revisions suggested by the referees. The review work was managed by the Review Committee:

Jorge Angeles, McGill University, Symposium Co-Chair and former CCToMM Chair; Roger Boudreau, Université de Moncton, CCToMM Secretary-General; Jozsef Kövecses, Canadian Space Agency; Leila Notash, Queen's University, CCToMM Communications Officer; Jean-Claude Piedboeuf, Canadian Space Agency, Symposium Co-Chair and CCToMM Chair; Ron Podhorodeski, University of Victoria, CCToMM Treasurer. Reviewers were drawn mostly from CCToMM, but, due to the multidisciplinarity of some papers, we resorted also to external reviewers.

We take the opportunity here to acknowledge the fine review work of all the anonymous reviewers who provided their expertise and their time to make this symposium a technical success. The Canadian Space Agency is to be especially acknowledged for allowing us to use their facilities as the venue of the Symposium, while the Department of Mechanical Engineering and the Centre for Intelligent Machines, McGill University, helped us with the logistics. We also thank Ms. Irène Cartier, McGill University, who acted as an assistant to the Technical Committee and served as a liaison with authors, besides providing her translation expertise and her time to help us with local arrangements.. Scott Nokleby, outgoing CCToMM Student Representative, was instrumental in setting up and updating the Web site of the symposium, while Alexei Morozov, Design Engineer at McGill University's NSERC Design Engineering Chair, provided technical support in the editing of the Proceedings, for which we are deeply thankful.

*Jorge Angeles and Jean-Claude Piedboeuf*, Symposium Co-Chairs

Montreal and St.-Hubert, May 30, 2003

**PRÉFACE**

Nous vous souhaitons la bienvenue à l'édition 2003 du Deuxième Symposium sur les mécanismes, les machines et la mécatronique de CCToMM ou $M^3$. Le Symposium $M^3$ se tient à tout les deux ans, en alternance avec le Symposium du Forum CSME.

Les actes du symposium contiennent 29 communications qui furent toutes soumises au processus de revision de deux experts. Les corrections suggérées par les experts ont été incorporées dans la version finale des communications. Le travail de revision a été supervisé par le comité de revision composé des personnes suivantes :

Jorge Angeles, Université McGill, co-organisateur du symposium et ancien président de CCToMM; Roger Boudreau, Université de Moncton, secrétaire général de CCToMM; Jozsef Kövecses, Agence spatiale canadienne; Leila Notash, Queen's University, responsable des communications de CCToMM; Jean-Claude Piedboeuf, Agence spatiale canadienne, co-organisateur du symposium et président de CCToMM; Ron Podhorodeski, Victoria University, trésorier de CCToMM. Les experts provenaient pour la plupart de CCToMM, mais étant donné la multidisciplinarité de certains articles, nous avons eu recours à des experts externes.

Nous profitons de l'occasion pour exprimer notre appréciation pour l'excellent travail de revision anonyme de nos experts, qui ont apporté leur connaissance et leur temps pour faire du symposium un succès. Nous remercions tout spécialement l'Agence spatiale canadienne qui nous permet d'utiliser ses locaux pour la tenue du symposium. Nous reconnaissons également l'apport de services de logistiques du Département de génie mécanique et du Centre de recherches sur les machines intelligentes de l'Université McGill. Nous remercions également Irène Cartier pour son travail d'assistante au comité organisateur, et d'intermédiaire auprès des auteurs. Elle s'est aussi chargée de la traduction de textes. Nous voulons de même souligner le travail de Scott Nokleby, représentant sortant des étudiants à CCToMM, qui s'est occupé de la création et mise à jour du site Web du symposium, tandis que Alexei Morozov, ingénieur de conception à l'emploi de l'Université McGill, dans le cadre de la Chaire CRSNG en génie de la conception, nous a apporté une aide précieuse au plan technique pour la conception et la production des actes du symposium.

