Identification of Moving Lightning Clouds Using Four-dimensional Electromagnetic Potential

Atanas G. Chervenkov, Member IEEE

*Technical University of Sofia, Department of Theoretical Electrical Engineering, 1000 Sofia, Bulgaria, e-mail: acher@tu-sofia.bg

***Abstract*—The case of identification of a moving lightning cloud is being considered. An application of the four-dimensional magnetic potential is given. Relativistic computational method and models are used for determination of the electromagnetic field. A four-dimensional magnetic potential in cases of arbitrary velocity of movement can be used. The cases of the electromagnetic field distribution in moving environments with arbitrary speeds are being investigated. The electromagnetic field vectors as elements of Maxwell's tensor are defined. The elements of four-dimensional magnetic potential are magnetic vector potential and scalar electric potential. Analytical expressions for the electromagnetic field vectors using the four-dimensional potentials have been obtained. The components of the electric field and magnetic field, created by motion from moving charge, presenting lightning cloud, have been found. By solution of inverse problem, the quality of charge, its coordinates and velocity have been determined.**

***Keywords*— identification, dual tensor, electromagnetic field, four-dimensional potential, lightning clouds, Minkovski space**

I. Introduction

There are about 45 lightning flashes per second around the world. Some of them disrupt the operation of overhead power lines [1]. Together with generation of the characteristic blue-white light, radio wave pulses are also produced. The frequent crackles are heard when radio station works during a thunderstorm due to the lightning discharges. Interference also occurs in sensitive electronic equipment used for navigation. For prevention in case of lightning it is necessary to identify the sources of these lightning - the electric charges accumulated in the clouds. The case of identification of a charged moving cloud, which can de make lightning is being considered. An application of the four-dimensional magnetic potential is displayed. Relativistic computational approach and electromagnetic model for determination of the electromagnetic field are used. The results for the electric field intensity, obtained using the four-dimensional potential, are used to identify the moving exciter by solving of the inverse problem. As results of solution of inverse problem, the quality of charge, its coordinate and velocity have been determined.

II. Describing of Proposed Method

Most similar phenomena can be considered and described on the basis of the theory of Einstein special relativity at high and low speeds of movement. The special theory of relativity covers the physical processes occurring in a medium of constant speed. In this theory, it is assumed that the coordinates of the three-dimensional space x, y, z, and time t form a four-dimensional vector.

Charge coordinates can be taken as the center of gravity coordinates of a concentrated exciter - lighting cloud with the total value of the electric charge q, located at the center $0'$ of the moving coordinate system. $x'y'z'$- Fig.1. It moves in a plane parallel to YOZ (where a stationary observer is located), at a speed *v=const*. The method of mirror images is used. The upper space (the atmosphere) has electromagnetic characteristics ε_1, μ_1 and down space (the earth) -ε_2, μ_2, respectively. A case in which $\varepsilon_1 = \varepsilon_0,\ \mu_1 = \mu_0$, and $\varepsilon_2 = \varepsilon_0 \varepsilon_r,\ \mu_2 = \mu_0$, (where $\varepsilon_0 = 8{,}8510^{-12}$ and $\mu_0 = 4\pi 10^{-7}$ are characteristics of the free space) is considered.

An unknown are:

- The cloud charge q (an exciter), which can be reduced to a point charge with the appropriate approximation;
- The height $h = x$, above the plane YOZ and the coordinates y and z of the point, at which the exciter is considered;
- The velocity of movement v.

This allows the application of basic dependencies in the Minkowski space [2], which are valid for a homogeneous medium. A four-dimensional magnetic potential $\overrightarrow{\Psi_\mu}$ is introduced. The elements of four-dimensional magnetic potential are magnetic vector potential $\overrightarrow{A_\mu}$ and scalar electric potential V_ε [3, 4, and 5].

