Influence of Magnet Dimensions on Torque Components and Cost of Synchronous Machine with Interior Magnets

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***Abstract*—The paper presents a study on interior permanent magnet (PM) synchronous machine (SM). The influence of PM dimensions on the electromagnetic torque and its components (torque from PM and reluctance torque) is investigated. The SM torque is calculated using an approach proposed by the authors based on FEM analysis and analytical formulae. The results show influence of the magnet width and magnet height on the torque of the machine. It is proven that the torque of the machine is affected much more by the width of the magnets than by their height. As a result from the study it is concluded that there are possibilities for significant reduction of the volume of magnets without sacrificing the machine torque. In the paper, it is also discussed the possible financial savings due to reduction of PM mass and price.**

Keywords—interior magnets synchronous machine, torque, reluctance torque, magnet price

I. Introduction

The synchronous machines with permanent magnets (PM) became very popular in recent years because of their excellent characteristics – high torque, low volume and mass, possibility of operation at high speeds. This success is due to the development of materials for permanent magnets with very good parameters such as NdFeB and SmCo. One of the main application area of high-performance PM synchronous machines (PMSM) is in electric and hybrid vehicles (EV and HEV) [1], [2]. It should also be noted the growing use of permanent magnet synchronous motors in servo drives for industrial automation, robots etc. The determination of parameters and characteristics using analytical and software methods is an important task, which gives possibility for optimization of the machine prior to real prototype production [3], [4]. One important aspect of machine optimization is to find construction with minimum magnet volume and mass still fulfilling the requirements for torque and power [5]. Thus, the machine price may be minimized because of relatively high prices of PM materials nowadays.

The main goal of this paper is to present a study on the influence of magnet dimensions on torque components and cost of machine with interior magnets. The study is based on multivariate calculations and involves 2D finite element analysis and determination of machine parameters with analytical formulae. After that, the electromagnetic torque is calculated with its two components – active torque (due to PM) and reluctance torque [6]. All these calculations are done for different PM dimensions (at constant machine length) and results for the torque components and total torque are compared.

II. Challenges in the Use of Rare-Earth PM

Much progress in the development of permanent magnets and electric machines with PM was made in the 1960s with the development of rare earth permanent magnets. Today, the magnets with the highest energy performance - about 370 kJ/m^3, are neodymium magnets NdFeB. The value of their residual induction is 1.0 ÷ 1.4 T, and their coercive intensity is significantly higher than that of other types of magnets - over 870 kA/m. The sintered NdFeB magnets contain 30 ÷ 32% rare earth metals (neodymium Nd and dysprosium Dy), 1% boron, 0 ÷ 3% cobalt and iron. Additives of small amounts of solid rare earth elements dysprosium, terbium, and cobalt are used in the alloy to increase temperature stability and resistance to demagnetization. Another great advantage is their linear demagnetizing characteristic.

The high energy performance of NdFeB magnets makes them the most widely used in the development and production of highly efficient electric motors. Permanent magnets have a significant influence on the characteristics of electrical machines. Although their weight is a small part of the total weight of the machine, their high price makes the whole product significantly more expensive. Ensuring a reasonable price and good competitiveness requires careful analysis of the cost of the magnets used in the machine construction. The prices are also influenced by the high demand for rare earth elements in recent years and the difficulties in securing their supply.

III. Machine Parameters and Torque Calculation

The machine under study is a three-phase synchronous motor with NdFeB permanent magnets of type N30UH. The construction is with interior magnets, based on a utility model of the company Almott [7]. The data for the machine is: number of poles 12, number of stator slots 72, rated current 180A, maximum current 360A. The study of the machine was performed using the finite element method (FEM) of a 2D model. Calculations of the distribution of the magnetic field in the machine are made under different conditions:

- Determining the magnetic field created by the currents in the phases, when rotor axes d and q are aligned with phase A at two current values 180 and 360A – in order to determine the inductances L_d, L_q of the machine.
- Determination of the distribution of the magnetic field created by the magnets - in order to calculate the flux linkage of the permanent magnets Ψ_m.

