Analysis of a serial circuit with two memristors and voltage source at sine and impulse regime

Valeri M. Mladenov
Dept. Theoretical Electrical Engineering
Technical University
Sofia, Bulgaria
valerim@tu-sofia.bg

Stoyan M. Kirilov
Dept. Theoretical Electrical Engineering
Technical University
Sofia, Bulgaria
s_kirilov@tu-sofia.bg

Abstract — **In the present paper the structure and principle of action of Williams's memristor are described. There are presented its basic parameters and the basic physical dependencies are confirmed. The analysis described here considers linear drift model of Williams's memristor. A SIMULINK model of circuit with two memristors is build with obtained formulae and Kirchhoff's voltage law. The basic results by the simulations organized in MATLAB and SIMULINK environment are given in graphical form. These results are associated with distortions of plateaus of impulses at different ratios between resistances of "opened" and "closed" states of Williams's memristor - R_{OFF} and R_{ON}. There are given also interpreting of results, which confirms that a memristor with high ratio *r* is better than a memristor with small value of *r*. In conclusion there are given basic deductions and perspectives for future applications of memristor circuits.**

I. INTRODUCTION

The memristor is proposed as a new basic two-port electrical element by the American scientist Leon Chua in 1971 [1]. The name of the element comes from the English combinations of words "memory resistor" and in translation means "resistor with memory". This is due to the fact that the memristor has a property of memorizing the final value of its resistance at interrupting the electromotive force sources. Initially it had not exist as an independent element, but it had been predicted and modeled by suitable combination of a nonlinear resistor and four-polar circuit based on dependent current source and dependent voltage source [1]. There has been realized functional nonlinear relationship between electrical charge $q(t)$ and magnetic flux $\psi(t)$. In this way there had been done completeness of relationships between the four basic electrical quantities. In fact, magnetic flux is not obtained in way, similar to producing the physical magnetic flux of the inductor. Here it is called "flux linkage" and it is integral of the voltage drop across the memristor with respect to time [2, 3, 4]. Because of the impossibility of the physical realization of the memristor, it had been forgotten for about 37 years. In 2008 a team of "Hewlett Packard" in the lead of Stanley Williams invents a physical prototype of memristor [10]. This prototype had been made of titanium dioxide [4, 5].

It is also called "Williams's memristor" and it will be object of examination in present paper. In the literature are encountered several types of memristors. There are polymeric memristors, manganese memristor systems, spintronic memristors and other types of memristors [7].

The resistance R participates as a coefficient of proportionality in the relationship between voltage u and current i. At the ideal capacitive element voltage and charge q are associated with the capacitance C. In the inductor magnetic flux ψ is expressed by current i and the coefficient of inductance L. If these elements are linear the presented relationships are sufficient for full description of the dependencies between electrical quantities. But if the elements are nonlinear there exists necessity of a new element which can associate charge and magnetic flux. This element is the memristor with coefficient of proportionality M. This coefficient has the same dimension as the dimension of the resistance – Ω [1, 4, 9] and is also called memristance. The dependencies between current i and charge q, and between voltage u and magnetic flux ψ are described with differentiating with respect to time t with accordance of its definition expressions.

The memristor proposed by the team of Hewlett-Packard is obtained by using of nanotechnology of crossing titanium rims or platinum rims with dimensions of nanometers. That technology uses inner layer of titanium dioxide [5, 9, 10].

The schematic image of memristor described here is given at fig. 1 [5]. A layer of titanium dioxide is placed between platinum rims. This layer consists of two sub-layers. The first sub-layer is doped by oxygen vacancies and has approximate chemical formula TiO_{2-x}, where $x \approx 0.03$. This layer has comparatively small specific electrical resistance. The other sub-layer consists of pure titanium dioxide and has high specific electrical resistance.

The width of doped layer is denoted with w and the width of whole memristor is signified by D. In the considered case the initial width of doped layer is w=1 *nm*. The width of whole memristor is D=10 *nm*. The fundamental parameter of semiconductor zone is the average value of ionic mobility of the oxygen vacancies μ_v. In the considered case its value is μ_v=1.10^{-14} $m^2/(V.s)$.

