RESEARCH OF VEHICLES DIRECTIONAL STABILITY

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Abstract: *In this work for the research of vehicles directional stability was used three different mechanical-mathematical models. The behavior of car stability has been simulated with MATLAB software.*

Keywords: DIRECTIONAL STABILITY, MECHANICAL-MATHEMATICAL MODEL, LATERAL ELASTICITY, SLIP ANGLE

1. Introduction

The tendency of a vehicle to return to its original direction when disturbed (rotated) away from that original direction is directional stability.

The goal of this work is research of vehicles directional stability. The situation is described by three different mechanical-mathematical models around to the axes O_y and O_z.

During the research we make the following considerations:

- The car and his models are symmetrical to the axe O_x;
- The dynamical process (displacement around to the axes O_z) isn't exanimated;
- The main factor of cars stability is the wheels slip angle;
- Working in the zone of pure rolling motion (the slip angle for the wheels was zero). Figure 1 shows the relationship between the lateral force a tire F_y and the slip angle of the tire δ_y.

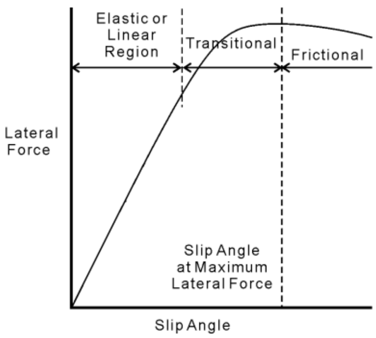

Fig. 1 *Relationship between the lateral force a tire F_y and the slip angle of the tire δ_y*

Mechanical-mathematical model examine the lateral elasticity

The behavior of the car is described using mechanical-mathematical model. Scheme of the model is shown to Figure 2 and the suspended masses include the masses of the elements of the car, passengers and load. If we consider that the slip angle for the wheels was zero we could examine the car as a mass witch point 1, 2, 3 and 4 around to the axe O_y are bending.

The motion of system is exanimated as function to the displacements are around the axes O_y and the angular displacement around the axes O_z.

In the center of gravity is fixed local coordinate system attached $O_0x_0y_0z_0$. All displacements of local coordinate systems are given to the absolute coordinate system $O_Ax_Ay_Az_A$. In the equilibrium position the axis of all coordinate systems are parallel.

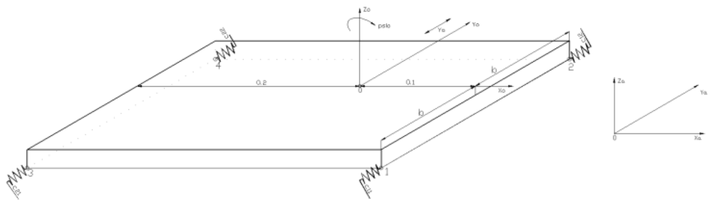

Fig. 2 *Mechanical-mathematical model, examined the zone of directional stability*

To find the lows motion to the absolute coordinate system $O_Ax_Ay_Az_A$ is necessary to define the transition matrices of each local coordinate systems to the absolute.

For generalized coordinate systems are assumed:

- y_0 – linear displacement of the local coordinate system $O_0x_0y_0z_0$ to absolute $O_Ax_Ay_Az_A$ around axis O_y;
- ψ_0 – angular displacement of the coordinate system $O_0x_0y_0z_0$ to absolute $O_Ax_Ay_Az_A$ around axis O_z;

Matrix of transition from $O_0x_0y_0z_0$ to $O_Ax_Ay_Az_A$ to the system is:

$$T_o^A = \begin{bmatrix} \cos\psi_o & -\sin\psi_o & 0 & 0 \\ \sin\psi_o & \cos\psi_o & 0 & y_0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

x_0 and z_0 are zero (we consider only linear displacement around axis O_y, i.e. only laterally).