*Jorge Angeles et Jean-Claude Piedboeuf*, co-organisateurs du Symposium

Montréal et St.-Hubert, 30 mai 2003

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# SYNTHESIS OF HYPOCYCLOIDAL GEARS

**R. Dolchinkov, V. Galabov*, N. Nikolov*, and V.N. Latinovic****

## 1. Abstract

The article presents the results of analytical study of hypocycloidal gearsets, generated by a known compound pinion consisting of a hub and cylindrical teeth that meshes with hypocycloidal gear with internal teeth; the number of teeth of the pinion being for one smaller than the number of teeth the hypocycloidal gear. Two gearsets are synthesized in which the compound pinion has been replaced by an integral pinion with tooth profile generated by an external and internal envelope resulting from successive positions of tooth profile of the hypocycloidal gear according to the conjugate action of the two in the plane perpendicular to the axes of centrodes.

## 2. Introduction

In spite of undisputable advantages of the involute gearsets due to their insensitivity to center distance mounting error, constant pressure angle and easy machining, cycloidal gears are becoming more and more attractive to researchers in exploring their kinematic and load transmission characteristics. The cycloidal gears are characterized by a large gear ratio if they are designed in a planetary arrangement. This is due to the fact that the number of teeth of pinion is for one smaller than the number of teeth of gear. The other advantage is in their higher load rating and efficiency for equal size of the gearset. Because of these advantages the overall share in gear applications in practice increases every year in favor of the cycloidal gears [1]. This claim can be guessed based on fact that a convex and a concave surface are in contact during the meshing; yet it has to be proven.

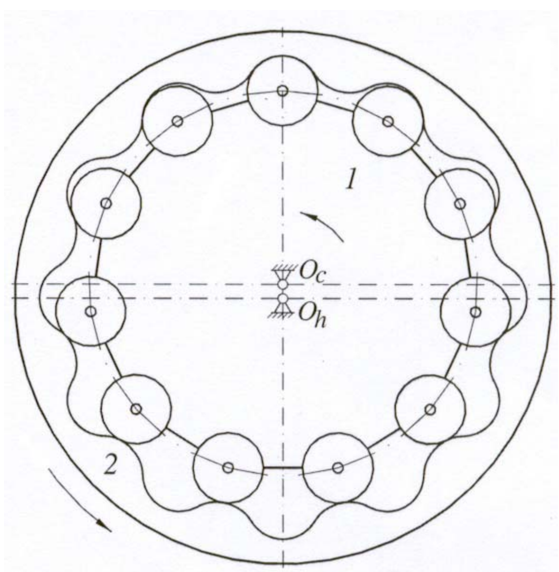


Fig. 1 Compound Pinion with Cylindrical Teeth and Epicycloidal Gear with m=3.5 mm, $N_1 = 11$, $N_2 = 12$, x =0.2 and $r_c^* = 1$

A well known hypocycloidal gearset with internal gear is one with compound pinion [2], [3]. The tooth profile of the internal gear is an equidistant curve of the shortened epicycloid, and the teeth of the pinion are cylinders; the number of cylinders being for one smaller that number of epicycloidal teeth. Fig. 1 reveals this type of gearset. At any instant a half of the meshing tooth pairs is in contact and transmits motion to the driven gear.

Dolchinkov [4] has synthesized and studied a new type of hypocycloidal gear , generated by a compound pinion with cylindrical teeth. In this case the compound pinion is replaced by an integral pinion with external tooth profile generated as an inner envelope curve of the hypocycloidal gear according to the conjugate action resulting from pure rolling of the centrodes. The hypocycloidal gear (with internal teeth) and the epicycloidal pinion constitute a new gearset. Compared to the gearset with the compound pinion and cylindrical teeth this gearset it is characterized by easy manufacturing and higher reliability and load rating.

In order to make a complete classification of hypocycloidal gearsets it is necessary to consider a new type of gearset, generated by a pinion with epicycloidal tooth profile. The tooth profile of the hypocycloidal gear with internal teeth is used instead of the compound pinion with cylindrical teeth. It is generated as an outer envelope curve of tooth profile of the hypocycloidal gear in the plane perprndicular to axes of the centrods that roll with no slippage on each other. In this articles the authors are presenting synthesis of a class of hypocycloidal gearsets. The synthesis is accomplished based on either the fundamental low of gearing [5] or on the conjugate action and the theory of envelopes [6].