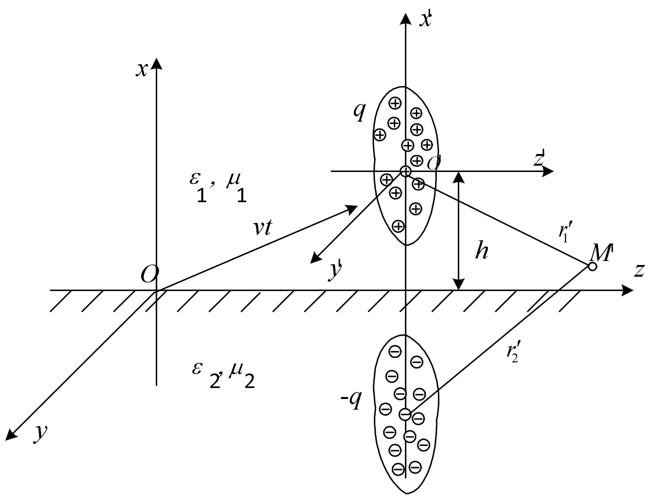

Fig. 1. Model of lightning cloud identification

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III. Determination of Electromagnetic Field

A. Four-Dimensional Magnetic Potential

The electromagnetic field vectors as elements of dual Maxwell's tensor are defined.

The components of the four-dimensional magnetic potential $\overrightarrow{\Psi'_\mu}$ in moving coordinate system *X'Y'Z'* are finding first [6].

$$\{\overrightarrow{\Psi'_\mu}\} = \begin{Bmatrix} \Psi'_{\mu_1} = A'_{\mu x'} = 0 \\ \Psi'_{\mu_2} = A'_{\mu y'} = 0 \\ \Psi'_{\mu_3} = A'_{\mu z'} = 0 \\ \Psi'_{\mu_4} = \frac{j}{c} V'_{\varepsilon} = \frac{j}{c}\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r'_1} + \frac{\varepsilon_0-\varepsilon_2}{\varepsilon_0+\varepsilon_2}\frac{1}{r'_2}\right) \end{Bmatrix} \quad (1)$$

where: $r'_1 = \sqrt{(x'_1 - x')^2 + (y'_1 - y')^2 + (z'_1 - z')^2}$;

$r'_2 = \sqrt{(x'_2 + x')^2 + (y'_2 - y')^2 + (z'_2 - z')^2}$.

Lorentz transformations on coordinates and electromagnetic potentials are used [2, 6]. For the components of the four-dimensional magnetic potential $\overrightarrow{\Psi_\mu}$ in stationary coordinate system *XYZ'* (for the stationary observer) is obtained

$$\{\overrightarrow{\Psi_\mu}\} = \begin{Bmatrix} \Psi_{\mu_1} = A_{\mu x} = 0 \\ \Psi_{\mu_2} = A_{\mu y} = 0 \\ \Psi_{\mu_3} = A_{\mu z} = \alpha\mu_0 v\frac{q}{4\pi}\left(\frac{1}{r_1} + \frac{\varepsilon_0-\varepsilon_2}{\varepsilon_0+\varepsilon_2}\frac{1}{r_2}\right) \\ \Psi_{\mu_4} = \frac{j}{c} V_{\varepsilon} = \frac{j}{c}\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} + \frac{\varepsilon_0-\varepsilon_2}{\varepsilon_0+\varepsilon_2}\frac{1}{r_2}\right) \end{Bmatrix} \quad (2)$$

where: $r_1 = \sqrt{[x_1 - (x - v_x t)]^2 + [y_1 - (y - v_y t)]^2 + [z_1 - (z - v_z t)]^2}$;

$r_2 = \sqrt{[x_2 - (x - v_x t)]^2 + [y_2 - (y - v_y t)]^2 + [z_2 - (z - v_z t)]^2}$.

At relatively low speeds, the relativity factor α acquires small values, such as at rest $\alpha = 1$.

A magnetic vector-potential $\vec{A}_\mu$ exists only in a stationary coordinate system. The moving cloud with charge *q* creates a magnetic field, that contains only one component of the magnetic vector-potential $A_{\mu z}$, directed along the z axis(in the direction of movement). The magnetic field induces an additionally electric field in the stationary observer, respectively.

B. An intensity of the electric field

The components of electric intensity vector $\vec{E}$ as elements of Maxwell tensor $\vec{F}_{jk}^{(\mu)}$ are determined, using the tensor dependences (4) [6].

