©Journal of the Technical University at Plovdiv
"Fundamental Sciences and Applications", Vol. 12, 2005
Series C - Electrical Engineering
BULGARIA

Electrical Engineering

COMPARATIVE ANALYSIS OF THE FORCE COMPUTATION METHODS IN THE 2D FEM

VASIL SPASOV, STANIMIRA SHISHKOVA, IVAN KOSTOV

Abstract. Three methods for electromagnetic force computation are analyzed - the virtual work method, the Maxwell stress tensor method and the nodal force method. The numerical analysis is carried out by the two-dimensional nodal finite element method with first order triangles. The methods are applied to compute the electromagnetic force at different air gaps of a non-linear permanent magnet with steel core. A comparison is made from the viewpoint of accuracy, speed and computer implementation.

Introduction.

The Maxwell stress tensor method (MSTM), the virtual work method (VWM) and the nodal force method (NFM) are among the most popular methods of electromagnetic force computation by the two-dimensional finite element method (FEM) with nodal elements [1], [2], [3]. Theoretically, these force computation methods are equivalent [2]. As the force, however, is calculated from an approximate finite element solution, the various methods can yield different forces for the same finite element mesh.

While the MSTM and VWM have been extensively studied with both edge and nodal finite elements, the NFM has been used mainly with edge-based formulations [2]. Moreover, the three above-mentioned methods have been applied mostly to devices with constant air gaps. Therefore, it is expedient to analyze the MSTM, VWM and NFM when computing force in devices whose air gaps vary within a wide range. For this purpose the electromagnetic force of a non-linear permanent magnet with a non-linear steel core is computed by the three methods in the present paper.

The MSTM, VWM and NFM are compared from the viewpoint of accuracy, speed and computer implementation. Finally, conclusions are made so as to help the user when choosing the most appropriate method of force computation.

ISSN 1310 - 8271

Description of the force computation methods.

In the virtual work method the force on an object is found as the derivative of the magnetic energy W_m with respect to the displacement s of the moving part:

$$F_s = -\left(\frac{\partial W_m}{\partial s}\right)_{\psi = const} . \qquad (1)$$

The flux linkage ψ in (1) should be kept constant.

Formula (1) can be approximated by the finite difference scheme:

$$F_s = \frac{W_2 - W_1}{\Delta s} , \qquad (2)$$

where W_2 is the energy after displacing the moving part at the distance Δs and W_1 is the initial energy.

Although easy to program, this modification has some serious disadvantages:

- Two solutions of the field problem are needed. This requires the generation of two meshes with the respective topological structures and two times longer computation time for the numerical solution.

- Additional numerical error is introduced when dividing the difference in energies $(W_2 - W_1)$ by the very small value Δs. This deteriorates the accuracy of results.

The global electromagnetic force $\boldsymbol{F}$ is computed by the Maxwell stress tensor method as follows [3]:

$$\boldsymbol{F} = \int_S \nu_0 \left[(\boldsymbol{B}\boldsymbol{n})\boldsymbol{B} - 0.5\,\boldsymbol{B}^2 \boldsymbol{n} \right] d\Omega , \qquad (3)$$

where S is a closed surface surrounding the moving object and $\boldsymbol{n}$ is an outward unit vector normal to this surface.

Thus the force of one finite element acting along the z axis is:

$$F_e^z = \nu_0 \left[\left(n_e^x B_e^x + n_e^y B_e^y + n_e^z B_e^z \right) B_e^z - 0.5\, B_e^2 n_e^z \right] S_e , \qquad (4)$$

where S_e is the area obtained when intersecting the element with the integration surface. The total force is equal to the sum of the forces of all intersected elements.

MSTM calculates the global force by integrating the force densities. These densities, however, have little meaning to the local force on a body [2]. Due to the discontinuity of the obtained field quantities, the results by MSTM need further processing such as averaging the field solution. Another difficulty of the MSTM is the proper choice of the integration surface S. Theoretically, the choice of S is arbitrary, provided it is situated in linear media and encloses the movable body. In practice, however, the location of the integration surface may strongly affect the accuracy of computed force.

A common disadvantage of MSTM and VWM is their restriction to global force computation. In the analysis and design of electromagnetic devices, however, it is often necessary to compute the local magnetic force. In this case the nodal force method can be used.

The force acting on node k in the nodal force method is computed as [2]:

$$F^k = -\iiint_{V_e} T_{ij} \frac{\partial N^k}{\partial x_j} dv, \quad (5)$$

where T_{ij} is the Maxwell stress tensor and N^k is the shape function at node k.

