Synthesis of Microwave Filters by Coupling Matrix Optimization

Marin V. Nedelchev, Ilia G.Iliev

Abstract - **This paper presents optimization method for synthesis of generalized microwave filters with arbitrary topology. The method utilizes Nelder-Mead local optimizer for coupling matrix determination. The synthesis procedure converges very fast as for a initial point is used a vector based on the Chebyshev all pole filter for the same degree of the filter. To validate the proposed synthesis method two numerical examples for resonant filters are computed. The frequency responses from the synthesis procedure and the theoretical responses show excellent agreement.**

Keywords - **microwave filter, Chebyshev filter, Nelder-Mead optimization, coupling matrix.**

I. Introduction

Microwave coupled resonator filters play important role in the modern communication systems. The constraint RF/microwave spectrum requires high attenuation in the stop band and low insertion loss in the passband of the filters. These requirements can be met only by cross-coupled microwave filters, realizing attenuation poles on finite frequencies. More over cross coupled filters can exhibit flat group delay, when realizing complex conjugate transmission zeroes. Cross-coupled resonator filters allow using various topologies with variety of frequency responses.

The microwave filter modelling is very important for the fast and accurate design.

In the early 1970's started the development of the theory of cross-coupled resonator filters by Atia and Williams in their basic paper [1]. Cameron extended the theory to general cross-coupled Chebyshev filtering functions synthesis in the papers [2, 3]. The synthesis procedure continues with deriving the transversal coupling matrix from the Chebyshev polynomials. Key point in the obtaining of the coupling matrix corresponding to the practical filter topology is to convert transversal form to folded form using matrix rotations. Folded form of the coupling matrix is starting point for matrix rotation sequences to derive the final coupling matrix. Most of the matrix rotation sequences are given in [4]. It is noticed that this method for synthesis suffers from generality, because the matrix rotations cannot be derived for every one practical filter topology. Some of the matrix rotation sequences cannot converge in order to find the coupling matrix. Some of the disadvantages in this method are solved if arrow form of the coupling matrix is used [5] or Pfitzenmeir method is used [6].

Marin Veselinov Nedelchev and Ilia Georgiev Iliev –are with Dept. of Radiocommunication and Videotechnologies in Faculty of Telecommunication in TU –Sofia, N8, Kliment Ohridski bul., 1700 Sofia, Bulgaria.

E-mails: mnedelchev@tu-sofia.bg, igiliev@tu-sofia.bg

In many practical cases, it is necessary to define the filter topology in order to satisfy some manufacturing or space requirements. In this case, the exact solution is hard to be found with the conventional synthesis methods.

Many commercially available design packages offer direct optimization of the physical dimensions of the filter topology. This method for synthesis cannot converge in the general case, because there is no "general" optimization method that is suitable for the optimization problem. If a local minimum is found, the sensitivity of this point is unknown. Starting from different points, the optimization process can lead us to different local minimum of the cost function. One of the solutions may be a global minimum, but there is no a priori guarantee for finding it. Some of the solutions may have very big sensitivities toward the manufacturing tolerances, temperature, or could not be realized in the practice. Therefore, the direct synthesis over the geometrical dimensions of the filter is not a good decision for general design method.

One possible general solution to the filter design for arbitrary topology is to apply direct local optimization over the coupling matrix with successive starting point. In the basic papers proposed optimization method for coupling matrix synthesis [7, 8], the starting vector is set to arbitrary values. This makes the local optimization very unstable method for cost function minimization. Another method is to use global optimization method for finding the coupling matrix for certain filter topology. They perform robust optimization, no matter about the starting point. Unfortunately the global optimizers such as genetic or stochastic have very slow convergence to the cost function minimum.

This paper presents optimization method for synthesis of microwave filters with arbitrary topology. The method uses Nelder-Mead local optimizer for coupling matrix determination. The synthesis procedure converges very fast as for a initial point is used a vector based on the Chebyshev all pole filter for the same degree of the filter. The cost function is based on amplitude of the transmission and reflection coefficient zeros and their values at the cut-off frequencies. To validate the proposed synthesis method two resonant filters are designed with asymmetrical responses. The frequency responses from the synthesis procedure and the theoretical responses show excellent agreement.

