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NONLINEAR STATE SPACE MODELS OF THE GENERALIZED INDUCTION ELECTROMECHANICAL CONVERTER BASED ON A STATIONARY COORDINATE FRAME

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Abstract. A mathematical description of the electromechanical energy conversion in induction motors is presented in the paper. The dynamic equations connecting the currents and fluxes in the electrical motors are built by taking the projections of the respective variables in a stationary orthogonal coordinate frame. Those equations are untied with regard to different variables by solving the equations for the derivatives of the respective variables. Orientated nonlinear non-autonomous state space models based on state variables choices stemming from the solutions are created, describing the electromechanical conversion of energy. The structural properties of the proposed nonlinear state space models are analyzed with regard to complexity, observability, and suitability for nonlinear observers design. The peculiarities of the models are considered from the viewpoint of non-autonomy, bound conditions, stiffness and numerical properties. The consistency of the models and their equivalence are confirmed by physical experiments and simulation.

Keywords: Induction motor control; Nonlinear systems; Park and Clark transformations; Electromechanical converter; Three-phase induction machine;

1. Introduction

The induction motors have a wide industrial application and more than half of the electrical energy consumed is converted through them into mechanical [7]. Along with the uncontrolled electrical drives the share of controlled electrical drives increasingly grows including the control of the starting and braking processes, and the spinning velocity in wide limits via semiconductor starters and frequency converters. The number of uncontrolled electrical drives with modern microprocessor-based protection devices increases continuously. The reliable electrical drives with high quality of control imply knowledge of the mathematical description of electromechanical energy conversion and the possibilities for closed-loop systems design based on state observers [2,3,4,6,11]. The present advances in computer technology permit the design of high quality systems for control and protection of electrical motors by the implementation of modern

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methods and techniques from electrical drive theory and control theory [7,9,10].

The objectives of the paper is to present the generalized electromechanical model of the electrical drive, build a series of nonlinear state space models with respective state vectors, prove their equivalence by a physical experiment, and perform observability, numerical and stiffness analysis of the state space models derived.

2. Generalized electromechanical model of the electrical drive

Electrical drive with an induction motor is considered in this section. The three-phase power voltage system is a symmetric voltage source (VS) with infinite power and constant angular frequency. The momentary values of the phase power voltages and currents are known. The induction motor is considered as a symmetric load also. The controllable variables are the electromagnetic torque and the angular velocity of the motor as shown on fig. 1. In the electrical drives theory the motor is considered as an electromechanical energy converter conditionally separated into electrical part (EP) and

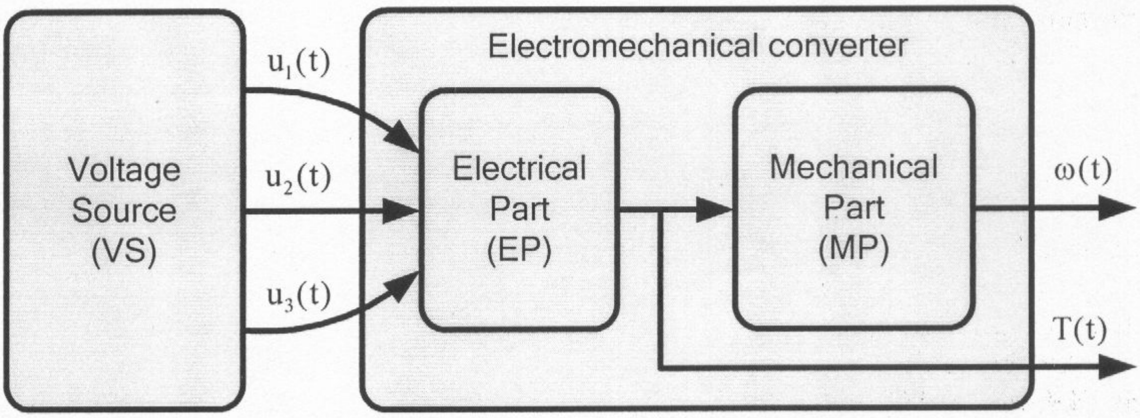

Figure 1: Generalized block diagram of the electrical drive

mechanical part (MP). The voltage and current equations of the voltage source (VS) are

$$\begin{aligned} u_1(t) &= u_{max}\sin(\omega t+\varphi_u), & i_1(t) &= i_{max}\sin(\omega t+\varphi_i) \\ u_2(t) &= u_{max}\sin(\omega t+2\pi/3+\varphi_u), & i_2(t) &= i_{max}\sin(\omega t+2\pi/3+\varphi_i) \\ u_3(t) &= u_{max}\sin(\omega t+4\pi/3+\varphi_u), & i_3(t) &= i_{max}\sin(\omega t+4\pi/3+\varphi_i) \end{aligned} \tag{2.1}$$

The electrical energy conversion in the induction motor is described by space vectors or their projections in orthogonal coordinate frames using appropriate transformations of the variables number (Clark transformation) and the angular velocity of the coordinate frame (Park transformation). These transformations usually impose the condition of power invariability. Generally, the voltage equations in orthogonal coordinate frames which can rotate with arbitrary angular velocities are [7]:

$$\begin{aligned} \mathbf{u}_s &= R_s\mathbf{i}_s + \frac{d\boldsymbol{\psi}_s}{dt} + j\omega_{k_1}\boldsymbol{\psi}_s \\ 0 &= R_r\mathbf{i}_r + \frac{d\boldsymbol{\psi}_r}{dt} + j\omega_{k_2}\boldsymbol{\psi}_r \end{aligned} \tag{2.2}$$

The variables in this vector equation system represent the mapping vectors of the motor voltage, current and flux, while the rotor parameters are reflected to the respective stator parameters. The angular velocities of the two coordinate frames ω_{k_1} and ω_{k_2} can be arbitrary unrelated functions of time, but they can be related as well. From the group of the related velocities the cases $\omega_{k_1} = \omega_{k_2} = \omega_e$ and $\omega_{k_1} = \omega_{k_2} = 0$ are used most often. For the first case the coordinate frame can be orientated along different vectors – most frequently fluxes.