The z component of the force of node k of a tetrahedral element becomes:

$$f_z^k = -\left(T_{31}b_k + T_{32}c_k + T_{33}d_k\right)V_e, \quad (6)$$

where b_k, c_k and d_k are the coefficients of the nodal shape functions.

It can be seen that the NFM is the easiest to program among the three methods. All quantities in (6) are already known from the finite element analysis. No integration surface as MSTM and no integration volume as WVM has to be defined. Thus NFM can be implemented fully automatically in models of arbitrary shape. These advantages make the nodal force method very attractive to use.

Comparison of the VWM, MSTM and NFM.

In order to compare the three methods, the force of an Alnico nonlinear permanent magnet with nonlinear steel core is computed [4]. The permanent magnet dimensions in centimeters are shown in Fig. 1.

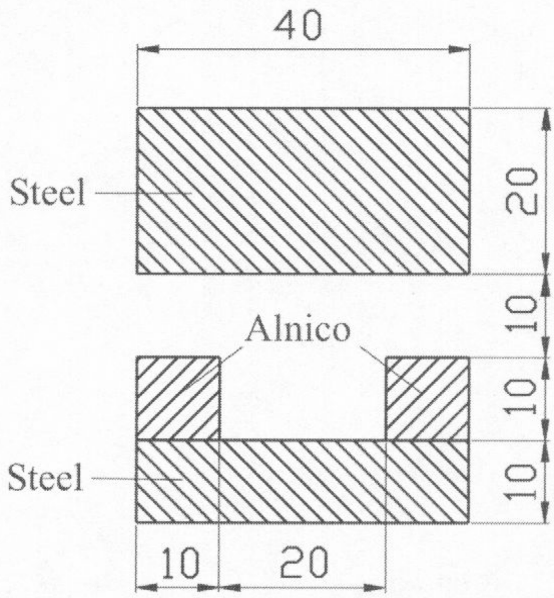

Fig. 1. Dimensions of the Alnico magnet.

The numerical analysis is performed by the software Finite Element Method Magnetics [5]. Magnetic vector potential formulation is used. The finite element mesh has 71394 nodes and 141915 triangles. The nonlinearity of the steel core and of the permanent magnet are taken into account in the finite element analysis. The computed electromagnetic forces at different air gaps of the magnet are given in Table 1 and shown in Fig. 2. The results for the VWM are obtained by (2) with Δs being $1\,\%$ of the air gap δ. The results with $\Delta s = 0.1\,\%$ of δ were similar. The integration contour for the MSTM encompasses two layers of finite elements in the air around the steel armature.

Table 1. Force by the NFM, MSTM and VWM

Air gap [cm]	Computed force [N]		
	NFM	MSTM	VWM
1	22,5240	22,5100	8,0300
2	13,5023	13,5000	6,7750
3	8,8606	8,8440	3,6500
4	6,1640	6,1710	3,5500
5	4,4759	4,4700	2,9700
6	3,3549	3,3620	2,0600
7	2,5741	2,5810	1,9400
8	2,0159	2,0170	1,6280
9	1,6031	1,6020	1,3397
10	1,2931	1,2900	1,1477

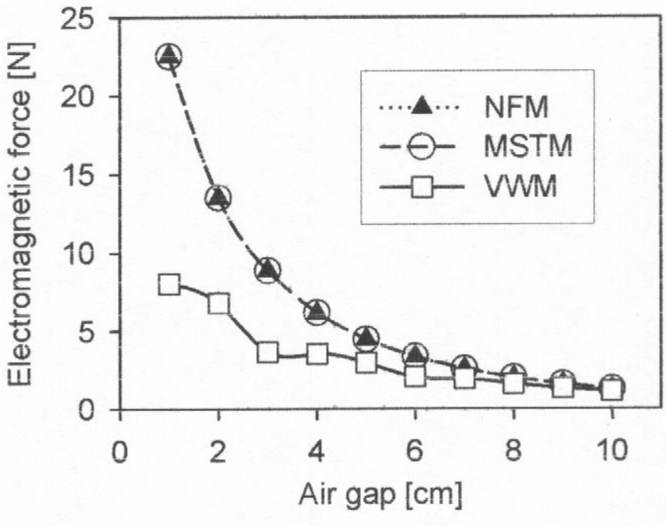

Fig. 2. Computed force at different air gaps.