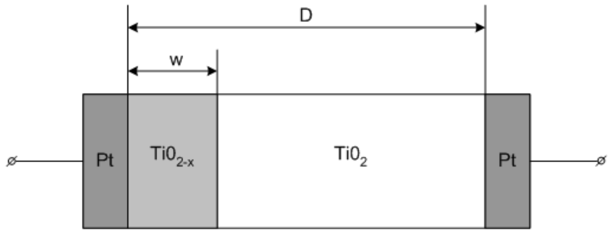

Figure 1. Scheme of titanium-dioxide memristor

At applying voltage across the memristor a movement of oxygen vacancies starts. When the potential of the electrode connected to the doped layer is positive, the oxygen vacancies begin to move to the other electrode. The charges pass in the sub-layer of pure titanium dioxide. Then the whole electrical resistance decreases. If at a fixed moment the voltage source is switched off, current stops to flow. But the charges of oxygen vacancies remain enveloped in the second sub-layer. In this way the memristor retains the last value of its resistance before interrupting the voltage source. If a voltage is applied again in the same polarity till full saturation of the volume of memristor with charges, memristor reaches to "fully closed" state. Then the memristor has minimal resistance R_{ON}. If the voltage is applied but in opposite direction, the oxygen vacancies return back partially in the first sub-layer. Then the whole resistance of the element increases. Hence the electric resistance of the memristor decreases or increases in dependence of polarity of applied voltage and with respect to the duration of interval in which voltage is applied. At the full moving of the charges in the initially doped sub-layer, the memristor becomes "fully opened". Then it has maximal resistance. When the boundary between the sub-layers is in the last left position or last right position, the memristor loose its nonlinearity. At different prototypes of Williams's memristor the ionic mobility and the width of the semiconductor zone remains permanent. For the present they have unique practical realized values. The resistances R_{ON} and R_{OFF} can have different values at the different prototypes of Williams's memristor [4, 6].

Essentially Williams's memristor is nonlinear element. Because of that memristor circuits are also nonlinear circuits. In theoretical point of view the distribution of currents and voltages in memristor circuits with two or more memristors with different parameters is interesting. Investigation of spectral structure of current in the memristor circuit is attractive task. This examination gives us an idea of extent of nonlinearity of the memristor circuit. The deduction of physical relationships at Williams's memristor is object of examination in the present publication. On this basis we can build SIMULINK model of memristor circuit with two memristors and voltage source for simulation in MATLAB & SIMULINK environment [8].

The paper is organized as follows. In section 2 there is presented the problem statement – deductions of physical dependencies of Williams's semiconductor memristor and building of SIMULINK model of circuit with two memristors with different parameters. Section 3 presents results of the simulation of circuit with two memristors and their interpretation. Section 4 concludes the paper with confirmation of the importance of memristors with high ratio *r* and the future applications of the Williams's memristor.

II. Problem Statement

Now we will deduce the physical dependencies between quantities in a simple memristor circuit with one memristor and a voltage source with voltage *e* (fig. 2). The analysis is associated with linear drift model of Williams's memristor. In the literature [2] are presented more accurate models of memristor where a boundary condition-based approaches are described.

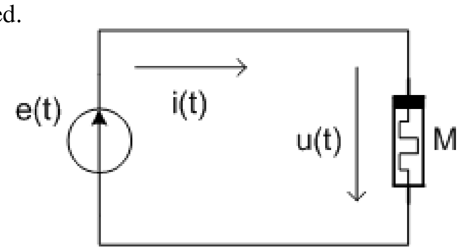

Figure 2. A circuit with one memristor and ideal voltage source

The flux linkage is presented with formula (1):

$$\psi(t) = \int_0^t u(t)dt \qquad (1)$$

The model used here for describing Williams's memristor is presented in details in [2, 4]. The basic dependencies and data obtained from Hewlett-Packard's team in 2008 are formulated in [4] and [8].