II. Resonator Filter Characteristics

The synthesis procedure starts with the low-pass prototype with normalized angular frequency of passband $\omega = 1$. The transfer and reflection coefficients may be expressed as a ratio of two N-th degree polynomials as follows:

$$S_{21} = \frac{P_N(\omega)}{E_N(\omega)},$$

$$S_{11} = \frac{D_N(\omega)}{\varepsilon E_N(\omega)}, \quad (1)$$

where ω is real angular frequency and $\varepsilon = \left(1/\sqrt{10^{RL/10}-1}\right).\left(D_N(\omega)/P_N(\omega)\right)\Big|_{\omega=1}$, RL is the prescribed value of the return loss in dB, in the passband of the filter. It is assumed that all polynomials are normalized to their highest degree coefficient. The reflection and the transfer coefficients must satisfy the unitary conditions of the scattering matrix.

$$S_{11}S_{11}^* + S_{21}S_{21}^* = 1. \quad (2)$$

It can be easily found that the transfer coefficient may be expressed in the following way:

$$S_{21}^2(\omega) = \frac{1}{1+\varepsilon^2 C_N^2(\omega)} \quad (3)$$

where $C_N(\omega) = P_N(\omega)/D_N(\omega)$ is the filtering function. For general Chebyshev characteristics, the filtering function is in the form:

$$C_N(\omega) = \cosh\left(\sum_{n=1}^{N} a\cosh(x_n)\right), \quad (4)$$

where $x_n = \frac{\omega - 1/\omega_n}{1-\omega/\omega_n}$, where ω_n is the angular frequency of the prescribed transmission zero.

In order to obtain the coupling matrix, it is necessary to consider the equivalent circuit of general coupled resonator filter shown on Fig.1.

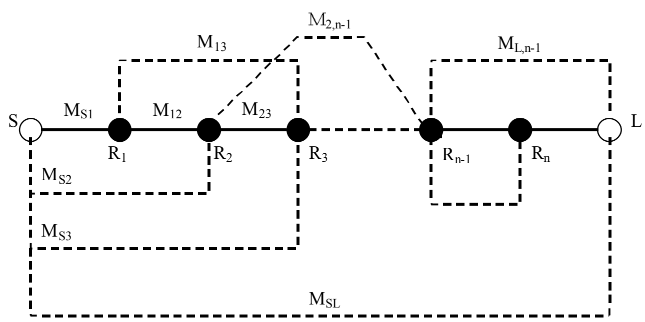

Fig.1. General coupled resonator filter

The equivalent circuit consists of N series coupled resonators with frequency independent couplings M_{ij} ($i \neq j$), between the *i-th* and *j-th* resonators. The circuit is driven by voltage source E with internal normalized resistance $R_1 = 1$ and loaded to normalized impedance $R_2 = 1$. There exists coupling between the Source and Load to perform all N finite frequency transmission zeroes. The resonant frequency of each resonator f_{0i} is represented by the self-coupling coefficient M_{ii} and the center frequency of the filter. They must satisfy the equation:

$$f_{0i}^2 - M_{ii} f_0 f_{0i} - f_0^2 = 0. \quad (5)$$

The transmission and reflection coefficients of a lossless filter of *N*-th order depend only of the coupling matrix $|M|$ (7):

$$S_{21} = -2j[A]_{N+2,1}^{-1},$$

$$S_{11} = 1 + 2j[A]_{11}^{-1}, \quad (6)$$

where $[A] = -j[R] + \omega[W] + [M]$, a $[R]$ e $(N+2)\times(N+2)$ matrix, which elements are zeroes except $R_{11} = R_{N+2,N+2} = 1$. $[W]$ is a $(N+2)\times(N+2)$ matrix, where the main diagonal elements are unity except $W_{11} = W_{N+2,N+2} = 0$. All remaining elements of $[W]$ are zeroes. $[M]$ is the coupling matrix, symmetrical around the main diagonal.

III. Synthesis of Microwave Filter with Coupling Matrix Optimization

The cost function used in the optimization process is based on the zeroes and poles of the filtering function C_N, assuming that the number of poles is P and zeroes N [8]:

$$Cost = \sum_{i=1}^{N}\left|S_{11}(\omega_{zi})\right|^2 + \sum_{i=1}^{P}\left|S_{21}(\omega_{pi})\right|^2 + \left(\left|S_{11}(\omega=-1)\right| - \frac{\varepsilon}{\sqrt{\varepsilon^2+1}}\right)^2 + \left(\left|S_{11}(\omega=1)\right| - \frac{\varepsilon}{\sqrt{\varepsilon^2+1}}\right)^2 \quad (7).$$

This cost function requires less computational efforts than using the theoretically derived S_{21} and S_{11} for pattern search.