## 3. Synthesis of Hypocycloidal Gearsets

The central curve of the cylinders of a compound pinion 1 in the plane of the hypocycloidal gear 2 is a shortened epicycloid described by a parametric equations (Fig. 2):

$$\xi_c = \frac{m}{2}[(N_2+1)\sin\varphi - (1-x)\sin(N_2+1)\varphi], \quad \eta_c = \frac{m}{2}[(N_2+1)\cos\varphi - (1-x)\cos(N_2+1)\varphi] \quad (1)$$

where $m$, $N_1$ and $x$ are a module, number of teeth and modification factor of the epicycloidal pinion respectively, and angle $\varphi$ assumes value within range $[0, \frac{\pi}{N_2}]$ .

The equations of the tooth-profile of the hypocycloidal gear that turns to be an equidistant curve of the shortened epicycloid (1), are:

$$\xi = \xi_c + r_c \frac{(1-x)\sin(N_2-1)\varphi - \sin\varphi}{\sqrt{1-2(1-x)\cos N_2\varphi + (1-x)^2}}, \quad \eta = \eta_c + r_c \frac{-(1-x)\cos(N_2-1)\varphi - \cos\varphi}{\sqrt{1-2(1-x)\cos N_2\varphi + (1-x)^2}} \quad (2)$$

where $r_c = m r_c^*$ is the radius of generating circle (equal to radius of the cylinder), $r_c^*$ is termed a coefficient of generating cirle, and $N_e$ is number of teeth of epicycloidal gear.

To replace the compound pinion with cylindrical teeth by an integral pinion its tooth-profile must be an envelope curve of the succesive positions of the tooth-profile of the epicycloidal gear in the plane perpendicular to axes of the centrodes that roll with no slippage on each other. The equations of a curve from the family of curves can be obtained by a kinematic inversion of the gear motion relative to the pinion motion. Then, commonly, one can assume that the fixed frame

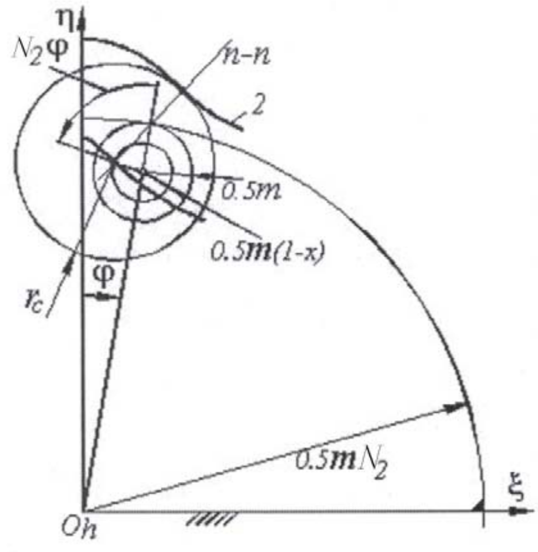


Fig. 2 Generating Tooth Profile of Epiocycloidal Gear

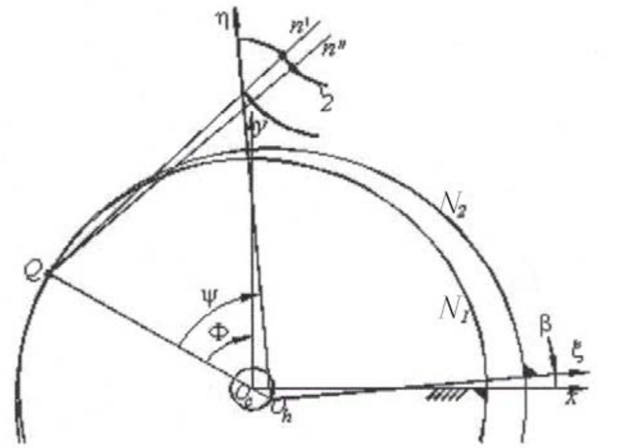


Fig. 3 Generating Two Families of Curves in Plane of Centrods

is attached to the pinion $O_c xy$ (does not move) (Fig.3). The hypocycloidal gear is fixed to frame $O_h\xi\eta$ and produces an angular displacement in plain $O_c xy$. Equations of a family of curves, corresponding to the succesive positions of the shortened epicycloid (1) and its equidistant tooth profile of the hypocycloidal gear are obtained by the following coordinate transformations:

$$x_c = \xi_c \cos\beta - \eta_c \sin\beta - a_\omega \sin\phi, \quad y_c = \xi_c \sin\beta + \eta_c \cos\beta - a_\omega \cos\phi \tag{3}$$

$$x = \xi\cos\beta - \eta\sin\beta - a_\omega \sin\phi, \quad y = \xi\sin\beta + \eta\cos\beta - a_\omega \cos\phi \tag{4}$$

where $\phi \in [0,2\pi]$ is an angle that defines the position of the centre $O_h$ of the hypocycloidal gear *1* (Fig.2), $\beta=\phi/N_1=\psi/N_2$ is an angle of rotation of the hypocycloidal gear relative to the pinion wheel, where $\psi$ defines the position of instantaneous center Q in the coordinate system of the hypocycloidal gear; $N_2 = N_1 + 1$; $N_1$ being the number of teeth (cylinders) of the compound gear, and $a_\omega = m(1-x)/2$ the center distance.

By substituting equations (1) and (2) into (3) and (4) the following is obtained:

$$x_c = \frac{m}{2}[N_1 \sin(\varphi + \frac{\psi}{N_2}) - (1-x)\sin(N_1\varphi - \frac{\psi}{N_1}) - \lambda \sin(\frac{N_2}{N_1}\psi)] \tag{5a}$$

$$y_c = \frac{m}{2}[N_1 \cos(\varphi + \frac{\psi}{N_2}) + (1-x)\cos(N_1\varphi - \frac{\psi}{N_1}) - \lambda \cos(\frac{N_2}{N_1}\psi)] \tag{5b}$$

$$x=\frac{m}{2}[N_1\sin(\varphi+\frac{\psi}{N_1})-\lambda\sin(N_1\varphi-\frac{\psi}{N_1})-\lambda\sin(\frac{N_2}{N_1}\psi)+2r_c^*\frac{\lambda\cos(N_p\varphi-\frac{\psi}{N_1})-\cos(\varphi-\frac{\psi}{N_1})}{\sqrt{1-2\lambda\cos N_2\varphi+\lambda^2}}] \quad (6a)$$

$$y=\frac{m}{2}[N_1\cos(\varphi+\frac{\psi}{N_1})+\lambda\cos(N_1\varphi-\frac{\psi}{N_1})-\lambda\cos(\frac{N_2}{N_1}\psi)+2r_c^*\frac{\lambda\cos(N_p\varphi-\frac{\psi}{N_1})-\cos(\varphi-\frac{\psi}{N_1})}{\sqrt{1-2\lambda\cos N_2\varphi+\lambda^2}}] \quad (6b)$$

where $\lambda$ designates factor *(1-x)*.

The envelopes of the family of curves, corresponding to the succesive positions of tooth-profile of the epicycloidal gear in the plane perpendicular to the axes of the centrods are defined by system:

$$y-y(\varphi(x),\phi_{1,0})=0 \quad (7a)$$

$$\frac{dy_c}{d\phi_{0,1}}=0,\ \text{where } \phi_{1,0}=\phi \quad (7b)$$

Equation (7a) is equivalent to equations $y-y(\varphi,\phi)=0$, $x-x(\varphi,\phi)=0$ and. Since the shortened epicycloid (1) and the tooth-profile of the hypocycloidal gear (2) are equidistant curves the slopes are the same, and hence $dy/d\phi\equiv dy_c/d\phi$. Equation (7a) can be rewritten as follows:

$$\frac{dy_c}{d\phi_{0,1}}=(y_c)'_\varphi\frac{d\varphi(x_c)}{d\phi_{0,1}}+(y_c)'_{\phi_{1,0}}\frac{d\phi_{0,1}}{d\phi_{0,1}}=-(y_c)'_\varphi\frac{d\varphi(x_c)}{d\phi_{1,0}}+(y_c)'_{\phi_{1,0}}=-(y_c)'_\varphi(\varphi)'_{x_c}\frac{dx_c}{d\phi_{1,0}}+(y_c)'_{\phi_{1,0}}=$$
$$=-\frac{(y_c)'_\varphi}{(x_c)'_\varphi}(x_c)'_{\phi_{1,0}}+(y_c)'_{\phi_{1,0}}=0 \quad .$$