The matrix of transition T_o^A can be simplified with the assuming that ψ_0 is small witch leads to the linearization of trigonometrically functions ($\sin\psi_0 \approx \psi_0$ and $\cos\psi_0 \approx 1$):

$$T_o^A = \begin{bmatrix} 1 & -\psi_o & 0 & 0 \\ \psi_o & 1 & 0 & y_0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Radius vector of the point gravity: $\rho_{mo} = [0, y_0, 0, 1]^T$

Radius vector of the point 1: $\rho_1^0 = [a_1, -b, 0, 1]^T$

$$\rho_1^A = T_0^A.\rho_1^0 \rightarrow \begin{pmatrix} a_1 + b\psi_0 \\ y_0 + a_1\psi_0 - b \\ 0 \\ 1 \end{pmatrix}$$

Radius vector of the point 2: $\rho_2^0 = [a_2, b, 0, 1]^T$

$$\rho_2^A = T_0^A . \rho_2^0 \rightarrow \begin{pmatrix} \mathrm{a}_1 - \mathrm{b}\psi_0 \\ \mathrm{y}_0 + \mathrm{a}_1\psi_0 + \mathrm{b} \\ 0 \\ 1 \end{pmatrix}$$

Radius vector of the point 3: $\rho_3^0 = [-a_2, -b, 0, 1]^T$

$$\rho_3^A = T_0^A . \rho_3^0 \rightarrow \begin{pmatrix} \mathrm{b}\psi_0 - a_2 \\ \mathrm{y}_0 - \mathrm{a}_2\psi_0 - \mathrm{b} \\ 0 \\ 1 \end{pmatrix}$$

Radius vector of the point 4: $\rho_4^0 = [-a_2, b, 0, 1]^T$

$$\rho_4^A = T_0^A . \rho_4^0 \rightarrow \begin{pmatrix} -a_2 - \mathrm{b}\psi_0 \\ \mathrm{y}_0 - \mathrm{a}_2\psi_0 + \mathrm{b} \\ 0 \\ 1 \end{pmatrix}$$

The components of the angular velocity of the system are set in advance:

$$\omega_{OX}^A = 0$$
$$\omega_{OY}^A = 0$$
$$\omega_{OZ}^A = \dot{\psi}$$

Kinetic energie of the systems is $T = \frac{1}{2} m_0 \dot{y}_0^2 + \frac{1}{2} J_{oz} \dot{\psi}_0^2$

Potential energie of the systems is

$$\varPi = \frac{1}{2} c_{y11} (y_0 + a_1\psi_0)^2 + c_{y12} (y_0 + a_1\psi_0)^2 + c_{y21} (y_0 - a_2\psi_0)^2 + c_{y22} (y_0 - a_2\psi_0)^2$$

Appling Lagrange's equation of 2nd kind

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right) - \left(\frac{\partial T}{\partial q}\right) = -\left(\frac{\partial \varPi}{\partial q}\right) - \left(\frac{\partial R}{\partial \dot{q}}\right)$$

The differentials equations of the system are

$$m_0 \ddot{y}_o + (c_{y11} + c_{y12} + c_{y21} + c_{y22}) y_o + ((c_{y11} + c_{y12}) a_1 - (c_{y21} + c_{y22}) a_2) \psi_0 = F_y$$
$$J_{oz} \ddot{\psi}_o + ((c_{y11} + c_{y12}) a_1^2 + (c_{y21} + c_{y22}) a_2^2) \psi_o + ((c_{y11} + c_{y12}) a_1 - (c_{y21} + c_{y22}) a_2) y_0 = M_\psi$$

For the differentials equations describing the system from figure 2 are valid:

$$[M]\ddot{q} + [B]\dot{q} + [C]q = [F]$$, където

[M] is the matrix of inertia witch is symmetrical to the main diagonal, with dimension 2x2 and has the following form:

$$[M] = \begin{pmatrix} m_0 & 0 \\ 0 & J_{oz} \end{pmatrix}$$

[C] is the matrix of elasticity which is also symmetric to the main diagonal and has dimension 2x2:

$$[C] = \begin{pmatrix} (c_{y11} + c_{y12} + c_{y21} + c_{y22}) & ((c_{y11} + c_{y12}) a_1 - (c_{y21} + c_{y22}) a_2) \\ ((c_{y11} + c_{y12}) a_1 - (c_{y21} + c_{y22}) a_2) & ((c_{y11} + c_{y12}) a_1^2 + (c_{y21} + c_{y22}) a_2^2) \end{pmatrix}$$

[B] is the matrix of dissipative forces, showing the influence of damper. Also symmetric with dimension 2x2, in our case [B] = 0.