The real variables and control inputs have upper and lower bounds imposed – currents, voltages, angular velocities, and torques. In other words they are nonlinearly bounded.

When the mathematical description is accomplished in a joint orthogonal coordinate frame with angular velocity $\omega_{k_1} = \omega_{k_2} = \omega_e$ then the equations (2.2) go into

$$\begin{aligned} \mathbf{u}_s &= R_s\mathbf{i}_s + \frac{d\boldsymbol{\psi}_s}{dt} + j\omega_e\boldsymbol{\psi}_s \\ 0 &= R_r\mathbf{i}_r + \frac{d\boldsymbol{\psi}_r}{dt} + j(\omega_e - \omega_r)\boldsymbol{\psi}_r \end{aligned} \tag{2.3}$$

Here ω_r denotes the electrical angular velocity of the variables in the rotor circuit. When the mathematical description is accomplished in a joint orthogonal coordinate frame with angular velocity $\omega_e = 0$ then the equations (2.3) read

$$\begin{aligned} \mathbf{u}_s &= R_s\mathbf{i}_s + \frac{d\boldsymbol{\psi}_s}{dt} \\ 0 &= R_r\mathbf{i}_r + \frac{d\boldsymbol{\psi}_r}{dt} - j\omega_r\boldsymbol{\psi}_r \end{aligned} \tag{2.4}$$

The relations between the variables in equations (2.1), (2.3) and (2.4) (Clark transformation) and the real variables of the induction machine are given by the expressions [2,6,7]:

$$\mathbf{u}_s = \begin{bmatrix} \sqrt{\frac{2}{3}} & -\sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{6}} \\ 0 & \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix}, \quad \mathbf{i}_s = \begin{bmatrix} \sqrt{\frac{2}{3}} & -\sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{6}} \\ 0 & \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} \end{bmatrix} \begin{bmatrix} i_1(t) \\ i_2(t) \\ i_3(t) \end{bmatrix}, \tag{2.5}$$

$$\boldsymbol{\psi}_s = L_s\mathbf{i}_s + L_m\mathbf{i}_r, \tag{2.6a}$$

$$\boldsymbol{\psi}_r = L_r\mathbf{i}_r + L_m\mathbf{i}_s. \tag{2.6b}$$

Different expressions [2] can be used for the determination of the electromagnetic torque momentary value $T(t)$ from the electrical part of the converter, but for the case $\omega_{k_1} = \omega_{k_2} = 0$ it is suitable to use the expression

$$T = K_d \operatorname{Im}[\boldsymbol{\psi}_r^* \mathbf{i}_s] \tag{2.7}$$

where $\boldsymbol{\psi}_r^*$ denotes the complex conjugate of $\boldsymbol{\psi}_r$ and K_d is the motor coeficient. The mechanical part of the electromechanical converter is most often represented by the simplified equation of motion

$$T - T_L = J\frac{d\omega}{dt}. \tag{2.8}$$

In (2.8) T_L and ω denote the momentary values of the load torque and the motor mechanical angular velocity, while J denotes the total inertia moment reflected to the motor shaft. The relation between the electrical frequency in the rotor ω_r and the mechanical angular velocity ω is given by the equation

$$\omega_r = z_p\omega. \tag{2.9}$$

The momentary value of the torque originating from the vector equation (2.7) is

$$T = K_d[\psi_{r\beta}i_{s\alpha} - \psi_{r\alpha}i_{s\beta}]. \tag{2.10}$$

The description considered is traditional and frequently used [1,4,7,8] for motor process analysis. It can be applied to explore electrical motors connected in star or delta scheme. The analysis is accomplished in a stationary coordinate frame and a recalculation of the vector coordinates in other coordinate frames is not necessary, which is an advantage of this description.

The main model considered is built on the basis of equations (2.4) by taking the projections of the respective variables on the real axis (α) and the imaginary axis (β) of a stationary orthogonal coordinate frame

$$\begin{aligned}
u_{s\alpha} &= R_s i_{s\alpha} + \frac{d\psi_{s\alpha}}{dt} \\
u_{s\beta} &= R_s i_{s\beta} + \frac{d\psi_{s\beta}}{dt} \\
0 &= R_r i_{r\alpha} + \frac{d\psi_{r\alpha}}{dt} - \omega_r\psi_{r\beta} \\
0 &= R_r i_{r\beta} + \frac{d\psi_{r\beta}}{dt} + \omega_r\psi_{r\alpha}
\end{aligned} \tag{2.11}$$

This model will be used in the next section to derive a series of nonlinear state space (SS) models defined by different choices of the state vector.

3. Nonlinear state space models of the induction motor

The general form of the nonlinear SS models that will be built is

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{u}),\ \mathbf{x}(t_0) = \mathbf{x}_0 \tag{3.1a}$$

$$\mathbf{y} = \mathbf{h}(\mathbf{x},\mathbf{u}) \tag{3.1b}$$

where $\mathbf{x} = [x_1, x_2, \ldots, x_n]^T$ is the state vector, $\mathbf{u} = [u_1, u_2, \ldots, u_r]^T$ is the input vector, and $\mathbf{y} = [y_1, y_2, \ldots, y_m]^T$ is the output vector. The functions $\mathbf{f}$ and $\mathbf{h}$ are real and nonlinear.

