AN INVESTIGATION OF THE MOTION OF A HELICOPTER ROTOR WITH FLAPPING AND LEAD/LAG HINGES IN HOVER

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Abstract:
A modelling of pattern helicopter rotor with flapping and lead/lag hinges in hover is presented in this paper. The coupled aeroelastic problem accounts for the mutual dependence between blade structure and rotor aerodynamics. The aerodynamic model uses Blade Element Momentum Theory (BEMT). BEMT gives good accuracy with respect to time cost. The structure model uses Finite Element Method (FEM). The investigation is based on the well-known solvers MATLAB and ANSYS. This modelling is also useful and applicable for airplane propellers, wind turbine rotors and airplane wings.

Keywords: helicopter, aeroelasticity, aerodynamics, structural dynamics.

1. Introduction

A large number of helicopters use mechanical flaps and lead/lag hinges. They are incorporated at the root of each blade. The hinges allow each blade to independently flap and lead/lag with respect to the hub plane under the action of lift and drag forces. In hover, the flow field is azimuthally axisymmetric and each blade encounters the same aerodynamic loads. The blades flap up and lag back with respect to the hub and reach a steady equilibrium position under the action of these steady aerodynamic and centrifugal forces.

The last years of research have provided significant successes in the prediction of airloads on the helicopter rotors. These predictions are based on a numerical approach, where the flow is simulated using Computational Fluid Dynamics (CFD) tools using prescribed moving boundary conditions. The computations normally include a comprehensive rotor code, coupled to Euler or Navier-Stockes solvers [4], [11], [13]. The examples for a successful application of CFD are the codes FLUENT, TURNS of NASA, FLOWer of Deutshes Zentrum für Luft und Raumfahr, elsA and WAVES of ONERA [13]. However, for the case of aeroelastic calculations the CFD method has to be very time consuming. Thus it can be replaced by the Blade Element Momentum Theory (BEMT) or the vortex wake method, which show a good accuracy with respect to time cost [8], [10]. In these methods, the helicopter blade is divided into a number of independent elements along the length of the blade. Each section of the blade acts as quasi 2-D airfoil, which produces aerodynamic forces and moments.

The structure of helicopter blade has been modelled in different ways but mostly relies on a modified beam model or one-dimensional finite elements [11].

The methods of analysis used for determination of the helicopter blade deformations under the effects of aerodynamic and inertia forces are described as “lumped-parameter” methods. There the continuous blade is presented by a number of discrete segments, so that the partial differential equations of blade deformations are replaced by a set of simultaneous ordinary differential equations. Such as the method of Holzer-Myklestad [3], collocation method [1] and Finite Element Method (FEM) [1], [5], [14]. The FEM solvers as ANSYS, ABAQUS, NASTRAN and...
ADAMS are often applied at the investigations of helicopter rotor dynamics.

The advanced helicopter code called UMARC is well validated and extensively used in the helicopter rotor dynamics investigations. The rotor-fuselage equations are formulated using Hamilton’s principle and are discretized using finite elements in space and time. The blade airloads can be computed using quasisteady aerodynamics, linear unsteady aerodynamics or nonlinear unsteady aerodynamics [2], [6].

Other successful helicopter code is CAMRAD. The used model is a combination of structural, inertial, and aerodynamic models. The rotor aerodynamic model is based on free wake model, which take into account the unsteady flow effects, including a dynamic stall [6], [7].

The aim of this work is to present an investigation of the motion of a helicopter rotor with flapping and lead/lag hinges flapping and lead/lag hinges in hover. The structural model uses FEM, and aerodynamic model uses BEMT. The structural model relies on ANSYS code, and aerodynamic model relies on MATLAB code.

2. Structural model

The sketch of the pattern rotor is shown in Fig.1.

An ANSYS code is used for the structural analysis. The structural model of the pattern helicopter rotor uses finite elements: BEAM44 and MPC184-Revolute (Table 1). The BEAM44 element has six degrees of freedom at each node: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z-axes. This element uses different asymmetrical geometry without coinciding centre of gravity and elastic centre. The MPC184-Revolute joint element is a two-node element that has only one primary degree of freedom, the relative rotation about the revolute (or hinge) axis. This element imposes kinematic constraints such that the nodes forming the element have the same displacements. Additionally, only a relative rotation is allowed about the revolute axis, while the rotations about the other two directions are fixed.

The rotor consists of 23 finite elements, as shown in Table 1. The material of the blade root and movable connection between hinge axes is Aluminum 7079, and the blade is from balsa wood. All necessary data are shown in Table 1. The transient analysis solution method is used and gyroscopic or Coriolis effects are included.

