# Adapted managerial mathematical model to study the functions and interactions between enterprises in high-tech cluster

Kiril P. Anguelov, and Vesela G. Kaynakchieva

Citation: AIP Conference Proceedings **1910**, 020019 (2017);
View online: https://doi.org/10.1063/1.5013956
View Table of Contents: http://aip.scitation.org/toc/apc/1910/1
Published by the American Institute of Physics

# Adapted managerial mathematical model to study the functions and interactions between enterprises in high-tech cluster

Kiril P. Anguelov[1, a)] , Vesela G. Kaynakchieva[2, b)]

[1,2] *Faculty of Management, Technical University, Sofia, 1000, Bulgaria, Sofia, 8 Kliment Ohridski Blvd.*

[a)] Corresponding author: ang@tu-sofia.bg
[b)] vodenicharova@tu-sofia.bg

**Abstract.** The aim of the current study is to research and analyze Adapted managerial mathematical model to study the functions and interactions between enterprises in high-tech cluster, and his approbation in given high-tech cluster; to create high-tech cluster, taking into account the impact of relationships between individual units in the cluster–Leading Enterprises, network of Enterprises subcontractors, economic infrastructure.

## 1. INTRODUCTION

The term "cluster" is submitted for the first time from Prof. Michael Porter. According to him the cluster can be classified as "mixture", but this term is with wider application and is consisted with international competition [7]. Deployment of an enterprise's production near suppliers or next to big markets it is not necessary, since lowering of transport costs eliminates this need [7].

Other author, who examined, study and analyze clusters, is Robert Breo. He gives definition about the cluster. Breo stated few factors, which distinguished today's industrial cluster from these clusters in 20th century [9]:

- Formal Business Plan;
- Integration and modification of numerous foreign technologies;
- Development of the work force of all levels;
- Organizational skills;
- Competition of local level;
- Efficient transport of products;
- Cooperation public stakeholders;
- Educational system (programs, training, study).

**The aim of the current study** is to research and analyze Adapted managerial mathematical model to study the functions and interactions between enterprises inhigh-tech cluster, and his approbation in given high-tech cluster; to create high-tech cluster, taking into account the impact of relationships between individual units in the cluster–Leading Enterprises, network of Enterprises subcontractors, economic infrastructure.

**Subject of the study** are theoretical mechanisms for the definition of mathematical models for study the functions and interactions between enterprises in high-tech cluster.

**Object of the study** are production enterprises, members of cluster.

**The relevance of the study** is justified by the importance of the research problem − adapted managerial mathematical model to study the functions and interactions between enterprises in high-tech cluster. Advantages, which have enterprises when they cooperated themselves and simultaneously are competitors on the domestic and

*Proceedings of the 43rd International Conference Applications of Mathematics in Engineering and Economics*
AIP Conf. Proc. 1910, 020019-1–020019-17; https://doi.org/10.1063/1.5013956
Published by AIP Publishing. 978-0-7354-1602-4/$30.00

foreign markets – stimulation of the innovation actions; exchange of knowledge, experience, ideas and good practices; increasing competitiveness of manufactured products and services..

## 2. STAGING (DESCRIPTION) THE PROBLEM

Examine the structure of Cluster "Mechatronics and Automation".

Leading enterprises – high-tech enterprises are key enterprises, exporting stocks and services outside the region, i. e. their role is basic in the high-tech cluster.

**The aim of the problem** is to create high-tech cluster, for which to choose such business incubators of ideas and technologies the Leading Enterprises and Enterprises Subcontractors, in order the cluster to function optimally, i. e. the aim is maximum overall efficiency of the high-tech cluster.

The fact that all of this must comply with the costs that will be needed and with the potential return on the invested capital must not be ignored.

The solution of the problem consists of few stages – Data calculating, unification of criteria for optimality appropriately, solving of the problems from describing mathematical models.

**The mathematical model consists of two problems.**

### 2.1. Theoretical mathematical model for creating of high-tech cluster

To solve the problem, are described two mathematical models, i. e. the mathematical model consists of two problems.

The high-tech enterprises use the development and products, created in business incubators.