The next formula (2) is deduced with admission that doped and undoped layers are connected in series:

$$u(t) = \left(R_{ON} \frac{w(t)}{D} + R_{OFF} \left(1 - \frac{w(t)}{D} \right) \right) i(t) \qquad (2)$$

The rate of moving of boundary between the zones is presented as a derivative of width of doped region with respect to time and it is given with formula (3):

$$v_0(t) = \frac{dw(t)}{dt} = \mu_v \frac{R_{ON}}{D} i(t) \qquad (3)$$

After integration of formula (3) with respect to time we obtain (4):

$$w(t) = \mu_v \frac{R_{ON}}{D} q(t) \qquad (4)$$

The charge $q(t)$ is substituted in formula (4) with quantity of charge Q_D necessary for moving the boundary between the zones in the last right position. Then we obtain formula (5):

$$Q_D = \frac{D^2}{\mu_v R_{ON}} \qquad (5)$$

If we denote the ratio of the width of doped layer $w(t)$ in a given moment to the whole width of the memristor D with $x(t)$, then we obtain:

$$x(t) = \frac{w(t)}{D} = \frac{q(t)}{Q_D} \tag{6}$$

The quantity $x(t)$ is also known as a normalized width of the doped region.

After substituting of (4) in (2) and after additional processing we obtain:

$$M(q) = R_{OFF}\left(1 - \frac{\mu_v R_{ON}}{D^2} q(t)\right) = R_{OFF}\left(1 - \frac{q(t)}{Q_D}\right) \tag{7}$$

The quantity M_0 represents the value of the coefficient M at the initial "opened" state of the memristor. The ratio of R_{OFF} to R_{ON} is: $r = R_{OFF} / R_{ON}$.

After we know the approximate equality $\Delta R = R_{OFF} - R_{ON} \approx R_{OFF} \approx M_0$, we transform formula (3) and we obtain the next dependence between the charge $q(t)$ and the flux linkage $\psi(t)$:

$$q(t) = Q_D\left(1 - \sqrt{1 - \frac{2}{Q_D R_{OFF}}\psi(t)}\right) \tag{8}$$

The relationship between the quantities $x(t)$ and $\psi(t)$ is obtained with analogous transformation. The dependence obtained here is presented with the next formula (9):

$$x(t) = 1 - \sqrt{1 - \frac{2\mu_D}{rD^2}\psi(t)} \tag{9}$$

The current in the circuit is obtained with differentiating of equation (8):

$$i(t) = \frac{u(t)}{R_{OFF}\sqrt{1 - \frac{2\mu_D}{rD^2}\int u(t)dt}} \tag{10}$$

On the next figure (3) there is presented examined circuit with two memristors in series and ideal sine voltage source.

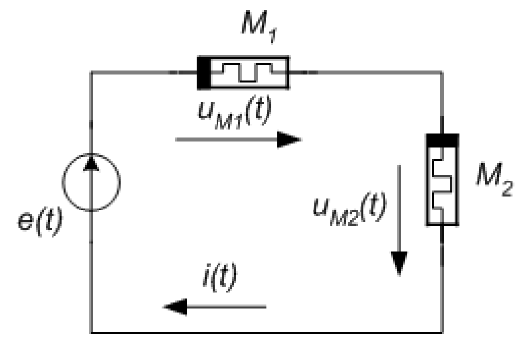

Figure 3. Circuit with two memristors in series and ideal sine voltage source

The memristors M_1 and M_2 have different parameters R_{ON} and R_{OFF}. The first memristor element has R_{ON1} = 200 *Ω* and R_{OFF1} = 60 *k Ω*. The second memristor element has resistances R_{ON2} = 100 *Ω* and R_{OFF2} = 16 *k Ω*. The other parameters of the memristors are the same as we noted above.

We construct the SIMULINK model of the circuit of fig. 5 as using formula (10) and the Kirchhoff's voltage law. We use also the structure elements of the MATLAB & SIMULINK libraries [6, 8]. After finishing the simulation we plot the graphical dependencies using the work memory of MATLAB. The model is given on fig. 4.