In Fig. 1 is shown a cross section of the machine with the distribution of the magnetic field created by the magnets in the

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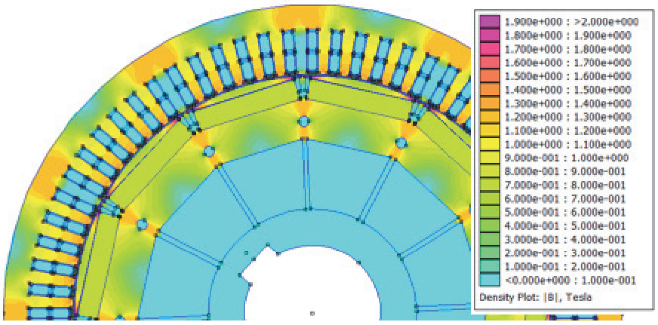

Fig. 1. Distribution of magnetic flux density in a PMSM with base construction – PM width *W* and height *H*.

basic construction of the machine with a width of magnets *W* and a height *H*. On the basis of FEM simulation is determined the magnetic flux density created by the magnets in the air gap. Using FFT the amplitude of the first harmonic of the flux density B_{1m} is determined.

Based on the results of the FEM studies, the inductances L_d, L_q of the machine and the flux linkage of the magnet Ψ_m are calculated according to the method described in [6]. The amplitudes of the first harmonics of the magnetic induction created by the magnet and by the current through the stator phases are used. The formulae for calculating the flux linkage from the PM are:

$$\Phi_m = \frac{2}{\pi} B_{1m} \tau l \qquad \Psi_m = w k_w \Phi_m \tag{1}$$

where Φ_m is the magnetic flux for one pole division, τ is the pole length, l is active machine length, B_{m1} is the magnitude of the first harmonic of the flux density, w is the number of turns of one phase, and k_w is the winding coefficient.

The formulae for calculating the synchronous inductances on both axes are:

$$L_d = \Psi_d / i_d \qquad L_q = \Psi_q / i_q \tag{2}$$

where Ψ_d, Ψ_q are calculated according to (1) using the first harmonic of the induction created by the currents components i_d and i_q through the stator phases, with the orientation of the spatial vector of the current along the axis of phase A and at 90° electrical from the axis of phase A.

Thus, the parameters L_d, L_q and Ψ_m are determined for each of the studied modifications.

Based on the obtained parameters, the characteristics of the electromagnetic torque of the machine are calculated depending on the torque angle in motor mode at a constant amplitude of the stator current. The electromagnetic torque is calculated by the formula:

$$T_e = \frac{3}{2} p_p \Psi_m i_q + \frac{3}{2} p_p \left(L_d - L_q\right) i_d i_q \tag{3}$$

where i_d, i_q are the components of the stator current along *d* and *q* axes, depending on the torque angle.

In order to determine the influence of the size of the magnets on the electromagnetic torque of the machine, multivariate calculations were made, and the described procedure was performed for different sizes of the magnets. The width *W* is changed by decreasing by 2, 4, 6, 8, 10, 12 and 14mm respectively:

$W-2$, $W-4$, $W-6$, $W-8$, $W-10$, $W-14$,

and the height *H* is reduced as follows:

$H-1$, $H-2$, $H-3$, $H-4$.

IV. Investigation of the Influence of Magnet Size on Torque Components

Exemplary forms of the components of the electromagnetic torque and the total torque calculated using described approach are shown in Fig. 2. The maximum torque from the PM T_{PMm}, the maximum reluctance torque T_{Rm} and the maximum total torque of the machine T_m are the maxima of the curves of the components of the electromagnetic torque and the total torque shown in Fig. 2.

In Fig. 3 are shown the calculated characteristics of the electromagnetic torque at different widths of the magnets and the height *H*. The significant decrease of the maximum torque at the reduction of the magnets' width is clearly seen. This is due to the decrease of the flux of the magnet, which is proportional to its area.

TABLE I. shows summarized results for the maximum values of the torques at different widths of the magnets. The dependence of the maximum value of the active, reluctance and total torque on the width of the PM at constant height is shown in Fig. 4 for a current of 180A. It can be seen that the reluctance torque is maximal at the width of the magnets *W*-6, *W*-4, as with decreasing and increasing the width of the magnets the reluctance torque decreases. On the other hand, the torque of the PM increases in proportion to the width of the magnet, which leads to an increase in the total torque.

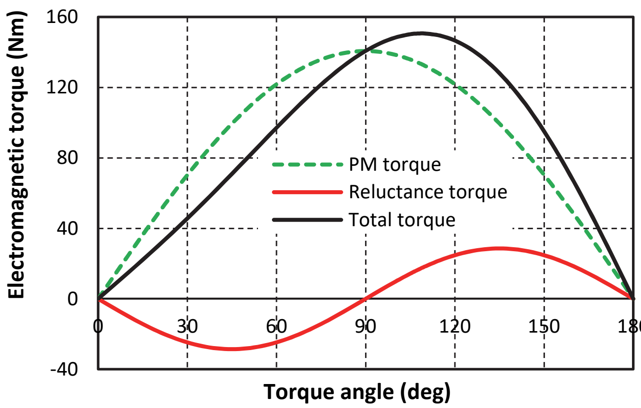

Fig. 2. Characteristics of electromagnetic torque and its components at current 180A for the base construction – PM width *W* and height *H*.