For the purpose of building those models equations (2.6) will be presented componentwise in the coordinates of the stationary orthogonal coordinate frame

$$\psi_{s\alpha} = L_s i_{s\alpha} + L_m i_{r\alpha} \tag{3.2a}$$
$$\psi_{s\beta} = L_s i_{s\beta} + L_m i_{r\beta} \tag{3.2b}$$
$$\psi_{r\alpha} = L_r i_{r\alpha} + L_m i_{s\alpha} \tag{3.2c}$$
$$\psi_{r\beta} = L_r i_{r\beta} + L_m i_{s\beta} \tag{3.2d}$$

Thus, our models will be built on the basis of equations (2.8), (2.9), (2.10), (2.11), and (3.2). The first model is determined by the choice of the state, input, and output vectors

$$\mathbf{x} = [x_1, x_2, x_3, x_4, x_5]^T = [i_{s\alpha}, i_{s\beta}, i_{r\alpha}, i_{r\beta}, \omega]^T, \tag{3.3a}$$
$$\mathbf{u} = [u_1, u_2, u_3, u_4]^T = [u_{s\alpha}, u_{s\beta}, J, T_L]^T, \tag{3.3b}$$
$$\mathbf{y} = [y_1, y_2]^T = [x_1, x_2]^T. \tag{3.3c}$$

The input and output vectors will be the same for all SS models. The state vector is defined by the components of the stator and rotor currents and the mechanical angular velocity. The SS model is composed from the joint system of the electrical and mechanical equations (2.11) and (2.8) which are transformed by a substitution of the algebraic equations (3.2) and (2.9) to eliminate the fluxes $\psi_{s\alpha}$, $\psi_{s\beta}$, $\psi_{r\alpha}$, $\psi_{r\beta}$, and ω_r, a subsequent solving of the resulting equations for the total time derivatives of the state vector (3.3a), and considering the vectors (3.3) which yields

$$\begin{aligned}
\dot{x}_1 &= k_1 u_1 - k_2 x_1 + k_3 x_3 - k_4 x_2 x_5 - k_5 x_4 x_5 \\
\dot{x}_2 &= k_1 u_2 - k_2 x_2 + k_3 x_4 + k_4 x_1 x_5 + k_5 x_3 x_5 \\
\dot{x}_3 &= k_6 u_1 - k_7 x_1 + k_8 x_3 - k_9 x_2 x_5 - k_{10} x_4 x_5 \\
\dot{x}_4 &= k_6 u_2 - k_7 x_2 + k_8 x_4 + k_9 x_1 x_5 + k_{10} x_3 x_5 \\
\dot{x}_5 &= [k_{11}(x_1 x_4 - x_2 x_3) - u_4]/u_3
\end{aligned} \tag{3.4a}$$

$$\begin{aligned}
y_1 &= x_1 \\
y_2 &= x_2
\end{aligned} \tag{3.4b}$$

The coefficients in these equations are defined as functions of the motor parameters L_s – stator inductance, L_r – rotor inductance, L_m – mutual inductance, R_s – stator resistance; R_r – rotor resistance, z_p – number of pole pairs as follows

$$k_1 = \frac{L_r}{L_r L_s - L_m^2},\ k_2 = \frac{R_s L_r}{L_r L_s - L_m^2},\ k_3 = \frac{R_r L_m}{L_r L_s - L_m^2},\ k_4 = \frac{L_m^2 z_p}{L_r L_s - L_m^2},$$
$$k_5 = \frac{L_m L_r z_p}{L_r L_s - L_m^2},\ k_6 = \frac{L_m}{L_m^2 - L_r L_s},\ k_7 = \frac{R_s L_m}{L_m^2 - L_r L_s},\ k_8 = \frac{R_r L_s}{L_m^2 - L_r L_s},$$
$$k_9 = \frac{L_m L_s z_p}{L_m^2 - L_r L_s},\ k_{10} = \frac{L_r L_s z_p}{L_m^2 - L_r L_s},\ k_{11} = K_d L_r,\ K_d = \frac{3}{2} z_p \frac{L_m}{L_r}.$$

The second SS model is defined by the following choice of the state vector

$$\mathbf{x} = [x_1, x_2, x_3, x_4, x_5]^T = [i_{s\alpha}, i_{s\beta}, \psi_{r\alpha}, \psi_{r\beta}, \omega]^T. \tag{3.5}$$

This model is obtained by solving (3.2cd) for $i_{r\alpha}$, $i_{r\beta}$ and substitution of the solution in the joint system of the electrical and mechanical equations (2.11) and (2.8) to eliminate these variables. By consideration of (2.9) the other superfluous variable ω_r is eliminated. After solving the resulting equations for the total time derivatives of the state vector (3.5), and considering the vectors (3.3bc) we arrive at the second model

$$\begin{aligned}
\dot{x}_1 &= k_1u_1 - k_2x_1 + k_3x_3 - k_4x_4x_5 \\
\dot{x}_2 &= k_1u_2 - k_2x_2 + k_3x_4 + k_4x_3x_5 \\
\dot{x}_3 &= k_5x_1 - k_6x_3 + k_7x_4x_5 \\
\dot{x}_4 &= k_5x_2 - k_6x_4 - k_7x_3x_5 \\
\dot{x}_5 &= [k_8(x_1x_4 - x_2x_3) - u_4]/u_3
\end{aligned} \tag{3.6a}$$

$$\begin{aligned}
y_1 &= x_1 \\
y_2 &= x_2
\end{aligned} \tag{3.6b}$$

where the coefficients are

$$k_1 = \frac{L_r}{L_rL_s - L_m^2},\ k_2 = \frac{R_rL_m^2}{L_r(L_rL_s - L_m^2)} + \frac{R_sL_r}{L_rL_s - L_m^2},\ k_3 = \frac{R_rL_m}{L_r(L_rL_s - L_m^2)},$$

$$k_4 = \frac{L_mz_p}{L_rL_s - L_m^2},\ k_5 = \frac{R_rL_m}{L_r},\ k_6 = \frac{R_r}{L_r},\ k_7 = z_p,\ k_8 = K_d,\ K_d = \frac{3}{2}z_p\frac{L_m}{L_r}.$$

The third SS model is based on the state vector

$$\mathbf{x} = [x_1, x_2, x_3, x_4, x_5]^T = [\psi_{s\alpha}, \psi_{s\beta}, \psi_{r\alpha}, \psi_{r\beta}, \omega]^T. \tag{3.7}$$