Finally, the following system is obtained:

$$x-x(\varphi,\phi_{1,0})\equiv x-x(\varphi,\psi)=0; \quad (8a)$$

$$y-y(\varphi,\phi_{1,0})\equiv y-y(\varphi,\psi)=0 \quad (8b)$$

$$-\frac{(y_c)'_\varphi}{(x_c)'_\varphi}+\frac{(y_c)'_{\phi_{1,0}}}{(x_c)'_{\phi_{1,0}}}\equiv\frac{(y_c)'_\varphi}{(x_c)'_\varphi}+\frac{(y_c)'_\psi}{(x_c)'_\psi}=0 \quad (8c)$$

After differentiating equations (5) with respect to two parametres $\varphi$ and $\psi$ and substituting into equation (8b) the following result is obtained:

$$\frac{\frac{m}{2}N_1[\sin(\varphi+\frac{\psi}{N_1})+\lambda\sin(N_1\varphi-\frac{\psi}{N_1})]}{\frac{m}{2}N_1[\cos(\varphi+\frac{\psi}{N_1})-\lambda\cos(N_1\varphi-\frac{\psi}{N_1})]}-\frac{\frac{m}{2N_1}[N_1\sin(\varphi+\frac{\psi}{N_1})-\lambda\sin(N_1\varphi+\frac{\psi}{N_1})-\lambda N_2\sin(\frac{N_2}{N_1}\varphi)]}{\frac{m}{2N_1}[N_1\cos(\varphi+\frac{\psi}{N_1})+\lambda\cos(N_1\varphi-\frac{\psi}{N_1})-\lambda N_2\cos(\frac{N_2}{N_1}\psi)}=0$$

The above equation can be reduced to the following simplified form:

$$\lambda\sin[(\psi-\varphi)+N_1\varphi]-\sin(\psi-\varphi)-\sin(N_2\varphi)=0 \tag{9}$$

which reduces to a quadric equation:

$$(\lambda^2-2\lambda\cos N_2\varphi+1)\cos^2(\psi-\varphi)+2\lambda\sin^2 N_2\varphi\cos(\psi-\varphi)+[2\lambda-(1+\lambda^2)\cos N_2\varphi]\cos N_2\varphi=0 \tag{10}$$

with the two roots:

$$\psi_{3,4}(\varphi)=\varphi+\cos^{-1}\frac{-\lambda\sin^2 N_2\varphi\mp\sqrt{\lambda^2\sin^4 N_1\varphi-[\lambda^2-2\lambda\cos N_2\varphi+1][2\lambda-(1+\lambda^2)\cos N_2\varphi]\cos N_2\varphi}}{\lambda^2-2\lambda\cos N_2\varphi+1} \tag{11}$$

The relationship (11) between parametres $\psi$ and $\varphi$ is substituted into the first two equations of system (8) and then into equations (6) to yield the outer and inner envelopes of successive positions of tooth profile of the hypocycloidal gear as a function of the parameter $\varphi$. The envelopes of the equidistant curve of the tooth profile of the epicycloidal gear for the shortened hypocycloid are obtained by substituting values (11) into equations (5). Fig.4 shows functions $\psi_3(\varphi)$ and $\psi 4(\varphi)$ and Fig.5 shows the tooth profile of the hypocycloidal gear 2 , cylinder 1 of the compound pinion and the tooth profiles of gears *3* and *4* $A_3BC_3$ and $A_4BC_4$, representing outer and inner envelope of the tooth profile of the epicycloidal gear in the plane perpendicular to axes of centrods for $N_1=11$, $N_2\equiv N_3\equiv N_4=12$ , x = 0.2 and $r_c^*=1$.