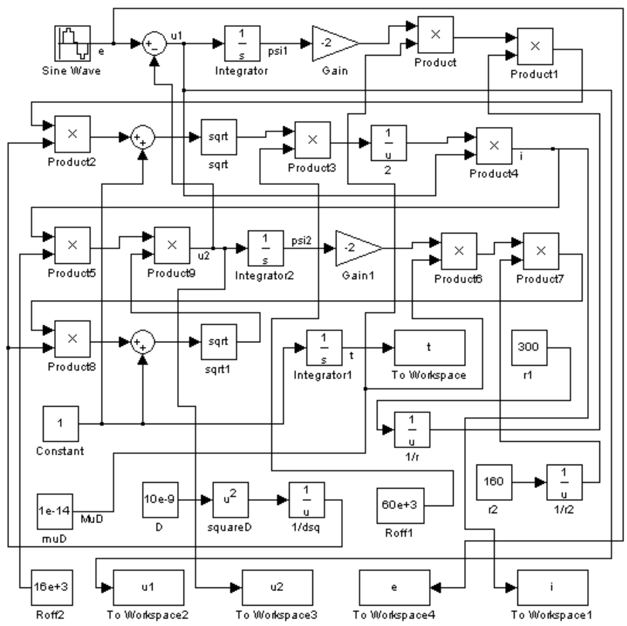

Figure 4. SIMULINK model of circuit with two memristors

In addition we use MATLAB code for fast Fourier transformation (FFT). Then we create the spectral structure of the current in the circuit. The same SIMULINK model is used with impulse voltage source in the investigating the circuit at impulse regime.

III. Results From Simulations of The Memristor Circuit

The circuit is examined at circular frequency $\omega = 4\ rad/s$ and electromotive force (EMF) e = 2,7 *V*. The curves of the electrical quantities EMF - e, the voltage drop u_1, the voltage drop u_2 and the current i in dependence of time are presented on fig. 5. The curve of electromotive force is ideal sine wave. It is obvious that the current and voltage drops over the memristors are non-sinusoidal. The voltage drop u_1 is higher than voltage drop u_2. That is due to the fact that the resistances of the first memristor *M1* are higher than the resistances of the second memristor *M2* at every moment of the examination of the circuit. The first memristor is more nonlinear than the second memristor. That is due to the fact that the deviation of the resistance of *M1* is from R_{ON1} = 200 *Ω* to R_{OFF1} = 60 *k Ω* but the deviation of the resistance of second memristor *M2* is only from R_{ON2} = 100 Ω to R_{OFF2} = 16 $k\ \Omega$. For estimation of extent of nonlinearity of the whole memristor circuit spectral analysis of the current is accomplished.

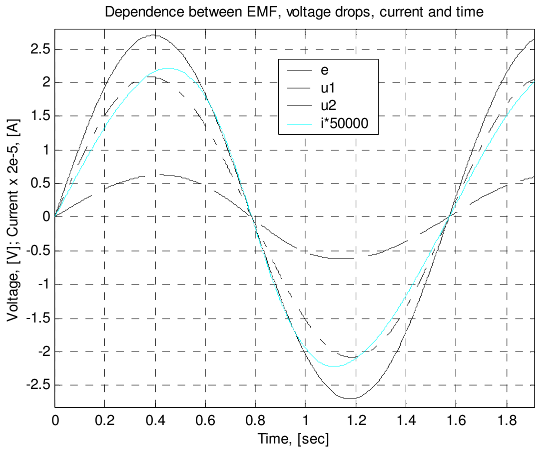

Figure 5. The EMF, the voltage drops over the memristors and the current in the circuit as functions of time at $\omega = 4\ rad/s$ and $u_m = 2,7\ V$

The spectral structure of the current is presented on fig. 6. It is obvious that only the second and the third harmonics are with significant magnitude. The other magnitudes of the harmonics are very small and might be ignored. Hence the memristor scheme is slightly nonlinear circuit.

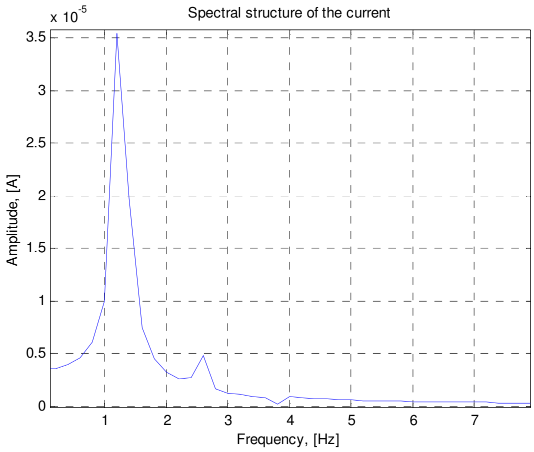

Figure 6. Spectral structure of the current

The volt-ampere characteristics of the memristors and of the whole circuit are given on the fig. 7. It is clear that both the characteristics of the memristors are "pinched hysteresis loops". The characteristic of the whole circuit have similar form. Volt-ampere characteristic of the first memristor M_1 covers more wide "area" than volt-ampere characteristics of the second memristor M_2. Volt-ampere characteristic of the whole memristor circuit covers most wide "area" over the coordinate system. Any of the characteristics has regions in which voltage decreases and current increases. In those regions the differential resistance is negative. The static resistances of both the memristors is always positive.

