Power Quality Analyzers Calibration on Total Harmonics Distortion of Voltage and Current by Reference Square Waveform Signal

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I. Itroduction

Total Harmonics Distortion (THD) of Voltage and Current respectively determined by the formula:

$$THD_U = \frac{\sqrt{\sum_{n=2}^{\infty} U_n^2}}{U_1} \qquad (1)$$

and

$$THD_I = \frac{\sqrt{\sum_{n=2}^{\infty} I_n^2}}{I_1} \qquad (2)$$

Calibration of the power quality analyzers (PQA) by THD of Voltage and Current usually performed using relatively expensive reference calibrators, designed to calibrate the PQA. For example, a calibrator Electrical Power Standard Fluke 6100A/80A, can generate in addition to the main network harmonic of the power supply network and a combination of other harmonics with frequencies multiples of the fundamental harmonic frequency. The present paper proposes an approach through which this difficulty can be overcome by using a simple calibrator of Square Waveform Signal of voltage and current for the purposes of calibration according to the THD parameter. In this case the Reference Square Waveform Signal, presented in Fourier series is a combination of harmonic signals with frequencies multiples of the main network harmonic.

An important parameter of the square waveform signal of the square waveform signal (fig.1) [1] is its duty cycle μ, which depends on the duration τ and the period T of the pulse and is determined by the expression $\mu = \frac{\tau}{T}$.

The reference square waveform voltage, presented in Fourier series has the following trigonometric description [1].

$$u(t) = 2U_m(\mu - 0,5) + \frac{4U_m}{\pi}\sum_{n=1}^{\infty}\frac{\sin(n\pi\mu)}{n}\cos\left(\frac{2n\pi t}{T}\right) \qquad (3)$$

Respectively the trigonometric Fourier description of reference square waveform current is

$$i(t) = 2I_m(\mu - 0,5) + \frac{4I_m}{\pi}\sum_{n=1}^{\infty}\frac{\sin(n\pi\mu)}{n}\cos\left(\frac{2n\pi t}{T}\right) \qquad (4)$$

The coefficient of total harmonic distortions for voltage and current, respectively, according to [2] is determined by the expressions

$$THD_U = \sqrt{\frac{\mu(1-\mu)\pi^2}{2\sin^2(\pi\mu)} - 1}\text{, for } 0 < \mu < 1, \qquad (5)$$

and

$$THD_I = \sqrt{\frac{\mu(1-\mu)\pi^2}{2\sin^2(\pi\mu)} - 1}\text{, for } 0 < \mu < 1, \qquad (6)$$

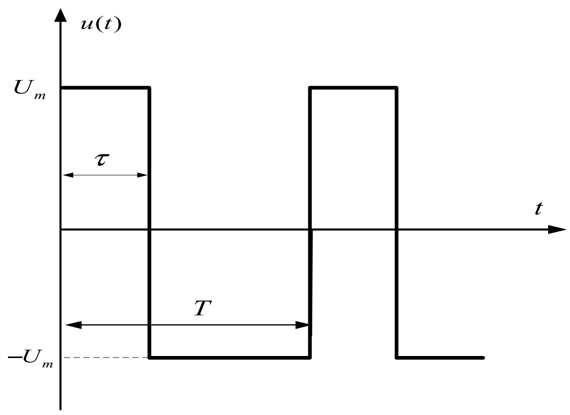

Fig.1. Timing diagram of bipolar periodic square waveform signal; [1, 2]

From expression (4) it is seen that the only parameter that affects the coefficient of total harmonic distortion of the Reference Square Waveform Signal is its duty cycle μ . Thus by changing the value of the duty cycle μ reference values of the coefficient parameter can be set of total harmonic distortions for PQA calibration, which are presented in Table.1.

TABLE 1

n	1	2	3	4
μ	0,25	0.3	0.4	0.5
THD_U	92.2253	76.3766	55.6227	48.3426
THD_I	92.2253	76.3766	55.6227	48.3426

II. MATHEMATICAL MODEL AND UNCERTAINTY OF THE CALIBRATION RESULT

The mathematical model for the actual value of the measured parameter THD of the voltage at which the PQA is calibrated is presented in [1] and has the form:

$$THD_{U,act} = THD_{U,cal} - \delta THD_{U,s.et} - \delta THD_{U,dr.et} + \delta THD_{U,res.cal} \quad (7)$$

where:

- $THD_{U,cal}$ is the total harmonic distortion coefficient of the voltage analyzer calculated by m number of independent measurements;
- $\delta THD_{U,s.et}$ the deviation at the calibration point from the set value by voltage from a calibrator;
- $\delta THD_{U,dr.et}$ - drift of the calibrator generated voltage value;
- \- $\delta THD_{U,res.cal}$ - the deviation from the resolution of the calibrated analyzer by voltage.