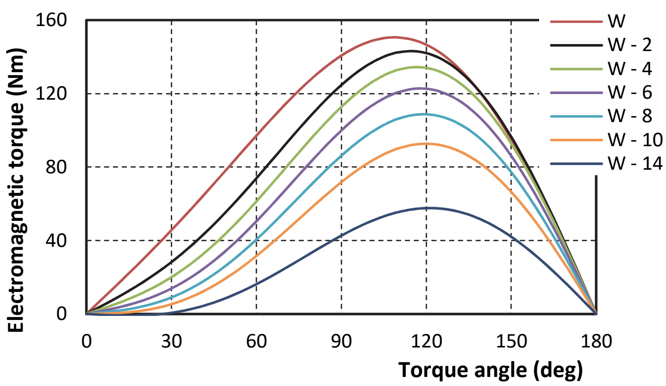

Fig. 3. Comparison of total electromagnetic torque characteristics at current 180A for different PM width and height *H*.

TABLE I. MAXIMUM VALUES OF THE TORQUE COMPONENTS AT DIFFERENT PM WIDTH AND HEIGHT *H* AT CURRENT 180A

Magnet width (mm)	Maximum PM torque T_{PMm} (Nm)	Maximum reluctance torque T_{Rm} (Nm)	Maximum total torque T_m (Nm)	Reluctance torque to total torque T_{Rm}/T_m
W	140.7	28.6	150.6	0.203
W-2	124.7	39.4	142.5	0.316
W-4	112.7	42.0	134.4	0.373
W-6	99.9	41.7	122.6	0.417
W-8	86.2	39.4	108.7	0.457
W-10	71.7	35.3	92.7	0.492
W-14	42.6	23.9	57.6	0.415

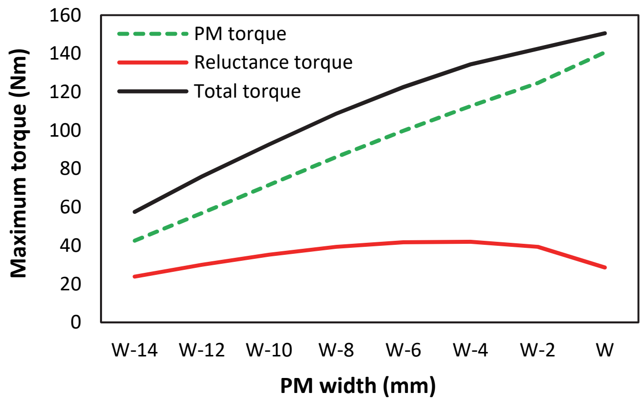

Fig. 4. Dependence of maximum values of torque components and total torque on PM width at current 180A and PM height *H*.

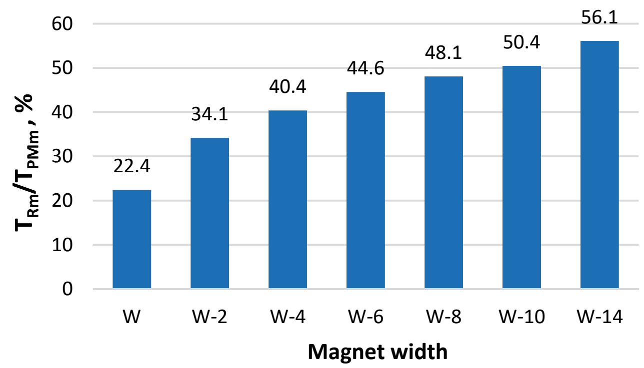

Fig. 5. The ratio of reluctance torque to PM torque at current 180A and PM height *H*.

In Fig. 5 is presented the percentage ratio of reluctance to the active torque for different widths of the magnets. This result shows that as the width of the magnets decreases, the ratio of reluctance to active torque, i.e. the share of the reluctance torque in the total torque increases. However, the reluctance torque decreases with a magnet width below *W*-6, and also the total torque decreases, as shown in Fig. 4.