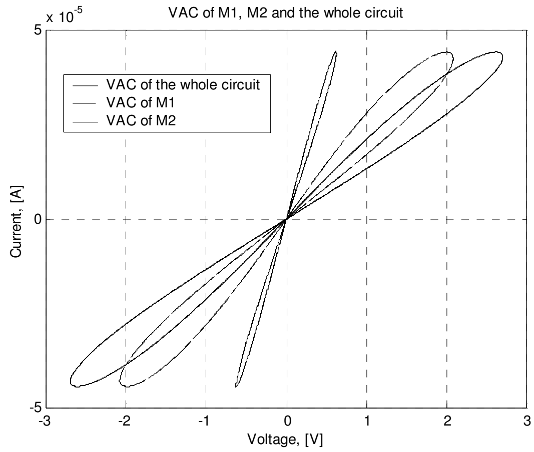

Figure 7. Volt-ampere characteristics of the memristors and of the whole circuit at $\omega = 4\ rad/s$ and $u_m = 2,7\ V$

Now the circuit will be examined at impulse regime. We use rectangular pulses consecution with magnitude 1 V, frequency 1 Hz and coefficient of filling 50 %. The described series don't contain dc component. It is presented on fig. 8.

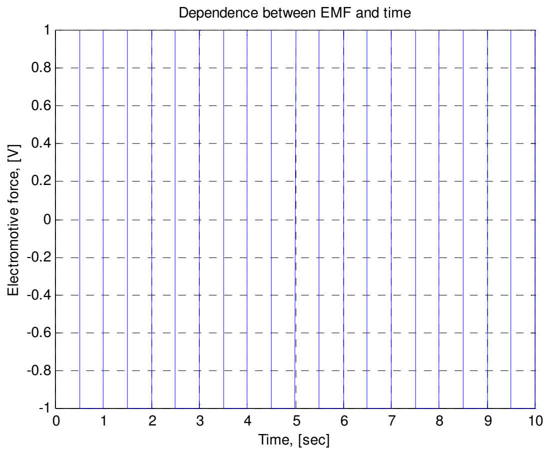

Figure 8. EMF of the impulse source as a function of the time with magnitude u_m = 1 V, frequency 1 Hz and coefficient of filling 50 %

The voltage drops u_1 and u_2 are given on fig. 9 and fig. 10, respectively. It is obvious that the plateaus of the impulses are slightly distorted with respect to the EMF impulses. The impulse series of the current in the circuit is shown on fig. 11. Its plateaus are deformed too, but the fronts of the impulses are visibly as the impulses of EMF are.

From these conclusions we may deduce that the examined circuit is slightly nonlinear. The first memristor M_1 is more nonlinear than the other memristor – M_2. This nonlinearity is more visible at low frequencies [5]. It is obvious that only the plateaus of the impulses are deformed, but the fronts which depends of the very high frequencies remains almost unaffected.

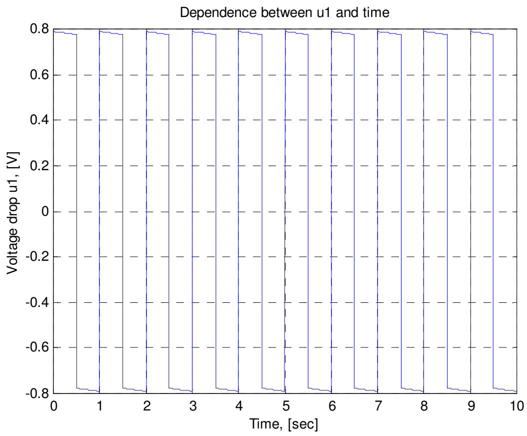

Figure 9. Impulse series of the voltage drop u_1 as a function of time at applied rectangular EMF of fig. 8