The generalized coordinates and their derivatives are:

$$\{q\} = \begin{bmatrix} y_0 \\ \psi_0 \end{bmatrix}; \{\dot{q}\} = \begin{bmatrix} \dot{y}_0 \\ \dot{\psi}_0 \end{bmatrix}; \{q\} = \begin{bmatrix} \ddot{y}_0 \\ \ddot{\psi}_0 \end{bmatrix};$$

To obtain natural frequencies of the system, the equations are represented in Cauchys' normal form: y+Ly = 0, where L has the following form:

$$L = \begin{bmatrix} M^{-1}B & M^{-1}C \\ I & 0 \end{bmatrix}$$

The output parameters of the system – displacement, velocity and acceleration are obtained from the equations: y+Ly = У, where У is:

$$Y = \begin{bmatrix} M^{-1} & F(t) \\ & 0 \end{bmatrix}$$

All solutions to the definite time interval are obtained after integration of the system using the method of Runge - Kutta.

Plane model examine the slip angle

Figure 3 shows the schema of the model exanimating the car stability with reporting the wheel slip angel. The model is known as Rocards model.

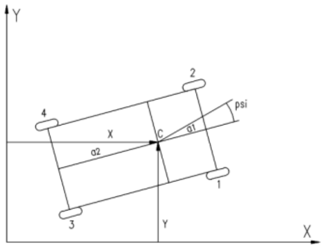

Fig. 3 *Plane model describing only the slip angle*

The initial conditions were assumed for modeling:

- The point 1, 2, 3 and 4 to the fig.3 are the projection of the wheels;
- Considering that the wheels are stiffs and can ignore their lateral elasticity around to the axis O_y;
- The car is shown at the angle ψ to the axis O_x.

From fig. 4 the slip angles are δ_1, δ_2, δ_3, δ_4 can be determinate by coordinate y, so we have:

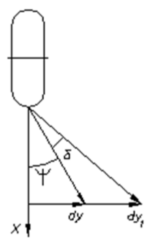

Fig. 4 *Slip angle*

$$tg(\psi) = \frac{dy}{dx} \rightarrow \psi = \frac{dy}{dx} \rightarrow dy = \psi . dx$$

$$tg(\psi + \delta) = \frac{dy_1}{dx} \rightarrow \delta = \frac{dy_1}{dx} - \psi \rightarrow \delta = \frac{dy_1 - dy}{dx} = \frac{dy_1 - \psi . dx}{v . dt}$$

If the vehicle moved with a pure rolling motion (the slip angle for the wheels were all zero) then the displacements of the point of contact of the wheels with the ground would be the same as the displacement of the center of gravity C in any interval δt: $dx = dx_1 = dx_2 = dx_3 = dx_4 = V.dt$, where V is the speed of the vehicle in direction O_x.

The equations of motion are:

$$m\ddot{y} + k_1(\delta_1 + \delta_2) + k_2(\delta_3 + \delta_4) = F_y$$
$$J_{OZ}\ddot{\psi} + k_1 a_1(\delta_1 + \delta_2) - k_2 a_2(\delta_3 + \delta_4) = M_\psi$$

Substituting for δ_1, δ_2, δ_3, δ_4 we obtain:

$$m\ddot{y} + \frac{2}{v}(k_1 + k_2)\dot{y} + \frac{2}{v}(k_1 a_1 - k_2 a_2)\dot{\psi} - 2(k_1 + k_2)\psi = F_y$$

$$J_{OZ}\ddot{\psi} + \frac{2}{v}(k_1 a_1^2 + k_2 a_2^2)\dot{\psi} - 2(k_1 a_1 - k_2 a_2)\psi + \frac{2}{v}(k_1 a_1 - k_2 a_2)\dot{y} = M_\psi$$

Taking F_y, M_ψ to be zero, $I = m.\rho^2$ and after substituting these relationships to the equations of motion. To solve the equations assume $y = Ae^{pt}$ and $\psi = Be^{pt}$ and substituting these value to the motions equation. To have a solution the equations system is necessary the matrix from the coefficient in front of the A and B to be equal to zero. So, we have:

$$\begin{vmatrix} p^2 + 2\dfrac{(k_1 + k_2)}{mv} p & 2\dfrac{(k_1 a_1 - k_2 a_2)}{mv} p - 2\dfrac{(k_1 + k_2)}{mv} \\ 2\dfrac{(k_1 a_1 - k_2 a_2)}{mv} p & p^2 + 2\dfrac{(k_1 a_1^2 + k_2 a_2^2)}{m\rho^2 v} p - \dfrac{(k_1 a_1 - k_2 a_2)}{m\rho^2} \end{vmatrix} = 0$$