Fig. 6 shows graphs of the maximum torque and the maximum value of the reluctance torque depending on the current at different widths of the magnets. It can be concluded that the maximum value of the machine torque depends almost linearly on the stator current at all widths of the magnets. For the reluctance torque, the dependence is also almost linear, as shown in Fig. 7. Theoretically, the dependence of the reluctance torque on the current should be quadratic, but this does not happen in practice. The reason is that as the current increases, the magnetic system becomes saturated and the values of the inductances on both axes decrease, as well as the difference between them, which according to (3) leads to a

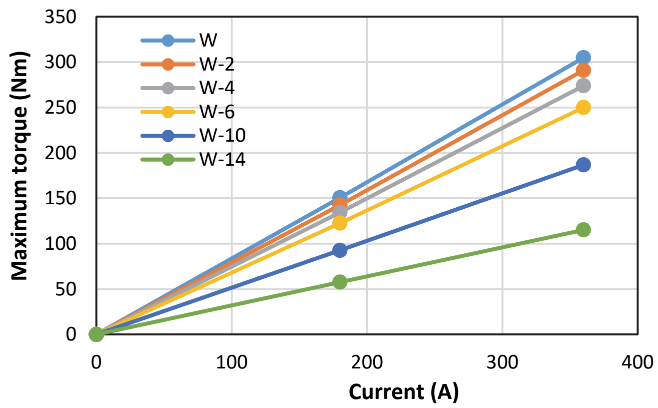

Fig. 6. Characteristics of the maximum torque depending on the current for different PM width and height *H*.

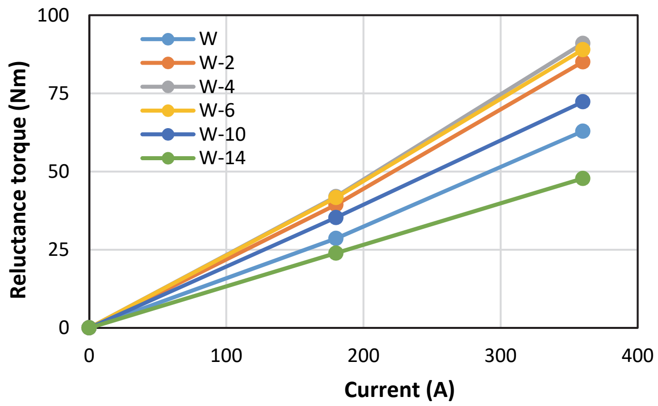

Fig. 7. Characteristics of the reluctance torque depending on the current for different PM width and height *H*.

decrease in the reluctance torque. Thus, due to the saturation, the reluctance torque in the tested machine does not increase in proportion to the square of the current, but almost linearly.

Given the economies of mass of the magnets, which could lead to a reduction in the price of the machines, it is interesting to examine the influence of the height of the PM on the machine electromagnetic torque. Below are presented the results of multivariate studies of the same synchronous machine with a magnet height reduced by 1 to 4mm and the same width *W*.

The results show a gradual decrease in the flux linkage of the magnet Ψ_m from 0.0615Wb to 0.0542Wb with a decrease in the height of the magnet from *H* to *H*-4 mm. At the same time, an increase in the inductances on both axes L_d and L_q is registered with a decrease in the height of the magnets. More noticeable is the increase in L_d, which is explained by the decrease of the magnetic resistance along the *d* axis with the decrease in the height of the magnet and its replacement by iron. The increase in L_q is less because there are no significant changes in the magnetic circuit along the transverse magnetic flux.

A comparison of the characteristics of the electromagnetic torque at different heights of the magnets and the width *W* is shown in Fig. 8. TABLE II. summarizes the results of the calculations of the electromagnetic torque of the machine at different heights of the magnets and at the same width *W*.

Fig. 9 graphically shows the dependences of the maximum values of the torque components at a current of 180A and different heights of the magnets - from *H* to *H*-4mm, at the same width *W*. The reduction of the height of PM as a whole

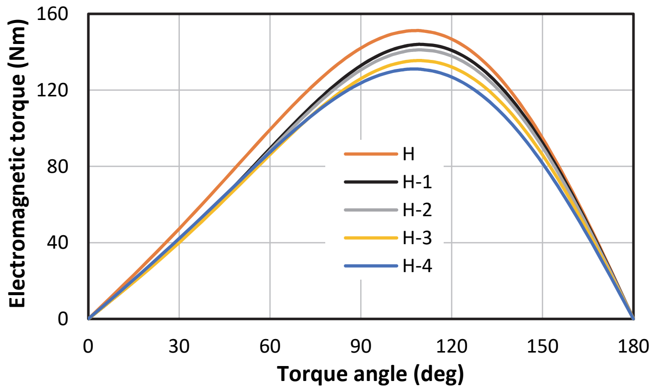

Fig. 8. Electromagnetic torque characteristics at current 180A, PM width *W* and different PM heights - from *H* to *H*-4.