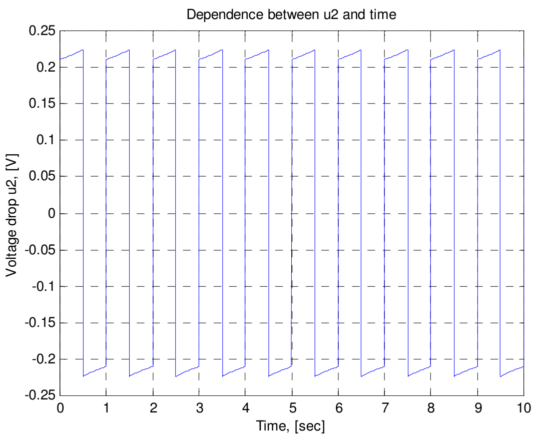

Figure 10. Impulse series of the voltage drop u_2 as a function of time at applied rectangular EMF of fig. 8

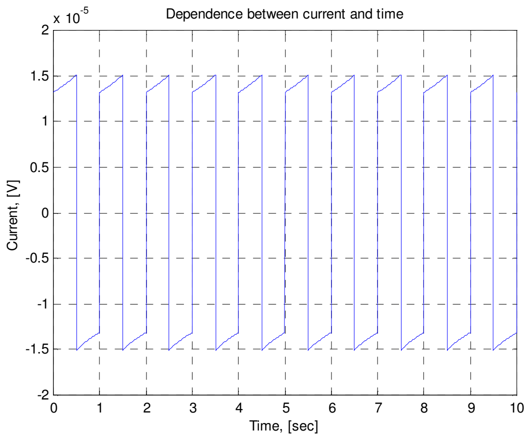

Figure 11. Impulse series of the current *i* as a function of time at applied rectangular EMF of fig. 8

A new experiment is made with examination of the two memristors separately. There is used rectangular impulse voltage source with magnitude $u_m = 1\ V$ and frequency 1 Hz.

The aim of analysis is to display how pulses are deformed and the extent of their distortion. The experiment is accomplished in MATLAB & SIMULINK environment.

The results of simulations for memristors M_1 and M_2 are shown on fig. 12 and fig. 13, respectively. Here it is made additional experiment with memristor M_3, which has many times higher resistances than the two other memristors. Its resistances are $R_{ON} = 100\ \Omega$ and $R_{OFF} = 1{,}6\ M\Omega$. The results of analyses of memristor *M3* is presented on fig. 14.

After comparison made of results shown above we could accomplish the following conclusions. When resistance R_{OFF} increases, impulses of current are less deformed with respect to their plateaus. This inference means that low frequency signals pass better through memristor with maximal ratio *r*. When ratio between resistances R_{OFF} and R_{ON} increases, impulses of current have almost the same form as EMF impulses. That result confirms the statement in [3] that achieving high ratio between R_{OFF} and R_{ON} is necessary for approving the quality and properties of Williams's memristor as almost ideal electronic switch.

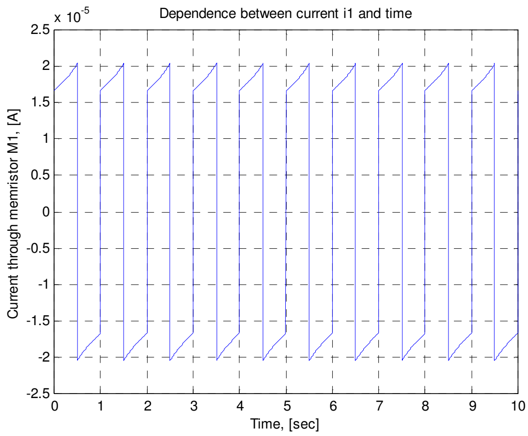

Figure 12. Impulse series of current i_1 through memristor *M1* as a function of time

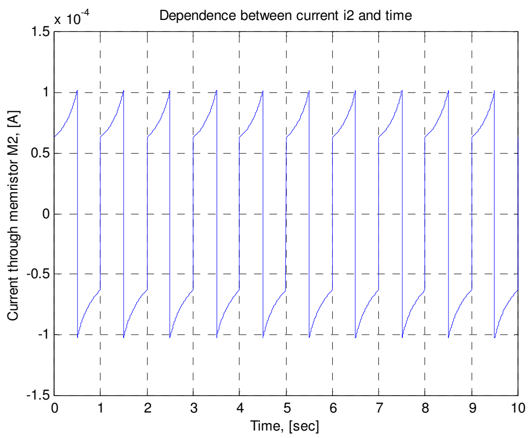

Figure 13. Impulse series of current i_1 through memristor *M2* as a function of time