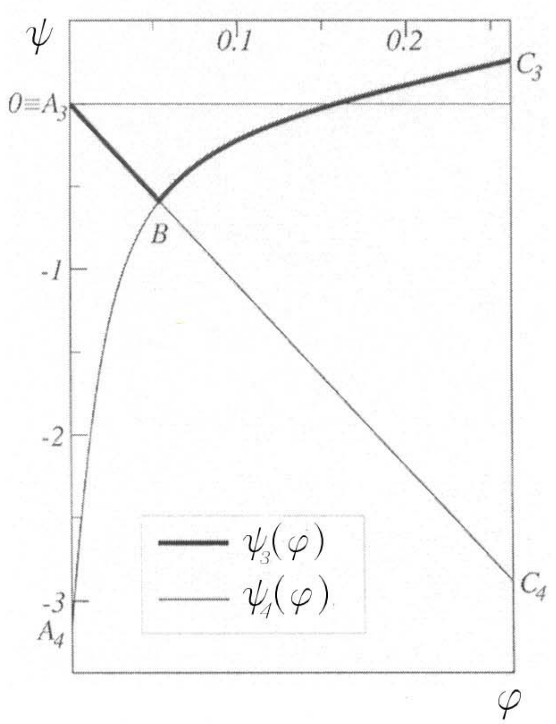


Fig. 4 Relationship Between $\psi$ and $\varphi$ for $N_1 = 11$, $N_2 = 12$ and x = 0.2

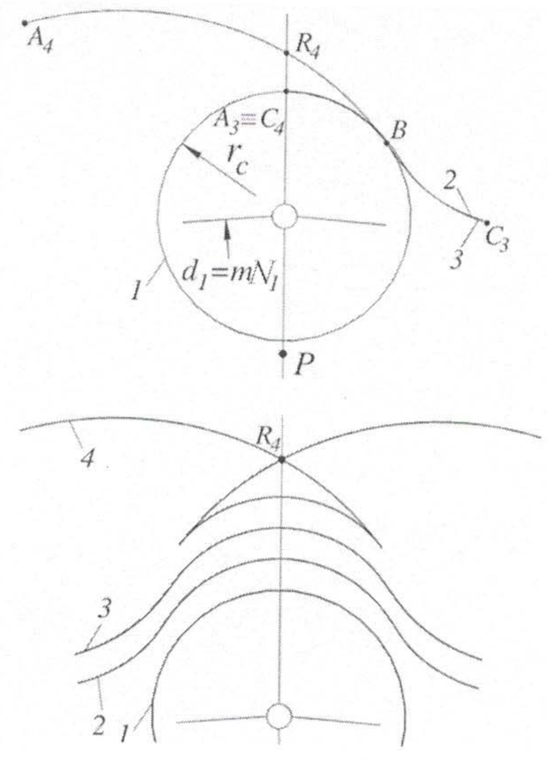


Fig. 5 Two Envelopes for $N_1 = 11$, $N_2 = N_3 = N_4 = 12$ , x = 0.2 and $r_c^* = 1$

Besides the approach, based on the theory of envelopes demonstrated above, it is also possible to use the approach reported in [4] and [7], based on the well known principle of kinematics the fundamental low of gearing for higher pairs in contact. According to this low the common normal at the contact point of the two profiles always passes through the instantaneous center on the line of centers and it is designated by P. Since the slope of the normal is –1/slope of curve at a point, it can be written in reference system of hypocycloidal gear $O_e\xi\eta$:

$$\frac{\eta_c-\eta_P}{\xi_c-\xi_P}=-\frac{1}{d\eta_c/d\xi_c}=-\frac{d\xi_c/d\varphi}{d\eta_c/d\varphi} \qquad (12)$$

Derivatives $d\xi_c/d\varphi$ and $d\eta_c/d\varphi$ are defined after differentiating equations (1). The coordinates of the instantaneous center (pole) $P$ in coordinate system $O_e\xi\eta$ are:

$$\eta_P=r_{\omega_e}\cos\psi; \qquad \xi_P=r_{\omega_e}\sin\psi, \qquad (13)$$

where $r_{\omega_e}=m\,N_e\,\lambda/2$ is a radius of the initial circle of epicycloidal gear.
Substituting equations (13), into equation (12) yields the following:

$$\frac{0.5m[N_1\cos\varphi+\lambda\cos(N_1\varphi)-\lambda N_2\cos\psi]}{0.5m[N_1\sin\varphi-\lambda\sin(N_1\varphi)-\lambda\sin\psi]}=\frac{0.5mN_1[\cos\varphi-\lambda\cos(N_1\varphi)]}{0.5mN_1[\sin\varphi-\lambda\sin(N_1\varphi)]}$$

(14)

Simplifying equation (14) yields equation (9), which shows that the two approaches are equally successfull in defining the tooth profile.