Respectively the mathematical model for the actual value of the measured current parameter THD includes the same components, the difference being expressed in index replacement U of the voltage with the index I for current and has the form

$$THD_{I,act} = THD_{I,cal} - \delta THD_{I,s.et} - \delta THD_{I,dr.et} + \delta THD_{I,res.cal} \quad (8)$$

Based on expressions (7) and (8) and according to [3] the estimates of input values in the model are determined (the individual components in the models) their standard uncertainty, the actual value, the combined standard uncertainty and the extended uncertainty, as follows:

Determining the estimates of the input quantities.

\- The estimate for the measured value of the coefficient of total harmonic distortions in voltage and current, reported by the analyzer are defined as the average value of m number of measurements, respectively

$$THD_{U,cal} = \frac{1}{m}\sum_{i=1}^{m} THD_{U,cal,i} , \quad (9)$$

and

$$THD_{I,cal} = \frac{1}{m}\sum_{i=1}^{m} THD_{I,cal,i} , \quad (10)$$

\- The evaluation of the deviation of the set value $\delta THD_{U,s.et}$ from the calibrator for standard voltage and the estimation of the deviation of the set value $\delta THD_{I,s.et}$ from the standard current calibrator is determined by its specification by the specified maximum error value.

\- The estimate of the drift of the coefficient of total harmonic distortion of the calibrator has zero value due to the discrete frequency spectrum of the square waveform signal, i.e. $\delta THD_{U,dr.et} = 0$ and $\delta THD_{I,dr.et} = 0$.

\- The evaluation of resolution $\delta THD_{U,res.cal}$ by voltage and resolution $\delta THD_{I,res.cal}$ by current of the calibrated analyzer have a rectangular distribution with zero value, i.e. $\delta THD_{U,res.cal} = 0$ and $\delta THD_{I,res.cal} = 0$.

Determination of evaluation of the actual value

The actual value of the measured parameter THD in voltage and current is determined by all estimates of the input values and from the expressions for both models (7) and (8) and obtained respectively

$$THD_{U,act} = THD_{U,cal} - \delta THD_{U,s,et} \quad (11)$$

and

$$THD_{I,act} = THD_{I,cal} - \delta THD_{I,s,et} \quad (12)$$

Determination of the standard uncertainty of the input quantities.

\- The standard uncertainty of the estimate of the measured values of the total harmonic distortion coefficient $u(THD_{U,cal})$ by voltage and $u(THD_{I,cal})$ by current are determined by the expressions respectively

$$u(THD_{U,cal}) = \sqrt{\frac{\sum_{k=1}^{m}(THD_{U.cal,i} - THD_{U,cal})^2}{m(m-1)}} \quad (13)$$

and

$$u(THD_{I,cal}) = \sqrt{\frac{\sum_{k=1}^{m}(THD_{U.cal,i} - THD_{I,cal})^2}{m(m-1)}} \quad (14)$$

\- The standard uncertainty of the estimation of the deviation of the set value of the THD parameter of the calibrator $u(\delta THD_{U,s,et})$ by voltage and $u(\delta THD_{I,s,et})$ by current are determined by the calibrator specification.

\- The standard uncertainty of the estimate of the deviation of the resolution of the analyzer $u(\delta THD_{U,res.cal})$ by voltage is determined by the expression

$$u\left(\delta THD_{U,res.cal}\right)=\frac{a_U}{2\sqrt{3}}, \quad (15)$$

where a_U is the value of the least significant digit of the analyzer by voltage. And accordingly the standard uncertainty of the estimate of the deviation of the resolution on the analyzer $u\left(\delta THD_{U,res.cal}\right)$, by current is determined by the expression

$$u\left(\delta THD_{I,res.cal}\right)=\frac{a_I}{2\sqrt{3}}, \quad (16)$$

where a_I is the value of the least significant digit of the analyzer by current.