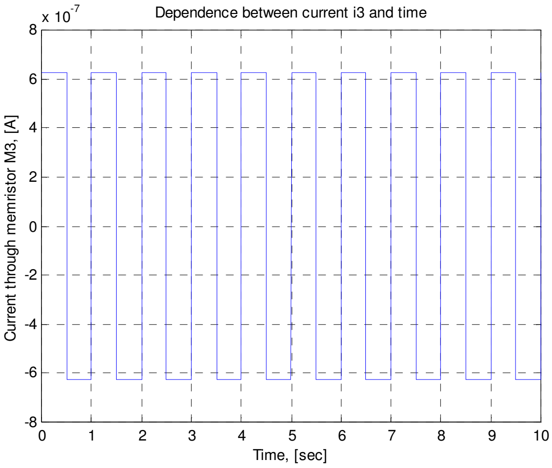

Figure 14. Impulse series of current i_3 through memristor *M3* as a function of time

IV. CONCLUSION

From the literature reference accomplished and from simulations realized here, there might be done several important conclusions. According to the year when it is invented the memristor is a new nonlinear two-port electrical element. It has an interesting property of remembering its resistance when the electrical sources are switched off. It retains its state for unlimited period. That makes it perspective element with memory. It could works in the impulse schemes with two states – opened and closed. The analysis described here considers linear drift model of Williams's memristor. In literature [2] are presented more accurate models of memristor where are considered boundary conditions of Williams's memristor. The memristor might be applied in nonvolatile computer memories. The memristor circuits are slightly nonlinear and they almost pass all the signals without distortions. In this paper there had been developed SIMULINK model for analyzing circuit with two memristors with different parameters. From the results presented here it is clear that changes of parameters R_{ON} and R_{OFF} affects on the extent of nonlinearity of Williams's memristor. The memristor pass high frequency signals without distortions. Achieving high ratio between R_{OFF} and R_{ON} approves the quality of Williams's memristor as electronic switch and memory element.

Due to memristor's small dimensions the operating memory and the hard drives of the computers might be made with very small dimensions. Their speed could be increased. The power consumption might be decreased several times. The Williams's memristor might be applied as basic structural element in the neural networks in automatics, in very high speed schemes for signal processing. The memristor also might be used in combination with CMOS transistors in hybrid memristor-transistor integrated circuits. Because of that the investigation of the memristors and memristor circuit is very important for many future applications.

ACKNOWLEDGEMENT

The scientific researches, which results are presented in this paper, are financed from the inner competition for scientific projects for help of PhD Students of Scientific-research sector of Technical University – Sofia for 2012 year.

REFERENCES

[1] Chua, L. O. Memristor – The Missing Circuit Element. IEEE Trans. on Circuit Theory, Vol. CT-18, pp. 507-519, September 1971.

[2] Corinto, F., A. Ascoli. A boundary condition-based approach to the modeling of memristor nanostructures. IEEE transactions on circuits and systems – I: Regular papers, Vol. 0, NO 0, 2012.

[3] Medha, H., S. Patil and T. Manjunath. Recent ATC's in the design of memristors. International journal of computer and electrical engineering, Vol. 2, № 3, pp. 575 – 582, June, 2010.

[4] Pazienza, G. E., J. Albo-Canals. Teaching Memristors to EE Undergraduate Students. IEEE Circuits and Systems Magazine, pp. 36-44, 22 November 2011.

[5] Strukov, D. B., G. S. Snider, D. R. Stewart, R. S. Williams. The missing memristor found. Nature, doi:10.1038/nature06932, Vol 453, pp. 80 – 83,1 May 2008.

[6] Zaplatilek, K. Memristor modeling in MATLAB & SIMULINK. Proceedings of the European Computing Conference, pp. 62 – 67.

[7] Hayes, B. The memristor. Computing science, American Scientist, Volume 99, pp.106 – 110, 2011.

[8] The Mathworks, Inc. SIMULINK – Dynamic system simulation for MATLAB, User Guide, version 3, January 1999.

[9] Tour, J. M., T. He. The fourth element. Nature, Vol. 453, pp. 42 – 43, 1 May 2008

[10] Williams, R. S. How we found the missing memristor. IEEE Spectr., vol. 45, pp. 28 – 35, 2008.