It is obtained by analogous elimination of the superfluous variables from the electrical and mechanical equations (2.11) and (2.8) and subsequent solving for the total time derivatives of the state vector components

$$\begin{aligned}
\dot{x}_1 &= u_1 + k_1x_1 - k_2x_3 \\
\dot{x}_2 &= u_2 + k_1x_2 - k_2x_4 \\
\dot{x}_3 &= -k_3x_1 + k_4x_3 + k_5x_4x_5 \\
\dot{x}_4 &= -k_3x_2 + k_4x_4 - k_5x_3x_5 \\
\dot{x}_5 &= [k_6(x_2x_3 - x_1x_4) - u_4]/u_3
\end{aligned} \tag{3.8a}$$

$$\begin{aligned}
y_1 &= x_1 \\
y_2 &= x_2
\end{aligned} \tag{3.8b}$$

with coefficients

$$k_1 = \frac{R_sL_r}{L_m^2 - L_rL_s},\ k_2 = \frac{R_sL_m}{L_m^2 - L_rL_s},\ k_3 = \frac{R_rL_m}{L_m^2 - L_rL_s},\ k_4 = \frac{R_rL_s}{L_m^2 - L_rL_s},$$

$$k_5 = z_p,\ k_6 = \frac{K_dL_r}{L_m^2 - L_rL_s},\ K_d = \frac{3}{2}z_p\frac{L_m}{L_r}.$$

The fourth SS model is defined by the choice of the state vector

$$\mathbf{x} = [x_1, x_2, x_3, x_4, x_5]^T = [\psi_{s\alpha}, \psi_{s\beta}, i_{r\alpha}, i_{r\beta}, \omega]^T. \quad (3.9)$$

The derivation of the fourth model is achieved by considering the system of algebraic equations (3.2) and (2.9) in the joint electromechanical system of equations (2.11) and (2.8) to eliminate the superfluous variables $\psi_{r\alpha}$, $\psi_{r\beta}$, and ω_r. The resulting system of equations is solved for the total time derivatives of the state vector components to yield

$$\begin{aligned}
\dot{x}_1 &= u_1 - k_1 x_1 + k_2 x_3 \\
\dot{x}_2 &= u_2 - k_1 x_2 + k_2 x_4 \\
\dot{x}_3 &= k_3 u_1 - k_4 x_1 + k_5 x_3 - k_6 x_2 x_5 + k_7 x_4 x_5 \\
\dot{x}_4 &= k_3 u_2 - k_4 x_2 + k_5 x_4 + k_6 x_1 x_5 - k_7 x_3 x_5 \\
\dot{x}_5 &= [k_8 (x_1 x_4 - x_2 x_3) - u_4] / u_3
\end{aligned} \quad (3.10a)$$

$$\begin{aligned}
y_1 &= x_1 \\
y_2 &= x_2
\end{aligned} \quad (3.10b)$$

with coefficients

$$k_1 = \frac{R_s}{L_s},\ k_2 = \frac{R_s L_m}{L_s},\ k_3 = \frac{L_m}{L_m^2 - L_r L_s},\ k_4 = \frac{R_s L_m}{L_s (L_m^2 - L_r L_s)},$$

$$k_5 = \frac{R_r L_s}{L_m^2 - L_r L_s} + \frac{R_s L_m^2}{L_s (L_m^2 - L_r L_s)},\ k_6 = \frac{L_m z_p}{L_m^2 - L_r L_s},$$

$$k_7 = \frac{L_m^2 z_p}{L_m^2 - L_r L_s} - \frac{L_r L_s z_p}{L_m^2 - L_r L_s},\ k_8 = \frac{K_d L_r}{L_s},\ K_d = \frac{3}{2} z_p \frac{L_m}{L_r}.$$

The SS models (3.4), (3.6), (3.8), and (3.10) derived in this section are in fact nonlinear and non-autonomous systems because the inputs have an explicit dependence on time.

4. Model equivalence and analysis

A physical experiment has been done with an induction motor of type 4AO90L4D and its time response with regard to the angular velocity has been recorded according to [1,5]. The four SS models derived have been simulated under the same conditions to confirm their equivalence to the induction motor of type 4AO90L4D which is investigated. Thus, all models are simulated on zero initial conditions and the following motor parameters $L_s = 0.263\ \text{H}$, $L_r = 0.251\ \text{H}$, $L_m = 0.24\ \text{H}$, $R_s = 4.8\ \Omega$, $R_r = 3.87\ \Omega$, $z_p = 2$. The input vector components are defined as

$$u_1 = u_{max} \sin(\omega_0 t),\ u_2 = u_{max} \cos(\omega_0 t),\ u_3 = J,\ u_4 = \begin{cases} T_{L_1} \text{sign}(x_5), & t < 0.53\ \text{s} \\ (T_{L_1} + T_{L_2}) \text{sign}(x_5), & t \geq 0.53\ \text{s} \end{cases}$$

where $u_{max} = 310\ \text{V}$, $\omega_0 = 100\pi\ \text{s}^{-1}$, $J = 0.038\ \text{kg.m}^2$, $T_{L_1} = 4\ \text{N.m}$, $T_{L_2} = 10\ \text{N.m}$. Figure 2 shows the angular velocity response ω of SS models (3.4), (3.6), (3.8), (3.10) and the measured response ω_{exp} obtained from the physical experiment carried out with