TABLE II. MAXIMUM VALUES OF THE TORQUE COMPONENTS AT DIFFERENT PM HEIGHTS AND WIDTH *W* AT CURRENT 180A

Magnet height (mm)	Maximum PM torque T_{PMm} (Nm)	Maximum reluctance torque T_{Rm} (Nm)	Maximum total torque T_m (Nm)	Reluctance torque to total torque T_{Rm}/T_m
H	142.1	27.4	151.2	0.181
H-1	132.8	29.8	144.0	0.207
H-2	130.6	28.6	141.1	0.203
H-3	126.0	26.5	135.5	0.196
H-4	123.7	22.8	131.0	0.174

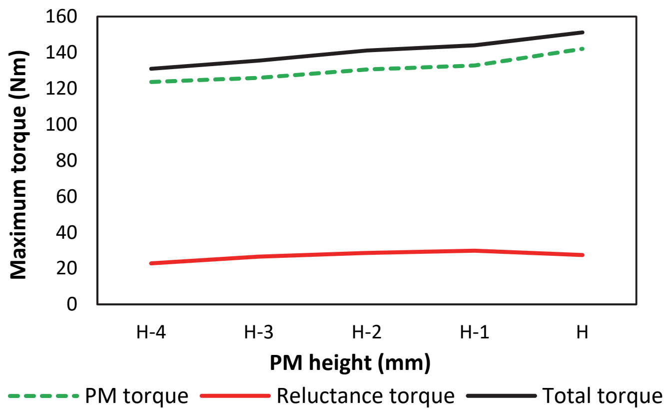

Fig. 9. Dependence of maximum values of torque components and total torque on PM height at current 180A and PM width *W*.

leads to a decreases in the total torque. This is due to the reduction of the active torque as a result of decrease of the magnets' flux. The results (TABLE II.) show that the reluctance torque reaches values up to 21% of the total torque. The reluctance torque does not change significantly with the height of the magnet. However, it can be noted that it is greatest at the height of the magnet *H*-1.

The results also show that the reduction in the total torque of the machine is not proportional to the reduction in the PM height. For example, decreasing the magnet height by 33% - from *H* to *H*-2 mm leads to a reduction in maximum torque by only 6.6% - from 151.2 to 141.1 Nm.

This trend is summarized in Fig. 10, where the ratio of the maximum torque to the total volume of the magnets at different heights is shown. It is clear how the decrease in height (which leads to a proportional decrease in volume) increases the torque produced per unit volume of the magnet. The volume of magnets V_{PM} is calculated by the formula:

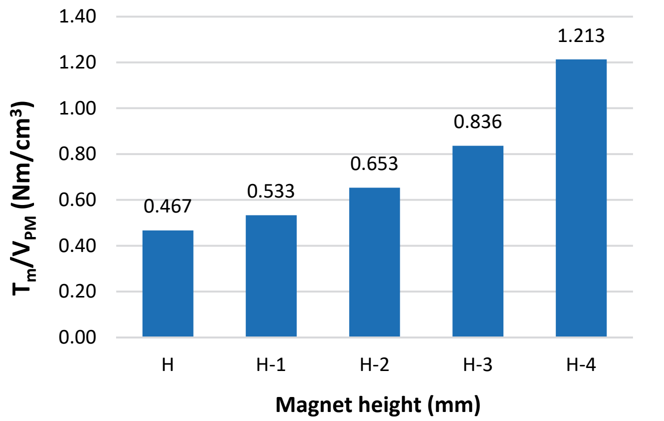

Fig. 10. Ratio of the maximum torque to the volume of the magnets when changing the PM height at constant width *W* and current of 180A.