- **First problem of the mathematical model**

Let leading high-tech enterprises have a number of $\boldsymbol{m-}\ \boldsymbol{K_1, K_2, \dots, K_m}$, and the goods and services offered from them have a number of $\boldsymbol{l-}\ \boldsymbol{S_1, S_2, \dots, S_l}$ . For every offered goods or services are provided costs and potential profit. How qualitative one good or service is, is influenced by different factors as invested capital, human resources, equipment, etc. The experts in the fields develop estimates all offered goods and services for each enterprise.

These assessments formed on the base some macroeconomic indicators:

- Manufacture;
- Labor market;
- International transactions.

Let these assessments are:

For enterprise $\boldsymbol{K_i, i = 1 \div m}$ , for offered goods and services $\boldsymbol{S_j, j = 1 \div l}$, assessments respectively $\boldsymbol{O_{ij}, i = 1 \div m,\ j = 1 \div l}$.

**TABLE 1**. Assessments of offered goods and services for given enterprise

| Goods and services / Enterprise | $S_1$ | $S_2$ | … | $S_l$ |
|---|---|---|---|---|
| $K_1$ | $O_{11}$ | $O_{12}$ | … | $O_{1l}$ |
| $K_2$ | $O_{21}$ | $O_{21}$ | … | $O_{2l}$ |
| … | … | … | … | … |
| $K_m$ | $O_{m1}$ | $O_{m2}$ | … | $O_{ml}$ |

$\boldsymbol{O_{ij}}$– assessment of leading $\boldsymbol{K_i}$ high-tech enterprise for offered from him $\boldsymbol{S_j}$ good or service, $\boldsymbol{i = 1 \div m,\ j = 1 \div l}$

Thus defined assessments are indicators for efficiency of each activity of the offered goods and services for each leading high-tech enterprise form the cluster.

Let potential forecast and plan costs for given leading high-tech enterprise for each from offered goods and services are $\boldsymbol{R_{ij,}\ i = 1 \div m, j = 1 \div l}$ – table 2

**TABLE 2**. Potential forecast and plan costs for offered goods and services of given leading high-tech enterprise (for 1 year)

| Goods and services / Enterprise | $S_1$ | $S_2$ | … | $S_l$ |
|---|---|---|---|---|
| $K_1$ | $R_{11}$ | $R_{12}$ | … | $R_{1l}$ |
| $K_2$ | $R_{21}$ | $R_{21}$ | … | $R_{2l}$ |
| … | … | … | … | … |
| $K_m$ | $R_{ml}$ | $R_{m2}$ | … | $R_{ml}$ |

$R_{ij}$ – forecast and plan costs of leading high-tech enterprise $K_i$ for single amount from offered from the enterprise $S_j$ goods and services (for 1 year)

Let potential forecast and plan profits for given leading high-tech enterprise for each from offered goods and services are $P_{ij,}\ i = 1 \div m, j = 1 \div l$ – table 3:

**TABLE 3**. Potential forecast and plan profits for offered goods and services of given leading high-tech enterprise (for 1 year)

| Goods and services / Enterprise | $S_1$ | $S_2$ | … | $S_l$ |
|---|---|---|---|---|
| $K_1$ | $P_{11}$ | $P_{12}$ | … | $P_{1l}$ |
| $K_2$ | $P_{21}$ | $P_{21}$ | … | $P_{2l}$ |
| … | … | … | … | … |
| $K_m$ | $P_{ml}$ | $P_{m2}$ | … | $P_{ml}$ |

$P_{ij}$– forecast and plan profits of given leading high-tech enterprise $K_i$ for single amount from offered form the enterprise $S_j$ goods and services (for 1 year)