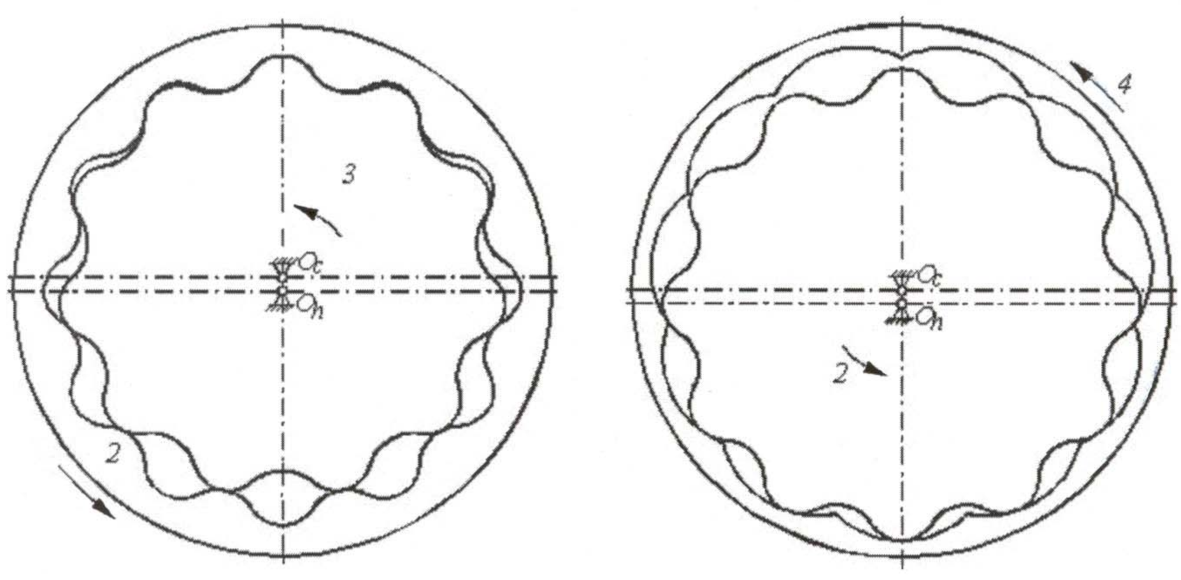


Fig. 6 Two Gearsets with m = 5 mm, $N_1 = 11$, $N_3 \equiv N_4 = 12$, x = 0.2 and $r_c^* = 1$

Fig.6 shown the two epicycloidal gearsets with m=5 mm, $N_1=N_3=N_4=11$, $N_2 = 12$ , x = 0.2 and $r_c^*=1$. The left and the right tooth profile of gear *3* in gearset *3-2* represent a smooth curve, with no undercutting while the left and the right tooth profile generated for gear *4* in gearset *2-4* is undercut and doubles over itself. The result is, after teeth generation, that tooth profile is pointed and formed only in region $A_4R_4$ (Fig.5). Despite of this undercut and unfavorable pressure angles, gearset *2-4* is of great interest, because gear 2 with external teeth has one tooth more than gear 4 with internal teeth, so the pinion becomes the gear and the gear becomes the pinion.

## 4. Conclusions

The article offers a classification generated hypocycloidal gearsets with internal meshing where a difference in tooth numbers is one. It has been shown that, besides a compound hypocycloidal gear with cylindrical teeth the two hypocycloidal gearsets are possible where the compound gear wheel is replaced by an integral gear. The tooth profile of the gears of the two gearsets is formulated analytically. The results obtained by use of the conjugate action and the theory of envelopes and those obtained by use of the instantaneous center of velocities and the fundamental law of gearing proved to be the same.

It is of interest to point out a new gearset *2-4*, consisting of a hypocycloidal gear 2 with external teeth and pinion *4* with internal teeth. Due to the undercutting of tooth profiles in gear *4*, this gearset is characterized by unfavorable pressure angles and could be used only for a kinematic transmission but not for power transmission. The fact that the gear with external teeth has one tooth more than the internal gear is a unusual phenomenon in the theory of toothed wheels.

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