In this way it is possible to formulate the local optimization problem for obtaining the coupling matrix.

The advantages of this method for synthesis are:

1. Design of filter with prescribed transmission zeros with symmetric or asymmetric response.
2. Design of filter with arbitrary topology, even or odd order.
3. Possibility of constraints for the magnitude and sign of the coupling coefficients if a given realization is intended.
4. Elimination of the similarity transformations for the coupling matrix and the extraction technique. There is no possibility for calculation errors or round off errors.

The main disadvantages for the optimization method are:

1. Exact solution is not guaranteed, especially for great number of variables.
2. The filter topology must be able to realize the desired filter response. Then a local minimum is reached by the optimization process.
3. If the initial guess is arbitrary, the global minimum cannot be reached in every filter design.
4. Some of the elements of the coupling matrix, derived in the optimization process may be impossible to realize.

The initial guess for the coupling matrix is very important for the reaching of the global minimum of the cost function (7). Having on mind that a local optimizer is used, the starting vector should be close to the target value in order to assure a fast convergence of the method. One of the possible starting coupling matrices is to set all self-coupling

couplings to zero ($M_{ii} = 0$) and all direct couplings to 1. The cross-coupling coefficients are all set to zero.
The second possible starting coupling matrix is to use classical Chebyshev filter from the same order. All self- and cross-couplings are set to zero. In order to find out which starting point is more computational efficient some numerical designs are investigated.

III. NUMERICAL RESULTS

For verification of the optimization method presented in this paper, it is applied to an asymmetric resonator filters.

A. Asymmetric Three Resonator Passband Filter

This filter is of Chebyshev type and it has return loss more than 20dB in the passband and rejection in the higher stopband better than -15dB. The transmission coefficient zero is placed on normalized frequency $\omega_p = 1.6$. The coupling scheme of the filter is shown on Fig.2.

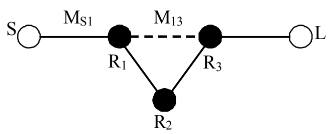

Fig.2. Coupling scheme of an asymmetric three pole filter

The reflection and transmission zeroes calculated and summarized in Table 1.

Table 1. Poles and zeros of asymmetric three resonator filter

№	Reflection zeros	Transmission zeros	Poles
1	*-j0.8061*	*j1.56*	*-0.9542-1.4447i*
2	*j0.9257*	*infinite*	*-1.1781+ 0.5923i*
3	*j0.2430*	*infinite*	*-0.2239 + 1.2150i*

The initial point for the coupling matrix elements for the optimization procedure is to set the values of the all pole three resonator Chebyshev filter $M_{S1} = 1.0825$, $M_{12} = M_{23} = 1.0303$.

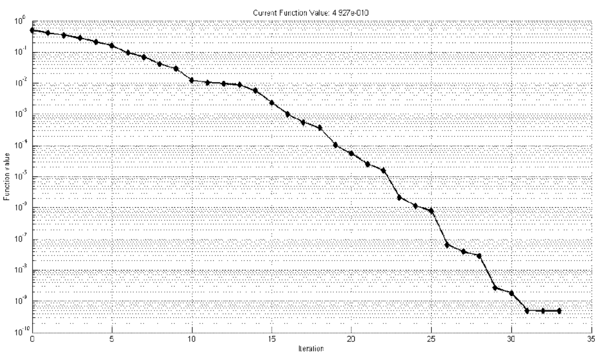

Fig.3 Cost function value for asymmetric three resonator filter

All self coupling and cross coupling coefficients are set to zero. The number of the independent values of the coupling matrix is 7.