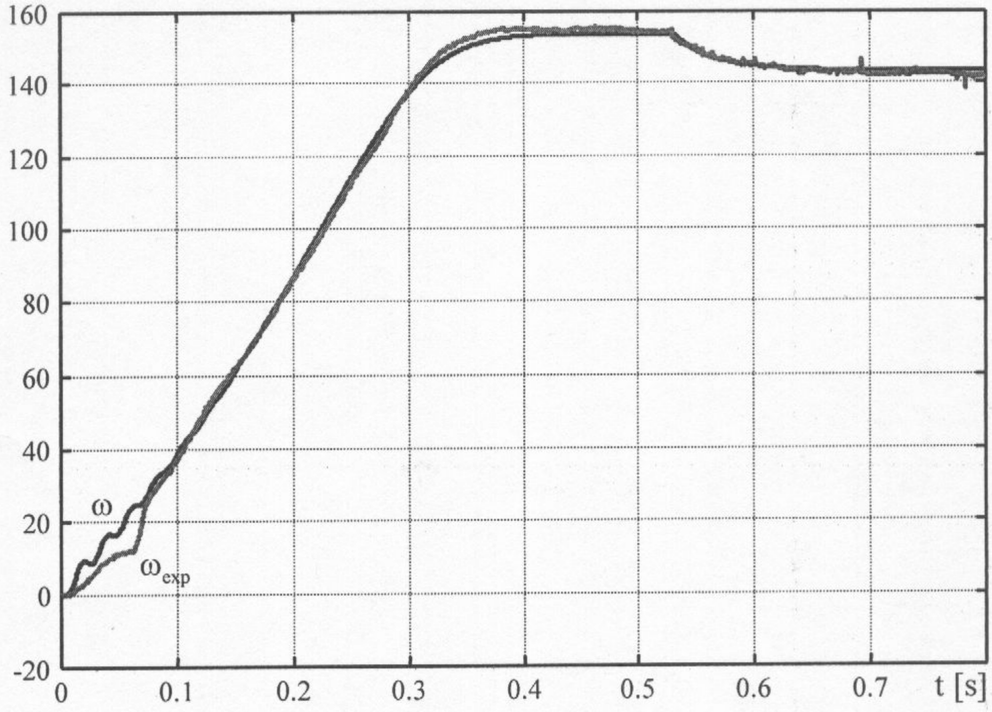

Figure 2: Angular velocity of the SS models and experimental response

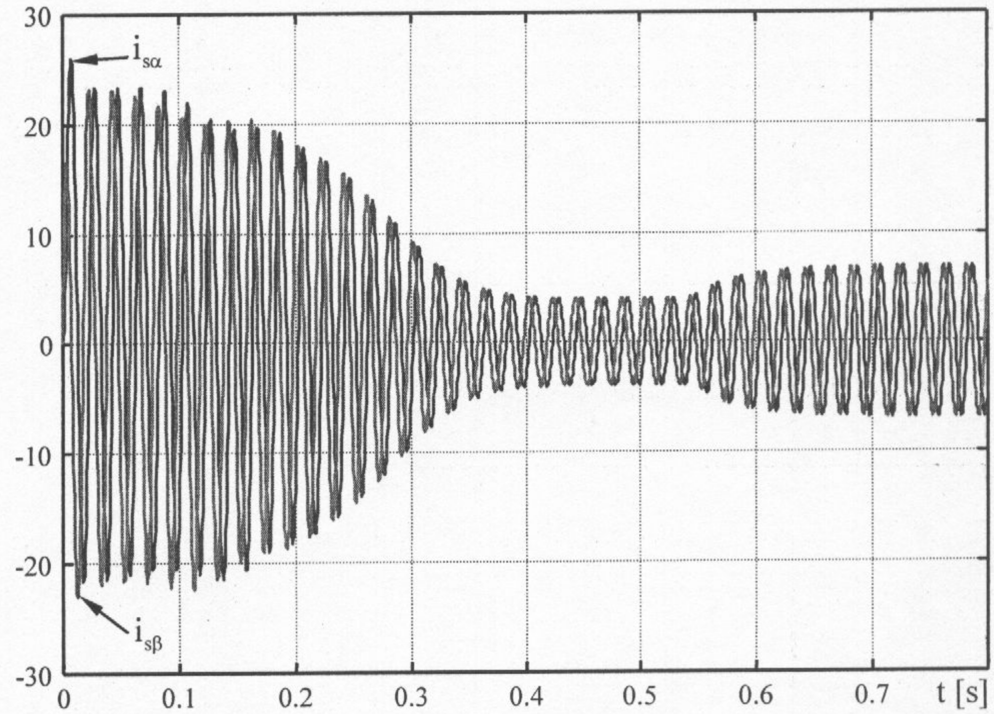

Figure 3: Stator currents response for SS models (3.4) and (3.6)

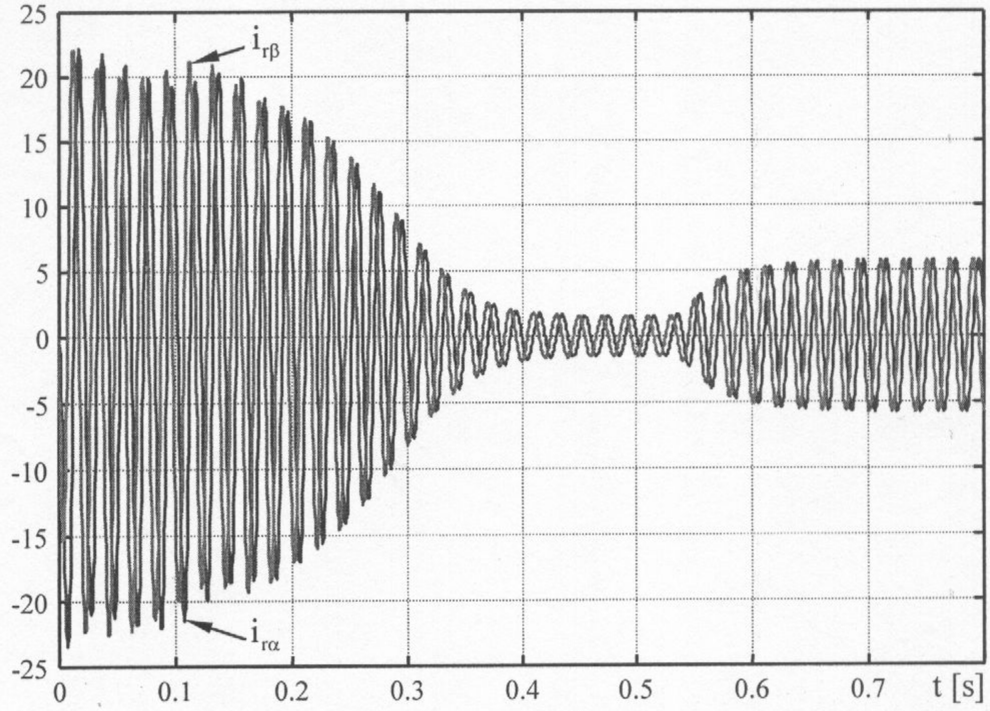

Figure 4: Rotor currents response of SS models (3.4) and (3.10)