**The mathematical model of the First problem**

The mathematical model of the task for distribution of goods and services between leading high-tech enterprises at limited general financial assets for costs and at least certain amount financial assets for profit is: Linear integer optimization problem with Boolean variables, i. e. $1 = trie, 0 = false$ [4, 1, 2, 3, 5, 6]:

$$max\left\{F(X) = \sum_{i=1}^{m}\sum_{j=1}^{l} O_{ij}\cdot x_{ij} = O_{11}\cdot x_{11} + O_{12}\cdot x_{12} + \cdots + O_{1l}\cdot x_{1l} + O_{21}\cdot x_{21} + O_{22}\cdot x_{22} + \cdots + O_{2l}\cdot x_{2l} + \cdots + O_{m1}\cdot x_{m1} + O_{m2}\cdot x_{m2} + \cdots + O_{ml}\cdot x_{ml}\right\}$$

subject to:

$$\begin{cases} \sum_{i=1}^{m}\sum_{j=1}^{l} R_{ij}\cdot x_{ij} \le R \\ \sum_{i=1}^{m}\sum_{j=1}^{l} P_{ij}\cdot x_{ij} \ge P \\ \sum_{i=1}^{m} x_{ij} \ge 1, j = 1 \div l \\ \sum_{j=1}^{l} x_{ij} \ge 1, i = 1 \div m \\ x_{ij} = \{0;1\} \end{cases} \quad (1)$$

where and $R$ is overall forecast and plan minimum guaranteed cost for offered goods and services from leading high-tech enterprises, and $P$ is overall forecast and plan maximum guaranteed profit. $x_{ij}$ is single quality of given good and services $S_j$, offered from leading high-tech enterprise $K_i$ in a period of time 1 year and

$$x_{ij} = \begin{cases} \mathbf{1}, \text{if instructedof the enterprise } \boldsymbol{K_i}\ offers\ the\ good\ and\ the\ service\ \boldsymbol{S_j}\ \boldsymbol{i = 1 \div m, j = 1 \div l} \\ \mathbf{0}, \text{if not instructed of the enterprise } \boldsymbol{K_i}\ offers\ the\ good\ and\ the\ service\ \boldsymbol{S_j}\ \boldsymbol{i = 1 \div m, j = 1 \div l} \end{cases}$$

- **Second problem of the mathematical model**

Indicators for the efficiency are seven factors, which indicate industrials cluster's influence on the competitiveness of enterprises, which are members of clusters. These indicators are as follows:

- Competitiveness of offered production $-\boldsymbol{A_1}$
- Labor productivity – $\boldsymbol{A_2}$;
- Financial results – $\boldsymbol{A_3}$ ;
- Growth of enterprises – $\boldsymbol{A_4}$
- Innovation of enterprises $-\boldsymbol{A_5}$
- Production and marketing flexibility $-\boldsymbol{A_6}$
- Adaptability of enterprises to market – $\boldsymbol{A_7}$

Each from the indicators is described quality. For the indicator $\boldsymbol{A_i, i = 1 \div 7}$, subset of elements of the indicator: $\{\boldsymbol{a_{ij}}\}, \boldsymbol{i = 1 \div 7, j = 1 \div 9}$ ($\boldsymbol{j}$ *is* **different number for different indicators** $\boldsymbol{A_i}$).

Indicators from $\boldsymbol{A_1 \div A_7}$ are evaluated degree of importance and determined by the degree of scale $\boldsymbol{1 \div 10}$ , as **1** is bad, and **10** is excellent, as the weight is assessed form results of the enterprises and expert evaluation of the researcher.

The experts in the given field determine the degree of importance, as they make an expert assessment, that transform quality characteristics of the factor **"a"** in quantitative ones.

For assessment of the indicator $\boldsymbol{A_i, i = 1 \div 7}$ , set the following table:

**TABLE 4**. Assessment of Indicator $\boldsymbol{A_i, i = 1 \div 7}$

| **Characteristics of the factor** / **Degree of importance** | $a_{i1}$ | $a_{i2}$ | $a_{i3}$ | $a_{i4}$ | $a_{i5}$ | $a_{i6}$ | $a_{i7}$ | $a_{i8}$ |
|---|---|---|---|---|---|---|---|---|
| **1÷ 10** | $b_{51}$ | $b_{52}$ | $b_{53}$ | $b_{54}$ | $b_{55}$ | $b_{56}$ | $b_{57}$ | $b_{58}$ |
| **Weight vector** | $u_{51}$ | $u_{52}$ | $u_{53}$ | $u_{54}$ | $u_{55}$ | $u_{56}$ | $u_{57}$ | $u_{58}$ |

Describe of the weight vector:

$\overrightarrow{u_i} = (u_{i1}, u_{i2}, \dots\dots\dots\dots, u_{i9})$, where $u_{ij} = \frac{b_{ij}}{\sum_{j=1}^{k} b_{ij}}$ (2)

Looking for distribution of goods and services between leading high-tech enterprises maximum overall efficiency from the implementation, as taking into account the following conditions – limited general financial assets for costs, at least certain amount financial assets for profit.