After matrix development is obtained:

$$p^4 + \frac{2}{mv}\left[k_1\left(1 + \frac{a_1^2}{\rho^2}\right) + k_2\left(1 + \frac{a_2^2}{\rho^2}\right)\right]p^3 + \left[\frac{4\left[(k_1 + k_2)(k_1 a_1^2 + k_2 a_2^2) - (k_1 a_1 - k_2 a_2)^2\right]}{m^2 \rho^2 v^2} - \frac{2(k_1 a_1 - k_2 a_2)}{m\rho^2}\right]p^2 = 0$$

This may be divides by p^2:

$$p^2 + \frac{2}{mv}\left[k_1\left(1 + \frac{a_1^2}{\rho^2}\right) + k_2\left(1 + \frac{a_2^2}{\rho^2}\right)\right]p + \left[-\frac{2(k_1 a_1 - k_2 a_2)}{m\rho^2} + \frac{4k_1 k_2 (a_1 + a_2)^2}{m^2 \rho^2 v^2}\right] = 0$$

The analysis of stability is made with the criteria of Raus-Kurvits by creating a square matrix from the characteristic equation $A(s) = a_0 s^n + a_1 s^{n-1} + \ldots + a_{n-1}s + a_n = 0$.

For being systems stability is necessary $a_0 > 0$ and $\Delta_1, \Delta_2, \ldots, \Delta_n$ to be always positive. Where $\Delta_1, \Delta_2, \ldots, \Delta_n$ are:

$$\Delta_1 = \|a_1\|, \Delta_2 = \begin{Vmatrix} a_1 & a_3 \\ a_0 & a_2 \end{Vmatrix}, \Delta_3 = \begin{Vmatrix} a_1 & a_3 & a_5 \\ a_0 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{Vmatrix}, \ldots, \Delta_n = \|\Gamma\|$$

In our case $a_0 > 0$, so to have a cars stability motion is necessary the moment of intertie of front wheels to be smaller than that of rear wheels.

For the vehicles speed is obtained:

$$v^2 \geq \frac{\left[\dfrac{4k_1 k_2 (a_1 + a_2)^2}{m^2 \rho_2}\right]}{\dfrac{2(k_1 a_1 - k_2 a_2)}{m\rho^2}} = \frac{2k_1 k_2 (a_1 + a_2)^2}{m[k_1 a_1 - k_2 a_2]}$$

Plane model examine lateral elasticity and slip angle

The initial conditions for this case of modeling were the same as these from fig.3 and this time the lateral elasticity of the wheel is added (fig.5).

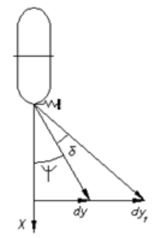

Fig. 5 *Plane model about lateral elasticity and slip angle*

Appling the models initial condition for the differentials equations describing the system are valid:

$$m\ddot{y} + k_1(\delta_1 + \delta_2) + k_2(\delta_3 + \delta_4) + 2(c_1 + c_2)y + 2(c_1 a - bc_2)\psi = F_y$$

$$J_{OZ}\ddot{\psi} + k_1 a_1(\delta_1 + \delta_2) - k_2 a_2(\delta_3 + \delta_4) + 2(c_1 a^2 + c_2 b^2)\psi + 2(c_1 a - bc_2)y = M_\psi$$

Substituting for δ_1, δ_2, δ_3, δ_4 we obtain:

$$m\ddot{y} + \frac{2}{v}(k_1 + k_2)\dot{y} + \frac{2}{v}(k_1 a_1 - k_2 a_2)\dot{\psi} - 2(k_1 + k_2)\psi + 2(c_1 + c_2)y + 2(c_1 a - bc_2)\psi = F_y$$

$$J_{OZ}\ddot{\psi} + \frac{2}{v}(k_1 a_1^2 + k_2 a_2^2)\dot{\psi} - 2(k_1 a_1 - k_2 a_2)\psi + \frac{2}{v}(k_1 a_1 - k_2 a_2)\dot{y} + 2(c_1 a^2 + c_2 b^2)\psi + 2(c_1 a - bc_2)y = M_\psi$$

2. *Computer simulation*

All vehicles models permit by variable of the input information to explore the influence of some construction parameters into the directional stability.