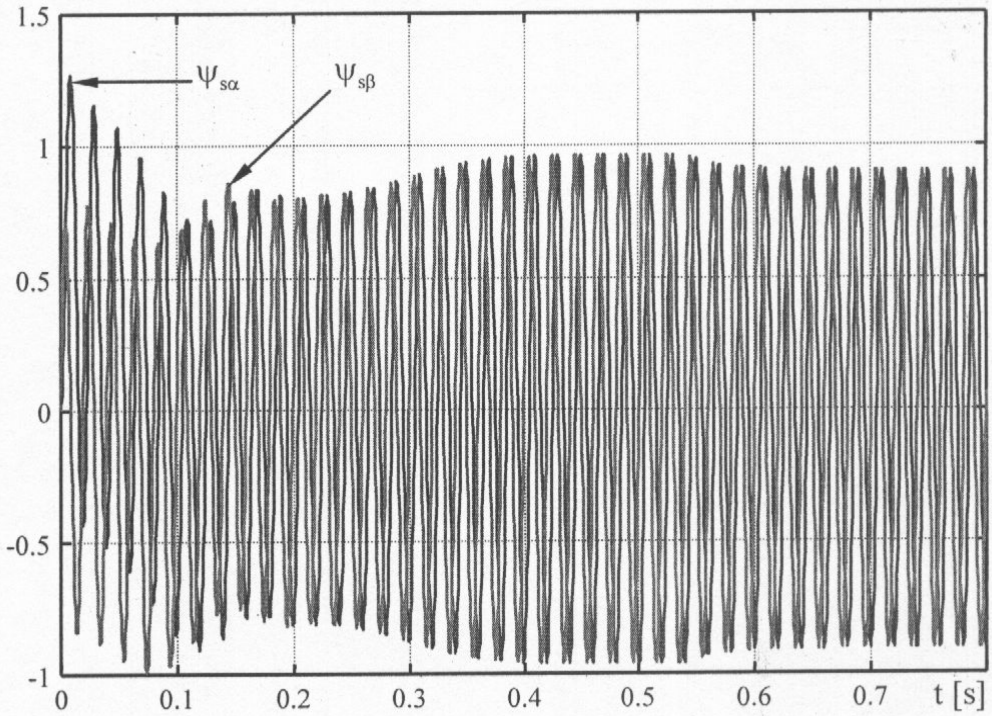

Figure 5: Stator fluxes response of SS models (3.8) and (3.10)

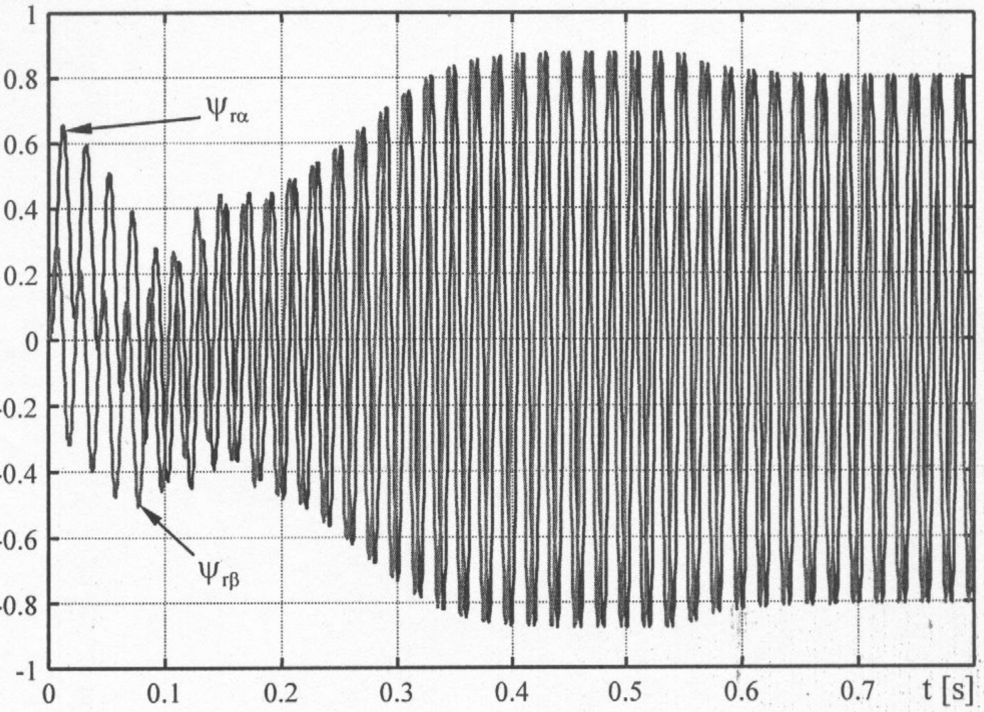

Figure 6: Rotor fluxes response of SS models (3.6) and (3.8)