It is necessary to determine Enterprise Subcontractors. The purpose is to experts determine what will be the activities as a whole, i. e. what delivery is necessary for raw materials and materials, equipment and service. Let these activities are denoted with $\boldsymbol{d_1, d_2, \dots, d_k}$. And Enterprise Subcontractors are $\boldsymbol{n}$ of number $-\boldsymbol{F_1, F_2, \dots, F_n}$ . Activities depend on the economic infrastructure of the subcontractor – factors as human resources, technologies, capital and finance, business climate, physical infrastructure. Let these factors are $\boldsymbol{s}$ of number $-\boldsymbol{f_1, f_2, \dots, f_s}$**.** Each factor must be assessed different indicators from experts in the relevant field. Let for factor $\boldsymbol{f_t, t = 1 \div s}$, the number of indicators is $\boldsymbol{e_t}$, and indicators are $\boldsymbol{a_{te_t}, t = 1 \div s.}$ Each indicator has particular importance (meaning).

***Weight method:*** For factor $\boldsymbol{f_t, t = 1 \div s}$, with indicators $\boldsymbol{a_{te_t}, t = 1 \div s}$, relevant assessments are$\boldsymbol{o_{te_t}, t = 1 \div s}$ , and their relevant importance (weights) are $\boldsymbol{u_{te_t}, t = 1 \div s}$ , where

$$\sum_{t=1}^{s}\sum_{l=1}^{e_t} u_{tl} \tag{3}$$

It's find efficiency of each factor as follows: For factor $\boldsymbol{f_t, t = 1 \div s}$, the efficiency is

$$q_t = \left(o_{t1}, o_{t2} \ldots o_{te_t}\right) \times \begin{pmatrix} u_{t1} \\ u_{t2} \\ \ldots. \\ u_{te_t} \end{pmatrix}, t = 1 \div s \tag{4}$$

The amount of resulting efficiencies gives the efficiency of the activity. In Table 5 are presented these efficiencies for each activity for each from enterprise subcontractors:

**TABLE 5**. Efficiencies for each activity of given enterprise subcontractor

| **Activities** / **Enterprises Subcontractors** | $\boldsymbol{d_1}$ | $\boldsymbol{d_2}$ | … | $\boldsymbol{d_k}$ |
|---|---|---|---|---|
| $\boldsymbol{F_1}$ | $Q_{11}$ | $Q_{12}$ | … | $Q_{1k}$ |
| $\boldsymbol{F_2}$ | $Q_{21}$ | $Q_{21}$ | … | $Q_{2k}$ |
| … | … | … | … | … |
| $\boldsymbol{F_n}$ | $Q_{n1}$ | $Q_{n2}$ | … | $Q_{nk}$ |

In order to deploy activities between enterprises subcontractors, looking for maximum overall efficiency from activities taking into account following conditions – limited overall financial assets for costs , let certain amount financial assets for profit.

Let forecast and plan costs for each enterprise subcontractor for each of activities are $\boldsymbol{r_{bc}, b = 1 \div n, c = 1 \div k}$ , and forecast and plan profits for each enterprise subcontractor for each of activities are $\boldsymbol{p_{bc}, b = 1 \div n, c = 1 \div k}$ – Table 6.