Vehicle characteristics and parameters' numerical values were taken directly from the literature.

To solve the equations was made a program with MATLAB software. It simulates the behavior of car with different speed, with different position of center of gravity and variable characteristic of wheels. The investigations were made in two parts – without and with disturbing force. This force works only for a few seconds.

m	Vehicle mass	1 500	kg
a	Distance from CG to front axel	1	m
b	Distance from CG to rear axel	1,5	m
I_{z0}	Moment of inertia to axe Oz	2 500	Kgm^2
c_y	Lateral stiffness of each tire to axe Oy	57 000	N/m

3. *Results*

Simulation results of ***the first model*** - displacements around the axes O_y and the angular displacement around the axes O_z when the center of gravity changes the position and for different value of tires lateral elasticity are given in table 1.

Simulations results of ***the second model*** shown in fig. 6…9 demonstrate the displacements are around the axes O_y and the angular displacement around the axes O_z as function of coordinate of center of gravity, the steering force characteristics for the front k_1 and rear tires k_2 and vehicles speed.

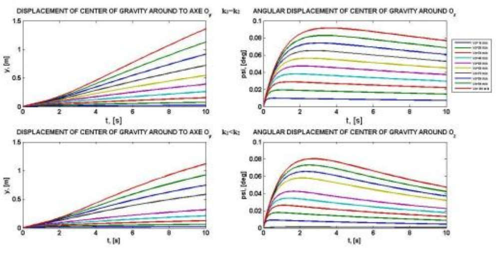

fig. 6 *Displacements are around the axes O_y and the Angular displacement around the axes O_z Distance from CG to front axel $a_1 = 1.2$, $V_{cr} = 0$ km / h,*

c_y, kN/m Front/rear wheel		Distance from CG to front axel a_1, m				
		1	1,2	1,25	1,3	1,5
50/50	Displacement around Oy, m	0,008714	0,008624	0,008658	0,008632	0,008694
	Angular displacement around O_z, grad	0,3924	0,3116	0,008931	0,2995	0,3895
60/60	Displacement around Oy, m	0,00796	0,007885	0,007903	0,007869	0,007943
	Angular displacement around O_z, grad	0,3586	0,2845	0,008154	0,2734	0,3556
70/70	Displacement around Oy, m	0,007391	0,007312	0,007311	0,007286	0,007354
	Angular displacement around O_z, grad	0,3321	0,2634	0,007553	0,2531	0,3288
80/80	Displacement around Oy, m	0,006928	0,006847	0,006843	0,006841	0,006898
	Angular displacement around O_z, grad	0,3105	0,2463	0,007066	0,2363	0,3076
90/90	Displacement around Oy, m	0,006534	0,006458	0,006454	0,006452	0,006487
	Angular displacement around O_z, grad	0,2921	0,2323	0,006661	0,2232	0,2902
50/60	Displacement around Oy, m	0,00831	0,008249	0,008232	0,008243	0,008256
	Angular displacement around O_z, grad	0,37	0,3661	0,3519	0,3111	0,3537
60/70	Displacement around Oy, m	0,007809	0,007592	0,007594	0,007583	0,007582
	Angular displacement around O_z, grad	0,3459	0,3331	0,3181	0,2553	0,3329
70/80	Displacement around Oy, m	0,007264	0,007076	0,007071	0,007067	0,00706
	Angular displacement around O_z, grad	0,3221	0,3101	0,2896	0,202	0,3084
80/90	Displacement around Oy, m	0,006807	0,06661	0,006646	0,006642	0,006637
	Angular displacement around O_z, grad	0,3009	0,2881	0,2655	0,1515	0,2936
90/100	Displacement around Oy, m	0,006364	0,006292	0,006287	0,006284	0,006312
	Angular displacement around O_z, grad	0,2859	0,27	0,2443	0,1057	0,2784
60/50	Displacement around Oy, m	0,008287	0,008219	0,008218	0,008231	0,008253
	Angular displacement around O_z, grad	0,3611	0,3021	0,3479	0,3599	0,3657
70/60	Displacement around Oy, m	0,007619	0,007577	0,007578	0,007573	0,007756
	Angular displacement around O_z, grad	0,3365	0,2442	0,3127	0,3307	0,345
80/70	Displacement around Oy, m	0,00708	0,007063	0,007061	0,00706	0,007225
	Angular displacement around O_z, grad	0,3154	0,19	0,2832	0,3052	0,322
90/80	Displacement around Oy, m	0,006658	0,006639	0,006637	0,006637	0,006774
	Angular displacement around O_z, grad	0,2976	0,1393	0,2576	0,2851	0,3008
100/90	Displacement around Oy, m	0,006332	0,006282	0,00628	0,00628	0,006332
	Angular displacement around O_z, grad	0,2815	0,09346	0,2376	0,2675	0,2844