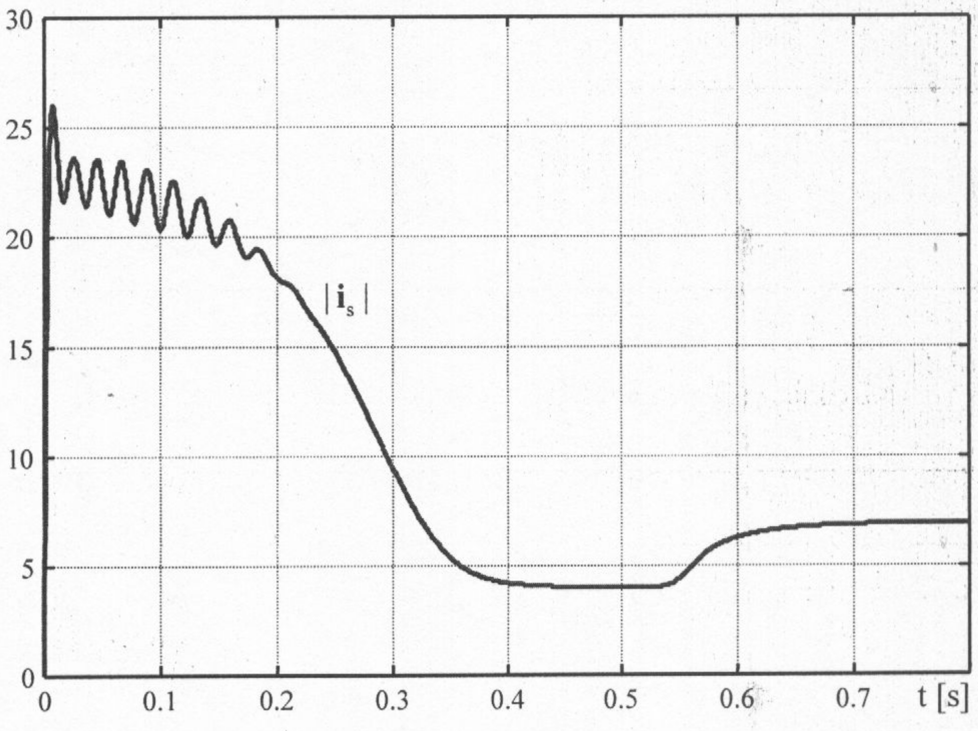

Figure 7: Module of the stator current vector

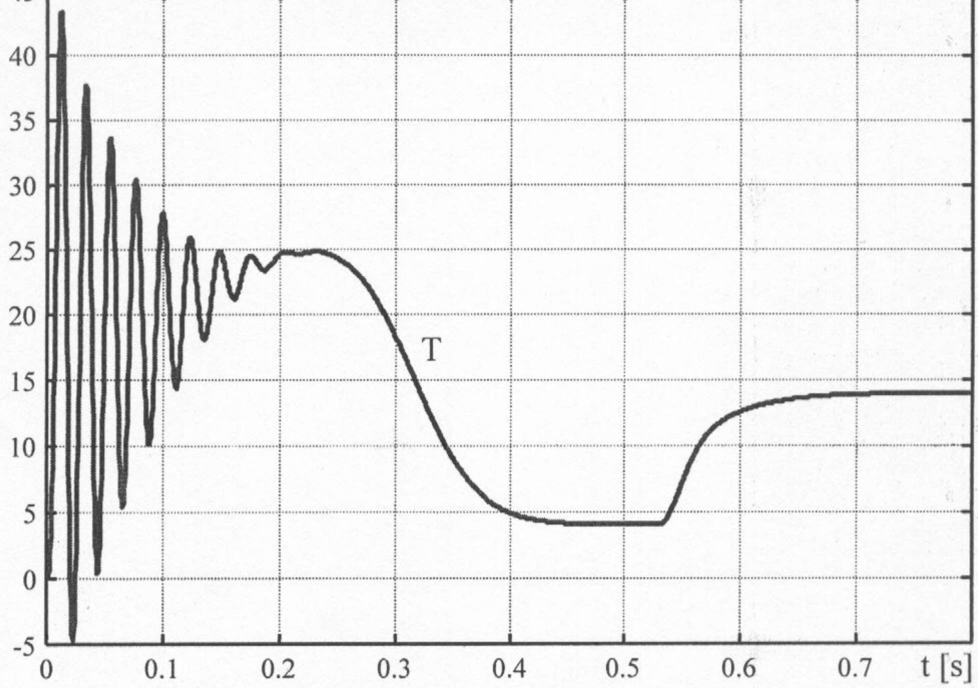

Figure 8: Motor torque response

the real system. Figures 3 and 4 depict the stator and rotor current components while figures 5 and 6 – the stator and rotor flux components obtained from the respective SS models which is a necessary confirmation of their equivalence. The module of the stator current vector is given on figure 7. The motor torque responses on figure 8, obtained from the four SS models, are one and the same, which is another confirmation of their equivalence.

The simulation experiment confirms that the four SS models derived are completely equivalent to each other with regard to the respective state vector components and the motor torque response. On the other hand they are equivalent with respect to the output ω to the measured response from the physical experiment. They have good numerical properties allowing smooth numerical integration via Runge-Kutta differential equation solvers. Fast and slow dynamics are observed in their time responses which is a prerequisite for stiffness of the equations. The fast dynamics is conditioned by the input signals having explicit time dependence. But luckily the level of stiffness is insignificant and the SS models can be successfully integrated with standard Runge-Kutta solvers by appropriate step selection.

The model analysis performed in this section includes also the investigation of the observability properties of the four SS models and the possibility to design nonlinear observers by transformation of the drive equations in reduced generalized observer canonical form (RGOCF) [8]. It is assumed that the stator current and the angular velocity are the measured output variables. By choosing observability indices

$n_1 = n_2 = 2$ and $n_3 = 1$ the system is split into three subsystems. Following [8] the observability matrix for the third SS model (3.8) reads

$$Q_s(\mathbf{x},\overline{\mathbf{u}}) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ k_1 & 0 & -k_2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & k_1 & 0 & -k_2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Its determinant is $\det[Q_s(\mathbf{x},\overline{\mathbf{u}})] = -k_2^2$ so the observability map is solvable with solution

$\mathbf{x}(\overline{\mathbf{y}},\overline{\mathbf{u}}) = [y_1, y_2, -k_2^{-1}(\dot{y}_1 - k_1 y_1 - u_1), -k_2^{-1}(\dot{y}_2 - k_1 y_2 - u_2), y_3]^T$.