3. Loads on the helicopter blade

In the BEMT method, the helicopter blade is divided into a number of independent elements along the length of the blade. At each section, a force balance is applied involving 2D section lift and drag with the thrust and torque produced by the section. At the same time, a balance of axial and angular momentum is applied. The force balances produce a set of non-linear equations which can be solved numerically for each blade section. The description follows [10].

Fig. 2 shows the incident velocities and aerodynamic forces at a blade element on the helicopter rotor.
Table 1. Description of the structural helicopter rotor FEM model

<table>
<thead>
<tr>
<th>Section properties:</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Section area, $A$, m$^2$</td>
<td>0.452×10$^3$</td>
<td>0.636×10$^4$</td>
<td>0.810×10$^4$</td>
<td>0.106×10$^3$</td>
</tr>
<tr>
<td>Inertia moment about the $x$ axis, $I_{xx}$, m$^4$</td>
<td>0.163×10$^7$</td>
<td>0.321×10$^9$</td>
<td>0.547×10$^9$</td>
<td>0.127×10$^9$</td>
</tr>
<tr>
<td>Inertia product, $I_{xz}$, m$^4$</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Inertia moment about the $y$ axis, $I_{yy}$, m$^4$</td>
<td>0.163×10$^7$</td>
<td>0.321×10$^9$</td>
<td>0.547×10$^9$</td>
<td>0.127×10$^9$</td>
</tr>
<tr>
<td>$x$ coordinate of shear center about c.g., $x_{sc}$, m</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.002</td>
</tr>
<tr>
<td>$z$ coordinate of shear center about c.g., $z_{sc}$, m</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Finite element model:

- Nodes: 1-23
- Elements: 1-21, 45, 67

Elements Type:

- Elements: 1-21
- Elements: 45 and 67

Material properties:

- Elements: 1-4
  - Aluminum 7079, isotropic material
  - Elastic modulus, $E_x$, GPa: 71.7
  - Elastic modulus, $E_y$, GPa: 0.156
  - Elastic modulus, $E_z$, GPa: 0.051
  - Shear modulus, $G_{xy}$, GPa: 26.9
  - Shear modulus, $G_{xz}$, GPa: 0.126
  - Shear modulus, $G_{yz}$, GPa: 0.017
  - Density, $\rho$, kg/m$^3$: 2800

- Elements: 5-21
  - Wood: Balsa, orthotropic material
  - Elastic modulus, $E_x$, GPa: 3.400
  - Elastic modulus, $E_y$, GPa: 0.156
  - Elastic modulus, $E_z$, GPa: 0.051
  - Shear modulus, $G_{xy}$, GPa: 0.183
  - Shear modulus, $G_{xz}$, GPa: 0.126
  - Shear modulus, $G_{yz}$, GPa: 0.017
  - Density, $\rho$, kg/m$^3$: 160

As is shown in Fig.2, the resultant flow velocity at each blade element at a radial distance $y$ is

$$U = \sqrt{U_T^2 + U_p^2}$$  \hspace{1cm} (1)

where $U_T = \Omega y$ and $U_p = v_T$. The induced angle of attack (inflow angle) at the blade element is

$$\phi = \tan^{-1} \left( \frac{U_p}{U_T} \right)$$  \hspace{1cm} (2)

The averaged induced velocity is connected with the inflow ratio

$$\lambda = \frac{V_i}{\Omega R} = \phi r$$  \hspace{1cm} (3)

where $r=y/R$.

The effective angle of attack is

$$\alpha = \theta - \phi$$  \hspace{1cm} (4)

where $\theta$ is the pitch angle at the blade element.
Fig. 2. Incident velocities and aerodynamic forces at a blade element

When there is torsional elastic deformations of the blade, \( \theta_e \), the effective angle of attack is

\[
\alpha = \theta - \phi + \theta_e
\]  

(5)

The resultant lift \( dL \) and drag \( dD \) per unit span on the blade element are

\[
dL = \frac{1}{2} \rho U^2 c CL dy \quad \text{and} \quad dD = \frac{1}{2} \rho U^2 c CD dy
\]

(6)

where \( \rho \) is the density of air, \( c \) is the local blade chord, \( CL \) and \( CD \) are the lift and drag coefficients.

Based on steady linearized aerodynamics, the local blade lift coefficient can be written as

\[
CL = CL_{0} (\alpha - \alpha_0)
\]

(7)

where \( CL_{0} \) is the 2-D lift-curve-slope of airfoil section, and \( \alpha_0 \) is the corresponding zero-lift angle. The effects of compressibility corrects the lift-curve-slope of each blade element according to Glauert’s rule

\[
CL_{0} = \frac{CL_{0,1}}{\sqrt{1 - M^2}}
\]

(8)

\( CL_{0,1} \) is the measured 2-D lift-curve-slope at \( M=0.1 \). The local blade Mach number is

\[
M = \frac{U}{c} = \frac{\Omega y}{a}
\]

(9)

where \( a \) is the sonic velocity.

