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Citation: AIP Conference Proceedings **2048**, 060030 (2018); doi: 10.1063/1.5082145
View online: https://doi.org/10.1063/1.5082145
View Table of Contents: http://aip.scitation.org/toc/apc/2048/1
Published by the American Institute of Physics

Model for Multi-Objective Optimization According to Pareto Principle of Business Interactions in a Production Cluster Composed by Participant Organizations with the Same Business Activities

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Abstract. The aim of the current study is to research and analyze a model for multi-objective optimization according to Pareto principle of business interactions between participants in a production cluster. Multi-objective optimization has non-uniform equivalent criteria. Present paper presents theoretical postulates related to finding the most appropriate business interactions in such a cluster - network of Enterprises subcontractors, production clusters, and optimal business interactions.

INTRODUCTION

The reasons to create industrial cluster are numerous but the main ones could be summarized as follows: specific technologies; united common efforts against strong competitors; access to markets; reduction of costs production etc. One of the main reason to create production cluster with participants in the same economic sector is to find the most appropriate and suitable business interactions between them which, on the one hand aims to reduce production costs, i.e. costs made by the enterprises during the process of production and on the other hand – increasing the profit from the business activity. In business interactions between cluster participants the above mentioned two criteria of optimality should be respected not only by minimizing the cluster total costs as well as maximizing the overall cluster profit, but the condition providing that each participant in the cluster should have good financial and economic results before joining the cluster also has to be met. This is a multi-objective task for finding optimal solution according to Pareto principle in non-uniform equivalent criteria. Multi-objective task for business cluster consisting of participants from different economic sectors will be a subject of another publication. Multi-objective optimization in non-uniform non-equivalent criteria will be presented in other paper.

The aim of the current study is to research and analyze a model for multi-objective optimization according to Pareto principle for business interaction between participants in a production cluster. Present paper presents theoretical postulates related to finding the most appropriate business interactions in such a cluster - network of Enterprises subcontractors, production clusters, and optimal business interactions.

Proceedings of the 44th International Conference on Applications of Mathematics in Engineering and Economics
AIP Conf. Proc. 2048, 060030-1–060030-6; https://doi.org/10.1063/1.5082145
Published by AIP Publishing. 978-0-7354-1774-8/$30.00

Subject of the study are theoretical mechanisms for finding the multicriterial optimal solution according to Pareto principle for business interaction between participants in a production cluster.

Object of the study are production enterprises, members of cluster.

The relevance of the study is justified by the importance of the research problem – through multi-objective optimization according to Pareto principle for business interactions each participant can predict its benefits or losses from membership in a potential business cluster even before its real foundation.

Task Staging

Let look at the task for production distribution in cluster with members (*r* number) from the same economic sector that produce *n* type of products. Let look of the general task, where each type of product could be produced in any enterprises from the cluster.

For each enterprises from the cluster are known: the cost price for one number of product type; production capacity (by types of operations) - b_i, where $i = 1,2,....m.$; cost norm for product production; the size of stocks for each type of resources - d_k, where $k = 1,2,....m.$ Resources are non-uniform and part of them form the production capacity of the enterprises. In the present task we accept as constraints factors: production capacity (is not possible for the enterprise to produce more than the capacity of machinery and technical equipment) and row materials needed for production process. Produced outputs from each type could not be negative. These constraints are presented in (2).

Is absolutely normal for management practice to be pursued maximum income for the business organization based on the profit maximization, generated by the realization of one product and at the same time – minimization of the costs for its production (1). The whole task acquires the following generalized type (1) – (2):

$$F = \left(\max : f_1 = \sum_{j=1}^{n} \sum_{s=1}^{r} P_j^s x_j^s, \quad \min : f_2 = \sum_{j=1}^{n} \sum_{s=1}^{r} R_j^s x_j^s \right) \tag{1}$$

Constraints:

$$S : \begin{cases} \sum_{j=1}^{n} a_{ij}^s x_j^s \le b_i^s \\ \sum_{j=1}^{n} c_{ij}^s x_j^s \le d_k^s \text{, за } \; i = 1,2,\dots\dots,m; \;\; ; \;\; s = 1,2,\dots\dots,r; \quad к = 1,2,\dots\dots,p \\ x_j^s \ge 0 \end{cases} \tag{2}$$

Where:

P_j^s - profit, obtained from the realization of one product from *j*-th type, made by *s*-th enterprise;

R_j^s - costs, made by the *s*–th enterprise for production of one product from *j*-th type;

x_j^s - quantity production from *j-th* type, produced by the *s*-th enterprise;

a_{ij}^s - machine time required for production operations from i-th type for one product from *j*-th type, produced by the *s*-th enterprise;

b_i^s - generalized maximum machine capacity from i-th type available for the *s*-th enterprise

c_{kj}^s - cost norm for raw materials from k-th type for production of one product from the *j*-th type, made by the *s*-th enterprise;

d_k^s - quantity resources from k-th type ти вид available for *s*-th enterprise.