Determination of the combined standard uncertainty of the input quantities

The combined standard uncertainty is determined by the standard uncertainty of the estimates of all inputs (for sensitivity factors 1) from the expressions:

- by voltage

$$u\left(THD_{U,act}\right)=\sqrt{u\left(\delta THD_{U,cal}\right)^2+u\left(\delta THD_{U,s.et}\right)^2+u\left(\delta THD_{U,res.cal}\right)^2} \quad (17)$$

- by current

$$u\left(THD_{I,act}\right)=\sqrt{u\left(\delta THD_{I,cal}\right)^2+u\left(\delta THD_{I,s.et}\right)^2+u\left(\delta THD_{I,res.cal}\right)^2} \quad (18)$$

Determination of the expanded uncertainty

The extended uncertainty is determined by the expressions:

- by voltage

$$U\left(THD_{U,act}\right)=k.u\left(THD_{U,act}\right) \quad (19)$$

and

- by current

$$U\left(THD_{I,act}\right)=k.u\left(THD_{I,act}\right) \quad (20)$$

where $k=2$ is the coverage factor with a probability of approx 95% and normal law of distribution.

III. ALGORITHM FOR CALIBRATION

Calibration algorithms represent the sequence of operations for control of the reference calibrator and the calibrated analyzer, generation of reference values for the input parameter THD the voltage / current from the calibrator, conducting multiple measurements at the calibration point, recording of analyzer results, collection of measurement results and their processing, the ultimate goal is to obtain an estimate of the actual value of the measured quantity and the expanded uncertainty at the point of calibration of the analyzer.

The algorithm for calibrating PQA by THD of voltage/current is based on calibration models (7) and (8), respectively. The sequence of operations is as follows:

- Enter data for the PQA calibration certificate: temperature, humidity, data for the standard and its calibration certificate, as well as data for the expert performing the calibration;
- Set the number of points n at which the calibrator is calibrated according to THD;
- Set the number of multiple measurements m in which the THD calibrator is calibrated;
- Set frequency values f and the amplitude U_m of the square waveform voltage and the amplitude, respectively I_m on the current;
- Set to the value of the duty cycle of the pulses μ which is predetermined by expression (5) or (6) according to Table 1, i.e. a reference value of the total harmonic distortion coefficient is set THD_U and respectively THD_I ;
- Enter data from the calibration certificate of the reference calibrator: the actual value of THD and its extended uncertainty, according to the range and the generated voltage / current value for each calibration point;
- Start the calibrator and analyzer, by successively performing m measurements for each of the selected n calibration points and visualizing the results obtained in tabular form.
- Calculation and visualization in tabular form of the results for the evaluations for each of the selected n number of calibration points, according to the mathematical models (7) and (8), using expressions (9) and (10), (11) and (12), (13) and (14), (15) and (16), (17) and (18), and (19) and (20),
- Archiving the results of the measured results and their subsequent processing.

IV. THE RESULTS OF THE CALIBRATION

For the purposes of this calibration, a Metrix CX1651 calibrator was used as a reference for the square waveform standard signal and Fluke 435 as a calibrated PQA, which is calibrated in n number of points, listed in Table 1.

The following reference calibrator outputs were used [4]:

- Voltage range $200V$
- Current range $2A$
- Frequency $f=50Hz$
- Deviation of the set point THD voltage at the calibration point $\delta THD_{U,s.et}=0.05\%$
- Uncertainty of the set point THD of the voltage $u\left(\delta THD_{U,s,et}\right)=0.3\%$
- Deviation of the set point THD of the current at the calibration point $\delta THD_{I,s.et}=0.2\%$
- Uncertainty of the set point THD of the current at the calibration point $u\left(\delta THD_{I,s,et}\right)=0.3\%$

The following outputs for the calibrated analyzer [5] were used:

- Voltage THD resolution $\delta THD_{U,res.cal} = 0.0000289$, resolution is 0,0001 and the measured values are reported via the PQA software.
- Current THD resolution $\delta THD_{U,res.cal} = 0.0000289$, resolution is 0,0001 and the measured values are reported via the PQA software.

The results of the calibration by the coefficient of total harmonic distortions by voltage are presented in Table 2 and Table 3.

The results of the calibration by the coefficient of total harmonic distortions by current are presented in Table 4 and Table 5.