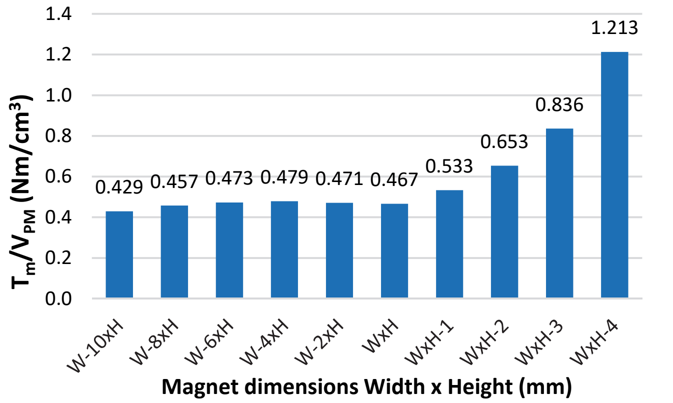

Fig. 11. Ratio of the maximum torque to the volume of the magnets at different dimensions of the PM and current of 180A.

$$V_{PM} = 12WHl \tag{4}$$

where: *W* is PM width, *H* – PM height, *l* – PM length, which is equal to the machine length.

Fig. 11 shows a comparison of the ratio of the maximum torque to the total volume of the magnets in the machine for some of the tested variants at current of 180A. The conclusion to be drawn is that reducing the width of the magnets only leads to a reduction in the electromagnetic torque, with an almost constant ratio of torque to the PM volume (differences are in the range of -8.1% to + 2.8% from the PM base size *W*x*H*). As the height of the magnet decreases, there is a significant increase in the torque produced per unit volume of the magnets. This increase is 2.78 times when the height of the magnet changes from *H* to *H*-4mm. Of course, this is accompanied by a reduction in the absolute value of the torque, as shown in Fig. 9.

Comparison of machine parameters for different sizes of magnets is shown in TABLE III.

Another interesting aspect of the study of the machine is the harmonic content of the magnetic induction created by the permanent magnets in the air gap. This study was performed by using the finite element method to find the distribution of the magnetic induction and then a harmonic analysis was performed by FFT. Exemplary results are shown in Fig. 12 and Fig. 13, where the calculated curve of magnetic induction and its decomposition in harmonic order is presented for the constructions with magnet dimensions *W* x *H* and *W*-8 x *H*.

TABLE III. COMPARISON OF MACHINE PARAMETERS FOR DIFFERENT SIZES OF MAGNETS AT CURRENT 180A

Magnet dimensions, mm	Maximum total torque T_{max}, Nm	Magnet volume V_{PM}, cm³	Specific torque T_{max}/V_{PM}, Nm/cm³	Reluctance torque to total torque T_{Rmax}/T_{max}
W x *H*	151.2	324	0.467	0.181
W x *H*-1	144.0	270	0.533	0.207
W x *H*-2	141.1	216	0.653	0.203
W x *H*-3	135.5	162	0.836	0.196
W x *H*-4	131.0	108	1.213	0.174
W-2 x *H*	142.5	302	0.471	0.276
W-2 x *H*-3	132.9	151	0.879	0.283
W-4 x *H*-3	124.0	140	0.883	0.315

In TABLE IV. , the data from the calculations of the harmonic content of the magnetic induction created by the PM are summarized.

In Fig. 14 is shown the dependence of the amplitude of the first and third harmonics of the magnetic induction on the width of the magnet at the same height *H*. In practice, the linear decrease in the amplitude of the first harmonic with a decrease in the width of the magnet can be seen. This is explained by the linear dependence of the flux of the magnet on its area - the reduction of the flux leads to a proportional reduction of the magnetic induction. The third harmonic decreases in the same way, which can also be explained by the general decrease in the magnetic flux and the change in the general shape of the magnetic induction curve.

TABLE IV. VALUES OF MAGNITUDES OF MAGNETIC INDUCTION HARMONICS [T] AT DIFFERENT MAGNET WIDTH

Harmonic No	Permanent magnet dimensions				
	W* x *H	***W*-2 x *H***	***W*-4 x *H***	***W*-6 x *H***	***W*-8 x *H***
1	0.77	0.675	0.605	0.53	0.461
3	0.22	0.15	0.093	0.05	0.019
5	0.1	0.02	0.028	0.02	0.08
7	0.045	0.023	0.06	0.051	0.04
9	0.001	0.056	0.052	0.047	0.005

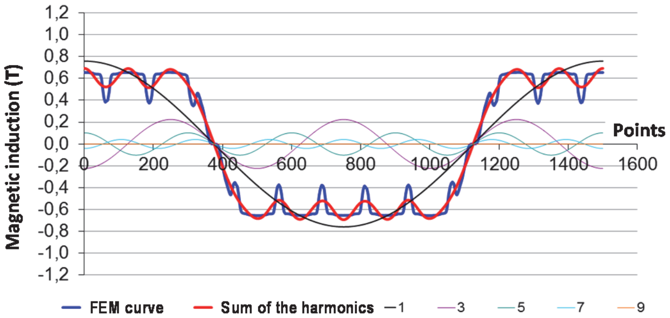

Fig. 12. Distribution of magnetic flux density created by PM with dimensions *W* x *H*.