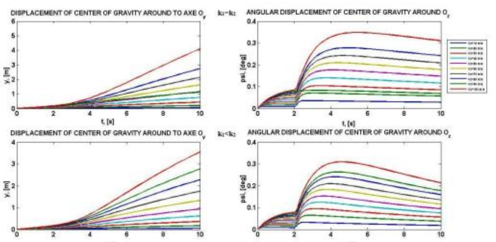

fig. 7 *Displacements are around the axes O_y and the Angular displacement around the axes O_z Distance from CG to front axel a_1 = 1.2, V_{cr} = 0 km / h, with disturbing force*

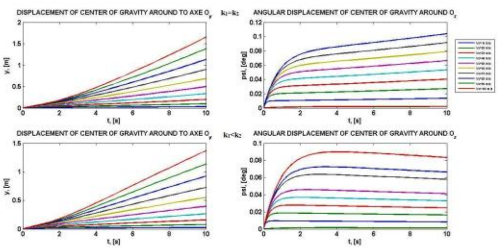

fig. 8 *Displacements are around the axes O_y and the Angular displacement around the axes O_z Distance from CG to front axel a_1 = 1.3, V_{cr} = 294 km / h*

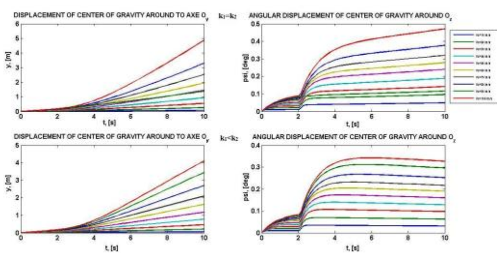

fig. 9 *Displacements are around the axes O_y and the Angular displacement around the axes O_z Distance from CG to front axel a_1 = 1.3, V_{cr} = 294 km / h, with disturbing force*

Simulations results of the ***third model*** shown in fig. 10…17 demonstrate the displacements are around the axes O_y and the angular displacement around the axes O_z as function of coordinate of center of gravity, the steering force characteristics for the front k_1 and rear tires k_2 and vehicles speed.

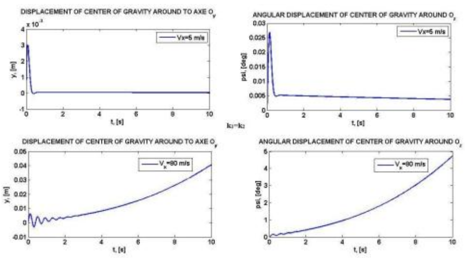

fig. 10 *Displacements are around the axes O_y and the Angular displacement around the axes O_z Distance from CG to front axel a_1 = 1.2, V_{cr} = 0 km / h*

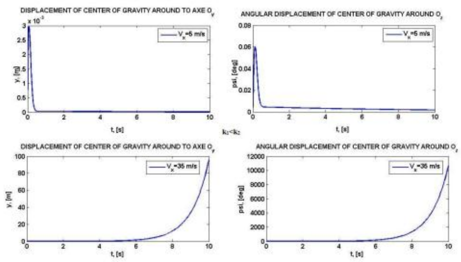

fig. 11 *Displacements are around the axes O_y and the Angular displacement around the axes O_z Distance from CG to front axel a_1 = 1.2, V_{cr} = 0 km / h*