The comparison between the computed forces in Table 1 and Fig. 2 shows that while NFM and MSTM compute similar forces for all air gaps, the accuracy of VWM rapidly deteriorates with the decrease of the air gap.

Next the three methods are compared with one another. The relative error of the NFM and the VWM is computed by the formula:

$$\varepsilon\% = \left[\left(F - F_{MSTM} \right) / F_{MSTM} \right] \cdot 100, \qquad (7)$$

where F is the force by the VWM or NFM and F_{MSTM} is the force by MSTM.

The force by the MSTM is used in (7) instead of measurements because the measured force was not available. In order to use MSTM as reference in (7), the integration contour has been adequately chosen [4], [5]. The relative error in force is shown in Fig. 3.

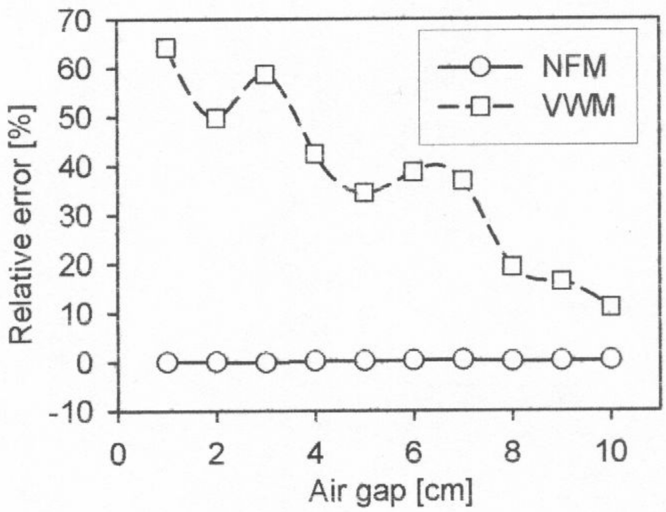

Fig. 3. Relative error of VWM and NFM.

Fig. 3 shows that the error of the VWM rapidly grows with the decrease of the air gap. Starting from $\varepsilon = 11\%$ at air gap $\delta = 10$ cm, the VWM underestimates force by 64% at $\delta = 1$ cm as compared to the NFM and MSTM.

The accuracy of the NFM is good at all air gaps, its maximum relative error being $\varepsilon = 0.24\%$ at $\delta = 10$ cm. The computer implementation of the MSTM, however, is more difficult than the NFM, since some manual labor is needed to define the integration surface. This shows that the NFM can be preferred to MSTM and VWM.

Finally, the speed of the MSTM, VWM and NFM is analyzed. The speed of the NFM is comparable with the speed of the MSTM, while the VWM is the most slowly due to the necessity of two solutions.

Conclusions.

Three methods of force computation using nodal finite elements are compared - the virtual work method, the Maxwell stress tensor method and the nodal force method. The methods are applied to compute the force at different air gaps of a non-linear permanent magnet. The accuracy of the methods is analyzed and their computer implementation and speed are discussed.

It is shown that the accuracy of the two-solution VWM deteriorates at smaller air gaps. This is due to the numerical error because of loss of significant figures in (2). Moreover, the VWM is the most slowly among the three methods.

The results show that the accuracy of the MSTM and NFM does not depend on the air gap length.

The NFM is easy to program and needs no integration contour. The speed of the NFM is competitive with the speed of the MSTM. These advantages, together with the possibility for visualization of the forces on the finite element nodes, make the NFM attractive to use. Based on the overall comparison, the nodal force method can be recommended as the method of choice for electromagnetic force computation by the 2D finite element method.

References

1. **S. J. Salon**. Finite element analysis of electrical machines, Kluwer Academic Publishers, 1995.
2. **I. Nishiguchi, A. Kameari and K. Haseyama.** On the local force computation of deformable bodies in magnetic field. *IEEE Trans. Magn.*, Vol. 35, No. 3, May 1999, 1650-1653.
3. **V. Spasov, So Noguchi and H. Yamashita.** Computation of electromagnetic force by the 3D nodal finite element method, *Proceedings of the Second Japanese-Greek Joint Workshop on Superconductivity and Magnetic Materials*, Oita, Japan, May 10-11, 2001, 80-85.
4. **K. Brandisky and I. Yatcheva.** *CAD systems in electromagnetism*. CIELA, Sofia, 2002 (in Bulgarian).
5. **D. Meeker.** Finite Element Method Magnetics 4.0 User's Manual, 2004.

Department of Electrical Engineering
Technical University
25, Tsanko Dystabanov Str.
4000 Plovdiv
BULGARIA