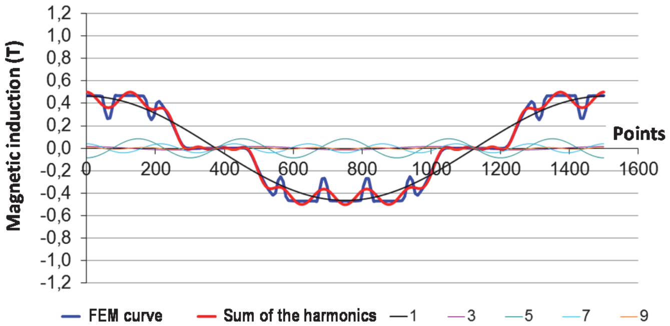

Fig. 13. Distribution of magnetic flux density created by PM with dimensions *W*-8 x *H*.

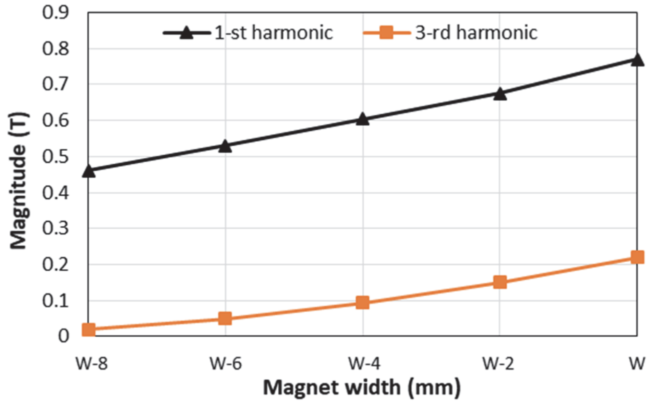

Fig. 14. Magnitude of the first and third harmonics of the magnetic induction created by the permanent magnets at a constant height *H* and different width.

With regard to the percentage of harmonics, the following results are obtained. There is also a linear dependence of the content of the third harmonic, which decreases from 28.6% at the base width *W* to 4.1% at the magnet with *W*-8. The fifth harmonic is highest at sizes *W* and *W*-8: 13 and 17.4%, respectively, while at *W*-6 to *W*-2 it is below 5%. For the other harmonics, no clear dependence on the width of the magnet can be found. Their content reaches a maximum of 10%.

Another valuable aspect of the study is the possibility of weakening the field of the magnets by the d-component of the stator current. Using FEM analysis, the resultant curve of the magnetic induction in the air gap of the machine at different heights of the magnets was calculated. The calculations are made at a stator current value of 180A and a fully demagnetizing armature reaction. The results are shown in Fig. 15.

Comparison of the resulting magnetic induction shows that there is a decrease in the amplitude of the first harmonic from 0.78T in the absence of demagnetization reaction to 0.58T at the base height of the magnet *H* and down to 0.42T at the height of the magnet *H*-4. These results mean that it is not possible to fully compensate for the magnetic field of the magnets even at greatly reduced heights *H*-4. However, there is a significantly bigger reduction in the resulting field (54% instead of 74%), which would lead to a wider speed range of the machine with the thinner magnet than the machine with the base height of the PM.

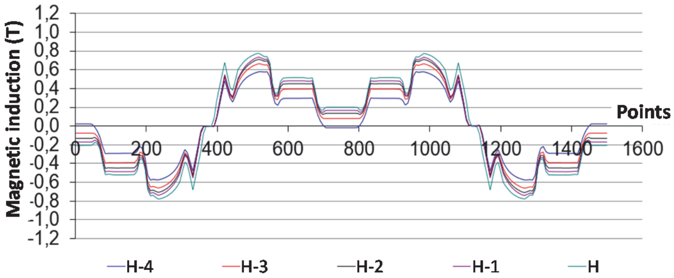

Fig. 15. Distribution of the air gap magnetic induction at PM flux weakening at different PM height and current of 180A.

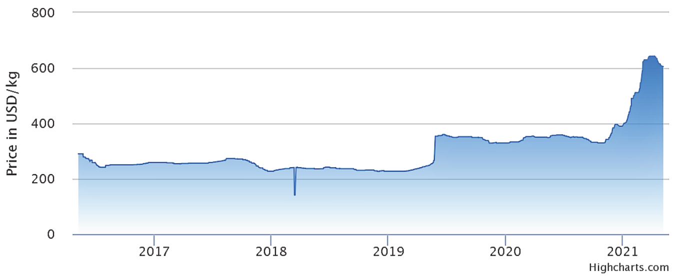

Fig. 16. Market price of dysprosium Dy for the last 4 years [8].