After 33 iterations for the optimization coefficient, the procedure converges. The values of the cost function vs the number of iterations is shown on Fig.3. The initial value of the cost function is 0.335 and the end value is $4.927.10^{-10}$. The optimization process stopped because of reaching local minimum of the cost function (7). The final coupling matrix is:

$$M = \begin{bmatrix} 0 & 1.0866 & 0 & 0 & 0 \\ 1.0866 & 0.1741 & 0.8076 & 0.7679 & 0 \\ 0 & 0.8076 & -0.7108 & 0.8076 & 0 \\ 0 & 0.7679 & 0.8076 & 0.1741 & 1.0866 \\ 0 & 0 & 0 & 1.0866 & 0 \end{bmatrix}. \quad (8)$$

The frequency response of the designed filter is shown on Fig.4. It is calculated by the derived in the optimization process coupling matrix (8) and (6). It is clearly seen that the normalized cut off frequency is $\omega_c = \pm 1$, while the transmission zero frequency is $\omega_p = -1.56$. The maximum value of the return loss is with the prescribed value of *-20dB.*

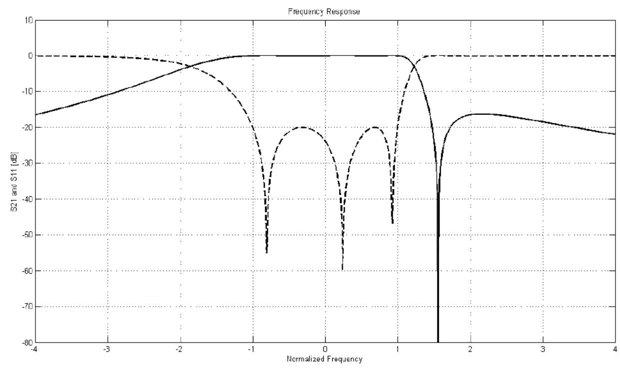

Fig.4 Frequency response of three resonator filter with asymmetric response. Solid line-S_{21}, dashed line- S_{11}

B. Asymmetric Five Resonator Passband Filter

The five resonator filter is formed by two cascaded trisections, each one performing one transmission zero. The filter is Chebyshev type and it has maximum return loss of -20dB. The transmission zeroes are placed on -2.3 and 1.6.

The coupling scheme of the filter is shown on Fig.5.

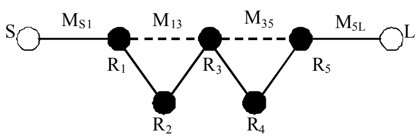

Fig.5. Coupling scheme of an asymmetric five pole filter

Both trisections share a common resonator in order to reduce the filter order. The roots of the polynomials in the numerator and denominator in (1) are shown in Table2.

Table 2. Poles and zeros of asymmetric five resonator filter

№	Reflection zeros	Transmission zeros	Poles
1	*-j0.9522*	*j1.6*	*-0.1838 - 1.1266j*
2	*-j0.5808*	*-j2.3*	*-0.5407 - 0.7163j*
3	*j0.0481*	*infinite*	*-0.6933 + 0.0655j*
4	*j0.6452*	*infinite*	*-0.1377 + 1.1036j*
5	*j0.9619*	*infinite*	*-0.4742 + 0.7961j*

The starting point for the optimization process is based on the Chebyshev coupling matrix elements $M_{S1} = 1.0137$,

$M_{12} = M_{45} = 0.8653$, $M_{23} = M_{34} = 0.6357$. The number of the independent values of the coupling matrix is 12. The optimization process converges very fast in 77 iterations of the optimizer with end cost function value $2.5953.10^{-12}$. The trials with setting of all main couplings to ones and all cross and self-couplings to zero as an initial point lead the optimizer to a local minimum with cost function value of order 1.10^{-3}. Fig. 5 shows the cost function value with respect to the iterations.

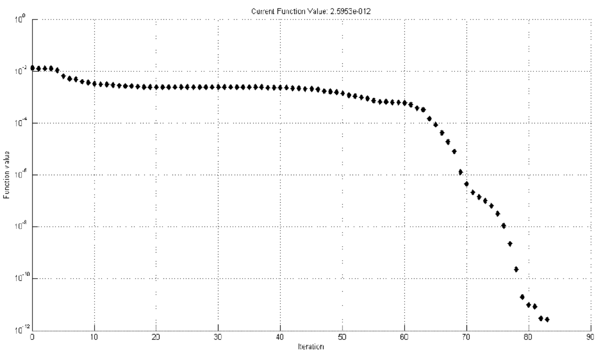

Fig.5 Cost function value for asymmetric five resonator filter

The coupling matrix derived in the optimization process is:

$$M = \begin{bmatrix} 0 & 1.0081 & 0 & 0 & 0 & 0 & 0 \\ 1.0081 & 0.0097 & 0.7590 & 0.3904 & 0 & 0 & 0 \\ 0 & 0.7590 & -0.5480 & 0.5411 & 0 & 0 & 0 \\ 0 & 0.3904 & 0.5411 & 0.0264 & 0.5952 & -0.2527 & 0 \\ 0 & 0 & 0 & 0.5952 & 0.3801 & 0.8152 & 0 \\ 0 & 0 & 0 & -0.2527 & 0.8152 & 0.0097 & 1.0081 \\ 0 & 0 & 0 & 0 & 0 & 1.0081 & 0 \end{bmatrix}$$

The corresponding frequency response calculated by the coupling matrix and Eq.(6) is shown on Fig.6.