By replacing this straight observability transformation in the derivatives of the outputs

$$h_1^{(2)}(\mathbf{x},\hat{\mathbf{u}}) = \dot{u}_1 + k_1(u_1 + k_1 x_1 - k_2 x_3) - k_2(k_4 x_3 - k_3 x_1 + k_5 x_4 x_5)$$
$$h_2^{(2)}(\mathbf{x},\hat{\mathbf{u}}) = \dot{u}_2 + k_1(u_2 + k_1 x_2 - k_2 x_4) - k_2(k_4 x_4 - k_3 x_2 - k_5 x_3 x_5)$$
$$h_3^{(1)}(\mathbf{x},\hat{\mathbf{u}}) = [k_6(x_2 x_3 - x_1 x_4) - u_4]/u_3$$

they are presented as functions of the outputs, the inputs and their derivatives

$$h_1^{(2)}(\overline{\mathbf{y}},\hat{\mathbf{u}}) = \dot{u}_1 + k_2 k_3 y_1 - k_4(u_1 + k_1 y_1 - \dot{y}_1) + k_1 \dot{y}_1 - k_5(u_2 + k_1 y_2 - \dot{y}_2) y_3$$
$$h_2^{(2)}(\overline{\mathbf{y}},\hat{\mathbf{u}}) = \dot{u}_2 + k_2 k_3 y_2 - k_4(u_2 + k_1 y_2 - \dot{y}_2) + k_1 \dot{y}_2 + k_5(u_1 + k_1 y_1 - \dot{y}_1) y_3$$
$$h_3^{(1)}(\overline{\mathbf{y}},\hat{\mathbf{u}}) = k_2^{-1} k_6 u_3^{-1}[(y_1 \dot{y}_2 - y_2 \dot{y}_1) + (u_1 y_2 - u_2 y_1)] - u_4 u_3^{-1}$$

which define the generalized observability canonical form (GObCF) and the possibility for transformation in RGOCF depends on them. The RGOCF for this system is

$$\begin{aligned} \dot{z}_1 &= a_1(\mathbf{y},\mathbf{u}) \\ \dot{z}_2 &= z_1 + a_2(\mathbf{y},\mathbf{u},\dot{\mathbf{u}}) \\ \dot{z}_3 &= a_3(\mathbf{y},\mathbf{u}) \\ \dot{z}_4 &= z_3 + a_4(\mathbf{y},\mathbf{u},\dot{\mathbf{u}}) \\ \dot{z}_5 &= a_5(\mathbf{y},\mathbf{u}) \end{aligned} \tag{4.1a}$$

$$\begin{aligned} y_1 &= g_1(y_1,\mathbf{u}) \\ y_2 &= g_2(y_2,\mathbf{u}) \\ y_3 &= g_3(y_3,\mathbf{u}) \end{aligned} \tag{4.1b}$$

The transformation in this form consists in the computation of the unknown functions taking part in it according to the necessary and sufficient conditions for transformability [8]. The last function $a_5(\mathbf{y},\mathbf{u})$ equals $h_3^{(1)}(\overline{\mathbf{y}},\hat{\mathbf{u}})$ or

$a_5(\mathbf{y},\mathbf{u}) = k_2^{-1} k_6 u_3^{-1}[(y_1 \dot{y}_2 - y_2 \dot{y}_1) + (u_1 y_2 - u_2 y_1)] - u_4 u_3^{-1}$.

It is seen that in this case $a_5(\mathbf{y},\mathbf{u})$ depends also on $\dot{y}_1$ and $\dot{y}_2$ which is a break of a necessary condition. The gradient of $a_2(\mathbf{y},\mathbf{u},\dot{\mathbf{u}})$ and its integrability matrix $\mathbf{J}_{ww}$ [8] are

$$\begin{bmatrix} \partial a_2/\partial y_1 \\ \partial a_2/\partial y_2 \\ \partial a_2/\partial y_3 \\ \partial a_2/\partial u_1 \\ \partial a_2/\partial u_2 \\ \partial a_2/\partial \dot{u}_1 \\ \partial a_2/\partial \dot{u}_2 \end{bmatrix} = \begin{bmatrix} k_1+k_4 \\ k_5 y_3 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{J}_{ww} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & k_5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The integrability matrix $\mathbf{J}_{ww}$ is not symmetric and therefore the function $a_2(\mathbf{y},\mathbf{u},\dot{\mathbf{u}})$ with the above gradient does not exist which is a second break of a necessary condition. The situation with the function $a_4(\mathbf{y},\mathbf{u},\dot{\mathbf{u}})$ is similar. Thus, the fact that some necessary conditions are not fulfilled means that this system is not transformable in RGOCF and the nonlinear observer based on this canonical form cannot be designed. The third SS model (3.8) is structurally the simplest. The other three SS models have also been checked and the results are analogous.

The observability and transformability in RGOCF of a nonlinear system are structural properties of the nonlinear system model and its generalized characteristic equation. The transformation in RGOCF and its nonlinear observer design are impossible for all the four SS models developed which is due to the specific crossed nonlinear dependencies in their equations leading to unsuitable structure of the respective generalized characteristic equations.

5. Conclusions

A mathematical description of the electromechanical energy conversion in an induction motor is presented in the paper based on a stationary coordinate frame for control purposes. The algebraic dependencies between the currents and the fluxes are revealed in vector and componentwise form. On this basis, the differential equations are untied with respect to different combinations of variables by solving them for the derivatives of those variables. A choice of state variables is done stemming from the solutions obtained resulting in the building of all possible orientated nonlinear state space models, describing the electromechanical converter based on a stationary coordinate frame. The consistency of the models and their equivalence are confirmed by physical experiments and simulation.

The structural properties of the nonlinear SS models derived are analyzed with regard to complexity, numerical properties and stiffness, non-autonomy, bound conditions, observability, and possibility for nonlinear observer design by RGOCF.

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