The scalar potential V_ε is 0 rank tensors (0-tensor). The vector potential $\vec{A}_\mu$ is first-rank tensors (1-tensor). The Maxwell tensor $F_{jk}^{(\mu)}$ is second rank tensors (2 tensor), which has 16 components where only six of them are independent, because it is an anti-symmetric tensor.

$$\left\{F_{jk}^{(\mu)}\right\} = \begin{Bmatrix} F_{11} = 0 & F_{12} = B_z & F_{13} = -B_y & F_{14} = -\frac{j}{c}E_x \\ F_{21} = -B_z & F_{22} = 0 & F_{23} = B_x & F_{24} = -\frac{j}{c}E_y \\ F_{31} = B_y & F_{32} = -B_x & F_{33} = 0 & F_{34} = -\frac{j}{c}E_z \\ F_{41} = \frac{j}{c}E_x & F_{42} = \frac{j}{c}E_y & F_{43} = \frac{j}{c}E_z & F_{14} = 0 \end{Bmatrix} \quad (3)$$

$$F_{jk} = \frac{\partial\Psi_k}{\partial x_j} - \frac{\partial\Psi_j}{\partial x_k} \quad (4)$$

For the electric field intensity components is obtained:

$$\vec{E} = \begin{Bmatrix} E_x = -\frac{q}{4\pi\varepsilon_0}\left[\frac{x_1-(x-v_x t)}{r_1^3} + \frac{\varepsilon_0-\varepsilon_2}{\varepsilon_0+\varepsilon_2}\frac{x_1+(x-v_x t)}{r_2^3}\right] \\ E_y = -\frac{q}{4\pi\varepsilon_0}\left[\frac{y_1-(y-v_y t)}{r_1^3} + \frac{\varepsilon_0-\varepsilon_2}{\varepsilon_0+\varepsilon_2}\frac{y_1+(y-v_y t)}{r_2^3}\right] \\ E_z = -\frac{q}{4\pi\varepsilon_0}\left[\frac{z_1-(z-v_z t)}{r_1^3} + \frac{\varepsilon_0-\varepsilon_2}{\varepsilon_0+\varepsilon_2}\frac{z_1+(z-v_z t)}{r_2^3}\right] \end{Bmatrix} \quad (5)$$

where: $r_1 = [x_1 - (x - v_x t)]^2 + [y_1 - (y - v_y t)]^2 + [z_1 - (z - v_z t)]^2$;

$r_2 = [x_2 - (x - v_x t)]^2 + [y_2 - (y - v_y t)]^2 + [z_2 - (z - v_z t)]^2$

When the exciter moves only in one direction - for example on the z axis, there are not both components of the velocity - v_x, v_y. In this case, for the electric field intensity components are obtained:

$$\vec{E} = \begin{Bmatrix} E_x = -\frac{q}{4\pi\varepsilon_0}\left[\frac{x_1-x}{R_1^{3/2}} + \frac{\varepsilon_0-\varepsilon_2}{\varepsilon_0+\varepsilon_2}\frac{x_1+x}{R_2^{3/2}}\right] \\ E_y = -\frac{q}{4\pi\varepsilon_0}\left[\frac{y_1-y}{R_1^{3/2}} + \frac{\varepsilon_0-\varepsilon_2}{\varepsilon_0+\varepsilon_2}\frac{y_1+y}{R_2^{3/2}}\right] \\ E_z = -\frac{q}{4\pi\varepsilon_0}\left[\frac{z_1-(z-v_z t)}{R_1^{3/2}} + \frac{\varepsilon_0-\varepsilon_2}{\varepsilon_0+\varepsilon_2}\frac{z_1+(z-v_z t)}{R_2^{3/2}}\right] \end{Bmatrix} \quad (6)$$

where: $R_1 = [x_1 - x]^2 + [y_1 - y]^2 + [z_1 - (z - v_z t)]^2$;

$R_2 = [x_2 - x]^2 + [y_2 - y]^2 + [z_2 - (z - v_z t)]^2$

The results for the electric field intensity, obtained through the described approach, are used to identify the moving exciter by solving of the inverse problem.

IV. Solution of the Inverse Problem

The number of unknown values is determined first. In the case of movement in an arbitrary direction, unknown values are: the charge *q*; the coordinate x_1; the coordinate y_1; the coordinate z_1; velocity of movement along the *x*-axis - v_x; velocity of movement along the *y* -axis v_y; velocity of movement along the *z* -axis v_z. It is possible to include the average dielectric constant of the lower half-space ε_2 as an unknown value, but this would further complicate the solution of the problem. When the charge is moving only in one direction along z-axis, the unknown values are reduced: the charge q; the coordinate x_1; the coordinate y_1; the coordinate z_1; velocity of movement along the *z* -axis v_z.