TABLE V. COMPOSITION AND PRICES PER KILOGRAM OF NEODYMIUM MAGNETS WITH DIFFERENT CONTENT OF DYSPROSIUM AND COBALT AS OF MAY 2021

Magnet type	NdFeB-SH	NdFeB-UH	NdFeB-EH	NdFeB-AH
Working temperature	***150º***	***180º***	***200º***	***230º***
Nd (%)	31	31	31	31
Fe (%)	63	61.5	60	58.5
B (%)	1	1	1	1
Co (%)	1.2	2	3	4
Dy (%)	3.8	4.5	5	5.5
Total (USD/kg)	**57.58**	**62.16**	**65.63**	**69.09**

V. PERMANENT MAGNETS' PRICES

Despite the problems caused by the COVID-19 pandemic, the demand for neodymium magnets is increasing. This leads to an increase in the prices of key materials for their production - neodymium and dysprosium. The prices of both metals have increased since the beginning of 2021 by more than 50% - see Fig. 16 [8]. Based on the prices of basic materials [8] and the composition of different types of neodymium magnets [9], the prices of materials for the production of magnets in USD/kg are calculated. The results are shown in TABLE V.

In TABLE VI. is shown an illustrative summary of the torque, volume and mass of the magnets and the approximate total cost of the magnets for a machine. The value for the NdFeB-UH type magnet from TABLE V was used to calculate the price. The material density of the magnet was assumed to be 7.5g/cm^3, according to the data cited in most literature sources.

TABLE VI. COMPARISON OF THE PARAMETERS OF THE MACHINE AND THE PRICE OF MAGNETS AT DIFFERENT PM SIZES

Magnet dimensions (mm)	Maximum torque (Nm)	Magnet volume (cm^3)	Magnet mass (kg)	Relative torque	Relative magnet mass	Sample price (USD)
W x *H*	151.2	324	2.43	1.00	1.00	151
W x *H*-1	144.0	270	2.03	0.95	0.83	126
W-2 x *H*	142.5	302	2.27	0.94	0.93	141
W x *H*-2	141.1	216	1.62	0.93	0.67	101
W x *H*-3	135.5	162	1.22	0.90	0.50	76
W-2 x *H*-3	132.9	151	1.13	0.88	0.47	70
W x *H*-4	131.0	108	0.81	0.87	0.33	50
W-4 x *H*-3	124.0	140	1.05	0.82	0.43	65

Also in TABLE VI. , the relative values of the machine torque and the mass of the magnets are shown, related to the values for the basic construction with magnet dimensions *W* x *H*. The results in the table show the significant possibilities for saving magnets with an acceptable reduction of the machine torque.

VI. CONCLUSION

The article presents an analysis of the influence of the dimensions of the permanent magnets on the torque of a synchronous machine with interior magnets. Detailed results on the influence of the sizes of the magnets on the two components of the electromagnetic torque - that of the permanent magnets and the reluctance torque - are presented. Comparisons are made with the parameters that are achieved with a basic design of the machine with certain sizes of magnets. The results show that as the width of the magnets decreases, there is an improvement in the harmonic composition of the distribution of the magnetic induction in the air gap, which would improve the operation of the machine when connecting the phases in a triangle. In addition, as the size of the magnets decreases, the torque of the machine also decreases, but not in proportion to the volume and mass of the magnets. This leads to the conclusion that it is possible to reduce the volume of magnets used without a significant loss of machine parameters. For example, reducing the volume of the magnets by 50% leads to a reduction in the maximum torque of the machine by only 10% compared to the basic design. The savings that can be obtained at current prices of neodymium magnets in this case reaches 75USD per machine. In large-scale production, this can lead to significant savings and lower costs of the product at competitive parameters. In addition, the achieved results make it possible to design a series of similar machines with different torque, power and speed range to cover a wide range of market requirements.

ACKNOWLEDGMENT

The authors would like to thank the Research and Development Sector at the Technical University of Sofia and ALMOTT Ltd. for financial support under the project BG16RFOP002-1.005-0013-C01 "Scientific industrial research to increase the energy efficiency of electric machines with rare earth permanent magnets" from the operational program "Innovation and competitiveness".

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