Private cases of this task are presented in the literature [1, 2] but in only-one constrained factor – the limitation of resources neither optimization function to take into account cost production, specific for concrete type of production. Here the authors present the general model of task.

Solving this task will allow us to determine maximum revenue for the business organization based on the profit maximization, generated by the realization of one product and minimization of the costs from its production. The above presented model considers all production constraints taking into account production capacity of the enterprises without conform to the range of market demands of these products. In order to comply with the consumption requirements two new constraints are added – minimum L and maximum K consumption on the market, where the business cluster operates. The new optimization task acquires the following type (3)-(4).

$$F = \left(\max : f_1 = \sum_{j=1}^{n} \sum_{s=1}^{r} P_j^s x_j^s, \quad \min : f_2 = \sum_{j=1}^{n} \sum_{s=1}^{r} R_j^s x_j^s \right) \tag{3}$$

Constraints:

$$S': \begin{cases} \sum_{j=1}^{n} a_{ij}^s x_j^s \le b_i^s \\ \sum_{j=1}^{n} c_{ij}^s x_j^s \le d_k^s \\ L_j \le \sum_{s=1}^{r} x_j^s \le K_j \\ x_j^s \ge 0 \end{cases}, \text{ за } i=1,2,\ldots\ldots,m;\ j=1,2,\ldots\ldots,n;\ s=1,2,\ldots\ldots,r;\quad к=1,2,\ldots\ldots,p \tag{4}$$

Where:

K_j - maximum volume production from *j*-th type that cluster can realize in the market on which operates;

L_j - minimum volume production from *j*-th type that cluster can realize in the market on which operates.

Solution of the tasks (1)-(2) or (3)-(4) allows to obtain optimal distribution of production programmes of enterprises - partners in the cluster. Due to the fact that these enterprises are independent economic entities it is really necessity the benefit for each of them to be guarantee. This implies the correct choice of concept of optimality.

Optimality of multi-objective model.

According to the applied concept for optimality tasks (1)-(2) and (3)-(4) will have different solutions. Several future papers will be dedicated to the tasks solutions with the concepts of optimality of Slater and Geoffrion. Present paper introduces the task`s solution through the optimality principles of Pareto.

For the tasks (1)-(2) and (3)-(4) we say that the point $X^* \in S$ is optimal according to Pareto principle, if there is no other point X', for which

$$f_k(X') \ge f_k(X^*), \forall k \in \{1,2\} \text{ и } X = \left\{ \left\{ x_j^s \right\}_{j=1}^{n} \right\}_{s=1}^{r} \tag{5}$$

where at least one strict inequality is met for the criterion. Graphically, for two optimization criteria, as it is for tasks (1)-(2) ang (3)-(4), this is presented in Figure 1. As can be seen from Figure 1, it is necessary that the area of permissible solutions to be limited on the top and on left – which in our tasks is represented by the requirements of production capacity, resource availability and possible sales. Below the set of acceptable solutions is limited by the requirement of non-negative value of production.

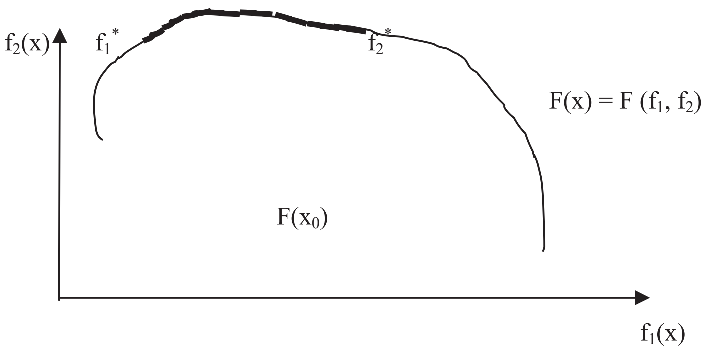

FIGURE 1. Optimization according Pareto principle in two criteria

From an economic point of view, taking into account tasks (1)-(2) and (3)-(4) Pareto optimality will give us the optimal Pareto frontier (between F1* и F2*), where all solutions will satisfy the following two conditions:

- The total cluster profit will be higher than the sum of the profits from clusters participants before its foundation;
- No one participant of the cluster has a lower profit than the one it would receive as single market player (without cluster membership).

These two conditions are really important in the phase of cluster`s foundation due to the fact that the main goal for the business organizations to become participants in the cluster is the potential profit. Through the received Pareto frontier it is ensured that at least one cluster participant will achieve better business result and not one member of cluster will generating a loss. This makes the Pareto optimality acceptable for use in tasks (1)-(2) and (3)-(4).

Solution of multi-objective model

The multi-objective model is characterized by the variety of criteria, therefore the crucial moment in its solution is the normalization of each criterion besides each of criterion $k \in K$ could be changed from 0 to 1 in its optimum point $X_k^*, k = \overline{1,2}$.

In order to execute vector optimization according Pareto principle over multi-objective model (1)-(2) and (3)-(4) we use [4].