The m numbers of measurements of the electric field intensity at different points in the space for a fixed moment of time t are made. Another variant is measurements to be made in limited points, but for different times$(t_1, t_2, t_3, \dots . t_m)$. This option reduces the number of measuring elements. The components of electrical field intensity $\vec{E}$, which has the highest value and these values are have critical with respect to the occurrence of lightning, are observed. The measured values are replaced in the equations of system (6), which correspond to the observed intensity component. These values of the unknowns q, x_1, y_1, z_1, v_z, are searched, for which equality is satisfied. A system of equations is compiled, which rank is equal to the number p of the unknowns. The measured values of electrical field intensity E in the left side of these equations are replaced. If unknown values are reduced to 3: charge q; the height above the ground h (the coordinate x_1) and the coordinate z_1, the problem is simplified and finding the solution makes real practical sense. In this case the solution of the inverse problem for identification of moving charge leads to solution of system (6), whereby the measured values of the electric field intensity - E_x, E_y, E_z in the left side are replace.

V. NUMERICAL RESULTS

The solution of the inverse problem is searched with next input data: observer coordinates (in point, where measurements are made) $x= 0, y = 0, z = 0$; cloud (charge) velocity $v=30m/s$; time $t=1s$; average permittivity of the lower half space (the atmosphere) $\varepsilon = \varepsilon_0$; average permittivity of the lower half space (ground)) $\varepsilon_2 = \varepsilon_0\varepsilon_r, \varepsilon_r = 3$. The solution to the inverse problem is made numerically using the software package Mathematica.

The described inverse problem is incorrect. The solution does not respond of the necessary and sufficient conditions for correctness i.e. to exist a solution, to be unique and to be sustainable. However, the condition of uniqueness cannot be fulfilled, because the equations (6) are nonlinear. For the considered case, there is a unique solution to the inverse problem exists. Due to the existence of non-unique, ambiguous solutions the regularization procedure is used. It is based on a optimization criterion

$$\min_s \sum_{k=1}^{p} \left(E_{js}^{(k)} - \hat{E}_{js}^{(k)}\right)^2 ; \; j1,2,3 ; \; s = 1,2,3, \dots, n \qquad (7)$$

where: $\{E_j\} = \{E_1, E_2, E_3\} = \{E_x, E_y, E_z\}$, p is the number of unknown quantities and the rank of the equation system, respectively, $E_{js}^{(k)}$ are calculated values, a $\hat{E}_{js}^{(k)}$ are measured values. Due to the existence of ambiguous solutions, parallel calculations and observations are made, which number is s. This solution, which satisfies the selected optimization criterion (3) is selected. This reduces the degree of freedom for the manifestation of incorrectness.

The results of the inverse problem solution are:

- charge value q=26.429 C;
- height above the ground h=7480m (coordinate x_1);
- distance to the lightning cloud (coordinate z_1) 2910m.

Always are exist conditions for misidentification, with insufficient number of points in the space or the time. About identification of a moving electric charge, connected with lightning clouds, the measuring problems of electrical field intensity there are no particular difficulties. Moving charged cloud excite a magnetic field, magnetic field induces an additionally electric field in the stationary observer. Of course, that this magnetic field can be registered and measured, using sensitive measuring equipment. This approach can be used in case of a motion with constant velocity of cloud.

The obtained solution makes sense and it is actually applicable, when the cloud charge creates a critical intensity at which a lightning discharge can occur. To choose a unique solution, multiple measurements of the electric field intensity must be made at several points at the same time. The solution must be sought at regular intervals, performing iterations until a procedure for convergence is obtained.

CONCLUSION

Using of a relativistic approach for the four-dimensional magnetic potential and its Lorentz transformation demonstrates, that moving electric cloud can excite a magnetic field. Analytical expressions for the electromagnetic field vectors using the four-dimensional potentials have been obtained. The components of the electric field and magnetic field, created by motion from moving charge, presenting lightning cloud, have been obtained. By the solution of inverse problem the quantity of equivalent charge of lightning cloud, its coordinates, velocity of movement and others parameters, are looked for. Identification can be useful for predicting the behavior of lightning clouds and preventing lightning strikes on sensitive electrical equipment. This approach can be used for an analysis and synthesis of the electromagnetic field in cases of movement of charged objects with high velocity.

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