The inflow ratio is obtained by the use of Prandtl’s tip-loss method

\[
\lambda = \frac{\sigma CL_{0}}{16F} \left( \frac{1 + 32F0r}{\sigma CL_{0} - 1} \right)
\]

(10)

where \( \sigma = \frac{N_s c}{\pi R} \) is the rotor solidity, and \( N_s \) is the number of rotor blades. The Prandtl’s factor, \( F \), corrects the induced velocity

\[
F = \left( \frac{2}{\pi} \right) \cos^{-1}\left( e^{-r} \right)
\]

(11)

and

\[
f = \frac{N_s}{2} \left( 1 - \frac{r}{r\phi} \right)
\]

(12)

Because \( F \) is a function of \( \lambda \), the Eq.11 must be solved iteratively. For first iteration \( F=1 \) (corresponding to an infinite number of blades) and then finding \( \lambda \) from Eq.11 and recalculating \( \lambda \) from the numerical solution to Eq.10. Convergence is obtained in 3 or 4 iterations.

Finally the forces perpendicular and parallel to the rotor disk are

\[
dF_{x} = dL \cos \phi - dD \sin \phi
\]

(13)

\[
dF_{y} = dL \sin \phi + dD \cos \phi
\]

(14)

The equilibrium of the blade depends by the balance of aerodynamic and centrifugal forces. In accordance with Figs. 3 and 4, a small element of the blade of length \( dy \) is considered. The mass of this element is \( m dy \). The centrifugal force acting in a parallel direction to the plane of rotation is

\[
d(F_{xy}) = m \Omega^2 y dy
\]

(15)
Because the aerodynamic center and shear center not coincident, reference to classical two-dimensional airfoil theory gives the section moment \( dM_x \), as taken about shear center, defined in Fig. 5

\[
dM_x = \frac{1}{2} \rho U^2 c_i x_c \, dy
\]  
(16)

Fig. 5. Section moment relevant geometry of a blade element

The helicopter airfoil is NACA 0012. The airfoil data for NACA 0012 are presented in [12]. The rotor angular velocity, \( \Omega \), is 50 rad/s. The blade chord is 0.032 m. The blade has not linear twist. The blade pitch angle is 5°. The aerodynamic loads are calculated by MATLAB code.

4. Numerical results

The algorithm of the coupling between the structural model and aerodynamic model is:

1. The aerodynamic forces are computed for an ideal rigid blade by MATLAB code;
2. The aerodynamic and inertia forces are applied on the blade FEM-model by code ANSYS;
3. The linear and angular blade deformations are computed by ANSYS code;
4. The new angle of attack for each section of the blade is computed and new distribution of the aerodynamic forces is received by MATLAB code for second iteration;
5. The new aerodynamic and inertia forces are applied on the FEM-model by ANSYS code;
6. The new linear and angular blade deformation are calculated by ANSYS code;
7. The new distributions of the aerodynamic forces and deformations are compared;
8. If they are changed more than a certain tolerance: go to step 4;
9. The iteration is repeated while a convergence is achieved.

The results are shown in Figs. 6-10. Fig. 6 shows the displacements \( x \), which define the lagging motion, Fig. 7 shows the displacements \( y \), which define the flapping motion, and Fig. 8 shows the torsional deformation of the helicopter blade in the nodes. The results are obtained by ANSYS code. Fig. 9 shows the lift distribution, and Fig. 10 shows drag distribution, as the results are obtained by MATLAB code.
Figs. 9 and 10 show that torsional deformations of the helicopter blade don’t effect on the lift and drag distribution. Therefore, the realization of the first three points of the algorithm is sufficient.

The coning (flapping up) angle $\beta$ is 0.86º, and the lag back angle $\zeta$ is 0.08º. The centrifugal forces are dominant and the coning angle of the helicopter rotor remains small. Because the drag forces are only a fraction of the lift forces and are overpowered by the centrifugal forces, the lag angle is smaller than the coning angle.

5. Conclusion and future works

A simple dynamic modelling of a pattern helicopter rotor with flapping and lead/lag hinges in hover, which uses well-known solvers MATLAB and ANSYS, was presented in this paper. The effective and fast BEMT method was applied to create an aerodynamic model. To analyse the aeroelastic behaviour of helicopter rotor the FEM method is used, as the rotor base, movable connection between hinge axes, and blade were presented with equivalent beam models. The hinges are presented as revolution joints.

The future work provides for this modelling to be made in forward flight. An experiment in a wind tunnel will also provided for.

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