The following steps have to be taken:

1. Optimization model. Corresponds to (1) or (3).

Consists of:

- vector criterion F(f1,f2) the components of which will be maximized – the profit of the enterprises (6);

$$\max : f_1 = \sum_{j=1}^{n} \sum_{s=1}^{r} P_j^s x_j^s \tag{6}$$

- vector criterion, the components of which will be minimized – production costs (7);

$$\min : f_2 = \sum_{j=1}^{n} \sum_{s=1}^{r} R_j^s x_j^s \tag{7}$$

2. Definition field compiled by (2) and (4)

3. We find a solution for each criterion individually, i.e. for (6) we are looking for the maximum, while for (7) we are looking for the minimum. In order to find the optimum according vector criterion for maximization we start

the solution from any non-maximum dimension $f_1^0 > 0$ to the optimal point X_1^*, and to find the criterion for minimization we have to start from any non-minimal point $f_2^0 > 0$ to the optimal point X_2^*. Then the set of all such points $f_k^0, k = \overline{1,2}$ is the worst invariable part of the criteria from the whole set of criteria *K*.

As a result of this we will eventually get the Pareto point $X_k^*, f_k^*, k = \overline{1,2}$

4. To normalize criteria f_k we take the following steps:

4.1. We define non-optimal invariable part of criterion and then we solve a task (6-7) for maximization criterion profit in the direction of minimization:

$f_1^0 = \min f_1(X)$ and satisfaction of conditions (2) or (4),

And for minimization criterion production costs in the direction of maximization:

$f_2^0 = \max f_2(X)$ and satisfaction of conditions (2) or (4).

4.2. From each criterion we remove its non-optimal invariable part:

$f_k(X) - f_k^0, k = \overline{1,2}$

As a result of which we obtain a criterion which in the transition from the non-optimal point to the optimal point lies within the limits

$$0 \le (f_1(X) - f_1^0) \le (f_1^* - f_1^0), \text{ for criterion profit} \quad (8)$$

$$0 \ge (f_2(X) - f_2^0) \ge (f_2^* - f_2^0), \text{ for criterion production costs} \quad (9)$$

4.3. After definition of the limits for changing the criteria we pass to the standard normalization of criteria:

$$\lambda_k(X) = \frac{(f_k(X) - f_k^0)}{f_k^* - f_k^0}, k = \overline{1,2}$$

Due to the (8) and (9) $\lambda_k(X) > 0$ and lies within the limits $0 \le \lambda_k(X) \le 1, k = \overline{1,2}$.

After this normalization we have not only normalized all criteria within the limits [0,1], but we have also eliminated the non-uniformity of the task, due to the fact that for all new criteria λ_k it is obviously that the maximum should be found.

5. Solution of MiniMax task (λ- task).

Formulating MiniMax task (λ- task)

In λ task, we search for $\lambda = \min \lambda_k(X), \forall X \in S$, which we maximized according to $X \in S$ and as a result we achieve MaxiMin optimization task with normalized criteria:

$$\lambda^0 = \max_X \min_k \lambda_k(X) = \left\{ \frac{(f_k(X) - f_k^0)}{f_k^* - f_k^0}, k = \overline{1,2} \right\}, \quad (10)$$

Under the defined constraints.

This could be defined as: $\lambda^0 = \max_X \lambda$, where $\lambda = \min_k \lambda_k$.

$$\lambda^0 - \frac{(f_k(X) - f_k^0)}{f_k^* - f_k^0} \le 0, k = \overline{1,2}$$

As a result of the solution of λ task we obtain optimal point and minimal relative estimate λ^0, such as:

$\lambda^0 \le \lambda_k(X^0), k = \overline{1,2}; X^0 \in S$.

Since λ^0 is the maximum low level for all relative estimates $\lambda_k(X), k = \overline{1,2}$, therefore $\{\lambda_k\}$ is the optimal Pareto point.

Un-normalized dimension of each of the criteria at this optimal point we can determined through the proportion:

$$\lambda_k(X^0) = (f_k(X^0) - f_k^0)/(f_k^* - f_k^0), k = \overline{1,2}$$

Hence

$$f_k(X^0) = f_k^0 + \lambda_k(X^0)/(f_k^* - f_k^0), k = \overline{1,2} \quad (11)$$

i.e. the dimension of any of the criteria at its optimum point is formed by the worst invariable part of $f_k^0, k = \overline{1,2}$ increased by the variable part of the criterion from the acceptable set of points *S*.

CONCLUSION

This paper presents an approach for modeling interactions in business clusters formed by participants with the same business activity. Multi-objective optimization is executed according to the Pareto principle. The optimization is performed in non-uniform but equivalent criteria. Through the proposed approach each participant in the potential business cluster could predict its benefits or losses even before its factual establishment. An opportunity is created to evaluate the usefulness for each newcomer in an existing cluster. This provides business managers with a mathematical tool that reduce erroneous business decisions in the process of interaction between participants with the same business activity.

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