TABLE 2

μ	THD_U %	$THD_{U,cal,1}$ %	$THD_{U,cal,2}$ %	$THD_{U,cal,3}$ %	$THD_{U,cal,4}$ %	$THD_{U,cal,5}$ %	$THD_{U,cal,6}$ %	$THD_{U,cal,7}$ %	$THD_{U,cal,8}$ %	$THD_{U,cal,9}$ %	$THD_{U,cal,10}$ %
0.25	92.2253	90,8312	90,8378	90,8456	90,8284	90,8236	90,8292	90,8392	90,824	90,836	90,8276
0.3	76.3766	75,0898	75,0894	75,0898	75,089	75,0894	75,0884	75,0896	75,0892	75,0898	75,0890
0.4	55.6227	54,3400	54,3398	54,3366	54,3380	54,3430	54,3416	54,3370	54,3392	54,3426	54,3376
0.5	48.3426	46,9970	46,9992	46,9938	47,0016	47,0078	47,0028	46,9978	46,9976	46,9974	46,9978

TABLE 3

μ	THD_U	$THD_{U,cal}$	$\delta THD_{U,s,et}$	$THD_{U,act}$	$u(THD_{U,cal})$	$u(\delta THD_{U,s,et})$	$u(\delta THD_{U,res.cal})$	$u(THD_{U,act})$	$U(THD_{U,act})$
0.25	92.2253	90,83226	0,04542	90,7868	0,00226	0,13625	0,0000289	0,13627	0,2725
0.3	76.3766	75,08934	0,03754	75,0518	0,00014	0,11263	0,0000289	0,11263	0,2253
0.4	55.6227	54,33954	0,02717	54,3124	0,00072	0,08151	0,0000289	0,08151	0,1630
0.5	48.3426	46,99928	0,02350	46,9758	0,00123	0,07050	0,0000289	0,07051	0,1410

TABLE 4

μ	THD_I %	$THD_{I,cal,1}$ %	$THD_{I,cal,2}$ %	$THD_{I,cal,3}$ %	$THD_{I,cal,4}$ %	$THD_{I,cal,5}$ %	$THD_{I,cal,6}$ %	$THD_{I,cal,7}$ %	$THD_{I,cal,8}$ %	$THD_{I,cal,9}$ %	$THD_{U,cal,10}$ %
0.25	92.2253	90,8762	90,8756	90,873	90,8768	90,872	90,8832	90,8808	90,8802	90,883	90,8836
0.3	76.3766	74,9808	74,9864	74,9852	74,9818	74,9836	74,983	74,9872	74,9854	74,986	74,9816
0.4	55.6227	54,3914	54,3928	54,3876	54,3916	54,3948	54,3918	54,3964	54,3928	54,3916	54,3944
0.5	48.3426	47,0590	47,0614	47,0598	47,0602	47,0614	47,0590	47,0612	47,0596	47,0622	47,0626

TABLE 5

μ	THD_I	$THD_{I,cal}$	$\delta THD_{I,s,et}$	$THD_{I,act}$	$u(THD_{I,cal})$	$u(\delta THD_{I,s,et})$	$u(\delta THD_{I,res.cal})$	$u(THD_{I,act})$	$U(THD_{I,act})$
0.25	92.2253	90,8784	0,18176	90,6967	0,00136	0,13632	0,0000289	0,13632	0,2726
0.3	76.3766	74,9841	0,14997	74,8341	0,00071	0,11248	0,0000289	0,11248	0,2250
0.4	55.6227	54,3925	0,10879	54,2837	0,00076	0,08159	0,0000289	0,08159	0,1632
0.5	48.3426	47,0606	0,09412	46,9665	0,00041	0,07059	0,0000289	0,07059	0,1412

V. CONCLUSIONS

1. Calibration of PQA by total harmonic distortion coefficient by setting the duty cycle of the pulse is justified, when is not available calibrator, which can generate two or more sinusoidal signals.
2. Through a square waveform standard signal can to set fixed reference values with high accuracy of the coefficient of total harmonic distortion.
3. The metrological results obtained during the calibration of Fluke 435 analyzer with Metrix CX1651 calibrator by coefficient of total harmonic distortions demonstrate the applicability of the square waveform standard signal for PQA calibration by coefficient of total harmonic distortions in metrological practice.

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