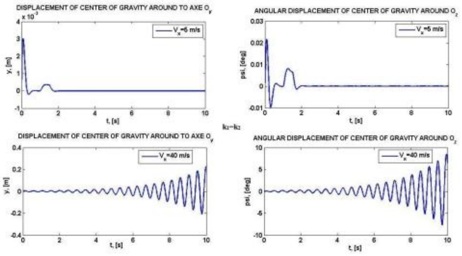

fig. 12 Displacements are around the axes O_y and the Angular displacement around the axes O_z Distance from CG to front axel a_1 = 1.2, V_{cr} = 0 km / ***h,*** *with disturbing force*

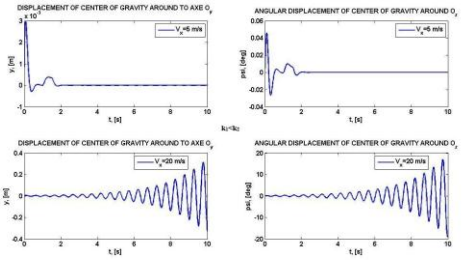

fig. 13 *Displacements are around the axes O_y and the Angular displacement around the axes O_z Distance from CG to front axel a_1 = 1.2, V_{cr} = 0 km / h, with disturbing force*

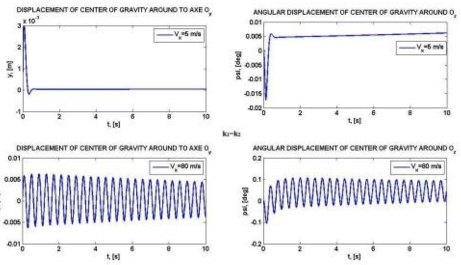

fig. 14 *Displacements are around the axes O_y and the Angular displacement around the axes O_z Distance from CG to front axel a_1 = 1.3, V_{cr} = 294 km / h*

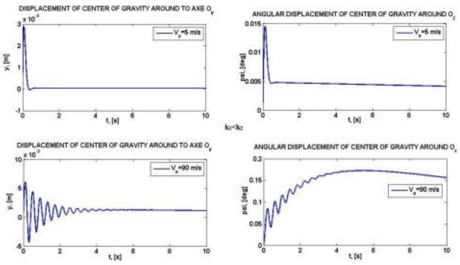

fig. 15 *Displacements are around the axes O_y and the Angular displacement around the axes O_z Distance from CG to front axel a_1 = 1.3, V_{cr} = 294 km / h*

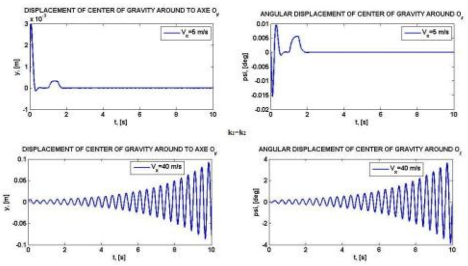

fig. 16 *Displacements are around the axes O_y and the Angular displacement around the axes O_z Distance from CG to front axel a_1 = 1.3, V_{cr} = 294 km / h, with disturbing force*

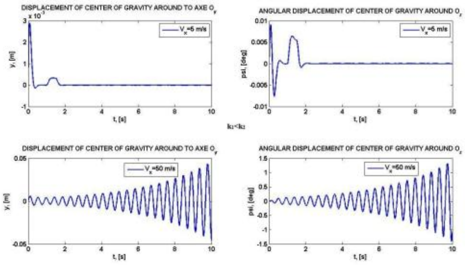

fig. 17 *Displacements are around the axes O_y and the Angular displacement around the axes O_z Distance from CG to front axel a_1 = 1.3, V_{cr} = 294 km / h, with disturbing force*

4. Conclusions

In this work was proposed vehicle dynamic models for research the car directional stability. The analysis shows the influence of some construction parameters into the directional stability as coordinates of center of gravity, cars speed, the steering force characteristic.

The model focused on the tires lateral elasticity shows that the system is unstable when the center of gravity changes his position.

The second model based on the tires slip angle shows that the cars directional stability concern lower diapason of speed.

The investigation of cars directional stability with the third model is most exact where the system is examined under the interaction between the elastics, inertials and slip angles force.

5. Acknowledgement

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