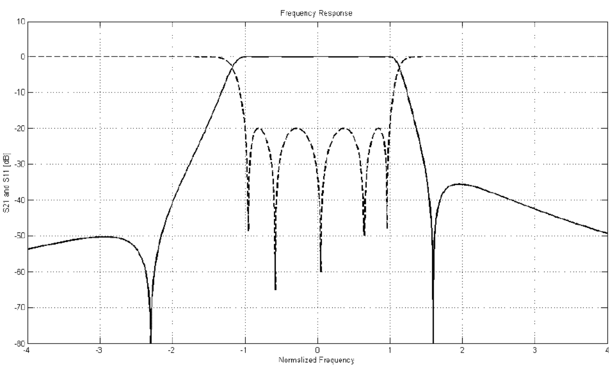

Fig.6 Frequency response of five resonator filter with asymmetric response. Solid line-S_{21}, dashed line- S_{11}

As it is clearly seen from Fig.6, the transmission zeros are placed on the prescribed values of -2.3 and 1.6. The maximum value of the reflection coefficient is -20dB.

Both presented examples show fast convergence of the cost function to a local minimum. In both cases this local minimum is found to be a global minimum corresponding to general Chebyshev filter. In both cases the starting point for the optimization process was the coupling matrix of classic Chebyshev filter. All the cross couplings and self-couplings were set to zero. During the tests of the optimization function, was experienced with another set of starting points. All the main couplings were set to ones and the rest of the couplings were set to zero. In this case the optimization process went to a minimum that was found to be local, but not a global one.

IV. Conclusion

This paper presents optimization method for synthesis of microwave filters with arbitrary topology. The method uses Nelder-Mead local optimizer for coupling matrix determination. The synthesis procedure converges very fast as for an initial point is used a vector based on the Chebyshev all pole filter for the same degree of the filter. To validate the proposed synthesis method two resonant filters are designed with asymmetrical responses. Both presented examples show fast convergence of the cost function to a local minimum. In both cases this local minimum is found to be a global minimum corresponding to general Chebyshev filter. The frequency responses from the synthesis procedure are within the expectations and found to be consistent with the theoretical responses and given filter specifications.

References

[1] A.E. Atia and A.E. Williams. "Narrow-Bandpass Waveguide Filters." 1972 Trans. on Microwave Theory and Techniques 20.4 (Apr. 1972 [T-MTT]): 258-265.

[2] Cameron, R., Advanced Coupling Matrix Synthesis Techniques for Microwave Filters, IEEE Trans on MTT-50, Jan.2003, pp.1-10.

[3] Cameron, R.J., General Coupling Matrix Synthesis Methods for Chebyshev Filtering Functions, IEEE Trans. on MTT, April 1999, pp.433-442.

[4] Rhodes, J.D., The Design and Synthesis of a Class of Microwave Bandpass Linear Phase Filters, IEEE Trans. on MTT, 1969 pp.189-204.

[5] Macchiarella, G, An Analytical Technique for the Synthesis of Cascaded N-Tuplets Cross-Coupled Resonators Microwave Filters Using Matrix Rotations, IEEE Trans. on MTT, May 2005, pp.1693-1698.

[6] G. Pfitzenmaier, "Synthesis and Realization of Narrow-band Canonical Microwave Bandpass Filters Exhibiting Linear Phase and Transmission Zeros," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp.1300–1311, Sep. 1982.

[7] Atia W.A., K.A. Zaki and A.E. Atia. "Synthesis of general topology multiple coupled resonator filters by optimization." 1998 MTT-S International Microwave Symposium Digest 98.2 (1998Vol. II MWSYM]): 821-824.

[8] Amari, S., Synthesis of Cross-Coupled Resonator Filters Using an Analytical Gradient-Based Optimization Technique, IEEE Trans on MTT Sept. 2000, pp.1559-1564.