**TABLE 6**. Forecast and plan profits for each activity of given enterprise subcontractor (for 1 year)

| **Activities** / **Enterprise Subcontractor** | $\boldsymbol{d_1}$ | $\boldsymbol{d_2}$ | … | $\boldsymbol{d_k}$ | $\boldsymbol{d_1}$ | $\boldsymbol{d_2}$ | … |
|---|---|---|---|---|---|---|---|
| $\boldsymbol{F_1}$ | $r_{11}$ | $r_{12}$ | … | $r_{1k}$ | $p_{11}$ | $p_{12}$ | … |
| $\boldsymbol{F_2}$ | $r_{21}$ | $r_{22}$ | … | $r_{2k}$ | $p_{21}$ | $p_{22}$ | … |
| … | … | … | … | … | … | … | … |
| $\boldsymbol{F_n}$ | $r_{n1}$ | $r_{n2}$ | … | $r_{nk}$ | $p_{n1}$ | $p_{n2}$ | … |

**Mathematical model of the Second problem**

The mathematical model of the problem for deployment of activities between enterprises subcontractors at limited overall financial assets for costs and at least certain amount financial assets for profit is: Linear integer optimization problem with Boolean variables, i. e. $1 = trie, 0 = false$ [4; 1; 2; 3; 5; 6]:

$$max\left\{G(Y) = \sum_{b=1}^{n}\sum_{c=1}^{k} Q_{bc} \cdot y_{bc} = Q_{11} \cdot y_{11} + Q_{12} \cdot y_{12} + \cdots + Q_{1k} \cdot y_{1k} + Q_{21} \cdot y_{21} + Q_{22} \cdot y_{22} + \cdots + Q_{2k} \cdot y_{2k} + \cdots + Q_{n1} \cdot x_{n1} + Q_{n2} \cdot x_{n2} + \cdots + O_{mk} \cdot x_{nk}\right\}$$

subject to:

$$\begin{cases} \sum_{b=1}^{n}\sum_{c=1}^{k} r_{bc} \cdot y_{bc} \leq r \\ \sum_{b=1}^{n}\sum_{c=1}^{k} p_{bc} \cdot y_{bc} \geq p \\ \sum_{b=1}^{n} y_{bc} \geq 1, c = 1 \div k \\ \sum_{c=1}^{k} y_{bc} \geq 1, b = 1 \div n \\ y_{bc} \in \{0; 1\} \end{cases} \quad (5)$$

Where $\boldsymbol{r}$ is overall forecast and plan minimum guaranteed cost for activities of enterprises subcontractors, and $\boldsymbol{p}$ is overall forecast and plan maximum guaranteed profit. $\boldsymbol{y_{ij}}$is single quality of given activities $\boldsymbol{d_c}$, implemented from enterprise subcontractor $\boldsymbol{F_b}$ in a period of time 1 year and

$$\boldsymbol{y_{ij}} = \begin{cases} \mathbf{1}, \text{if instructed of the enterprise } \boldsymbol{F_b} \text{ to perform activity } \boldsymbol{d_c}, & \boldsymbol{c = 1 \div k, b = 1 \div n} \\ \mathbf{0}, \text{if not instructed of the enterprise } \boldsymbol{F_b} \text{ to perform the activity } \boldsymbol{d_c}, & \boldsymbol{c = 1 \div k, b = 1 \div n} \end{cases}$$

## 2.2. Application of the Theoretical mathematical model in Cluster "Mechatronics and Automation"

Cluster "Mechatronics and Automation" was established on 22.12.2006 by Initiative Committee. The mechatronics and the automation suggest connection in manufacturing systems of mechanics, electronics and software. The development of the cluster is aimed at the demand for export-oriented niche market in order to realize the high-tech products and services with high value added. As products and services are realized mechatronic manufacturing systems, automation of continuous productions, software products for aggregates management, industries and factories (numerical control, MES, ERP and others).

Geographically the cluster is situated in the Town of Blagoevgrad, and it used with support of local municipal authority. Members of Cluster "Mechatronics and Automation" are high-tech enterprises, with scope of activity in the field of engineering, hardware and software[10].

The difference between ordinary and high-tech clusters is in the composition of development units, which are elements of the cluster. For them is characterized the presence of business incubator, where arise and developed high technologies. **Project of the structure Cluster "Mechatronics and Automation" (CMA) is shown in Fig. 1**

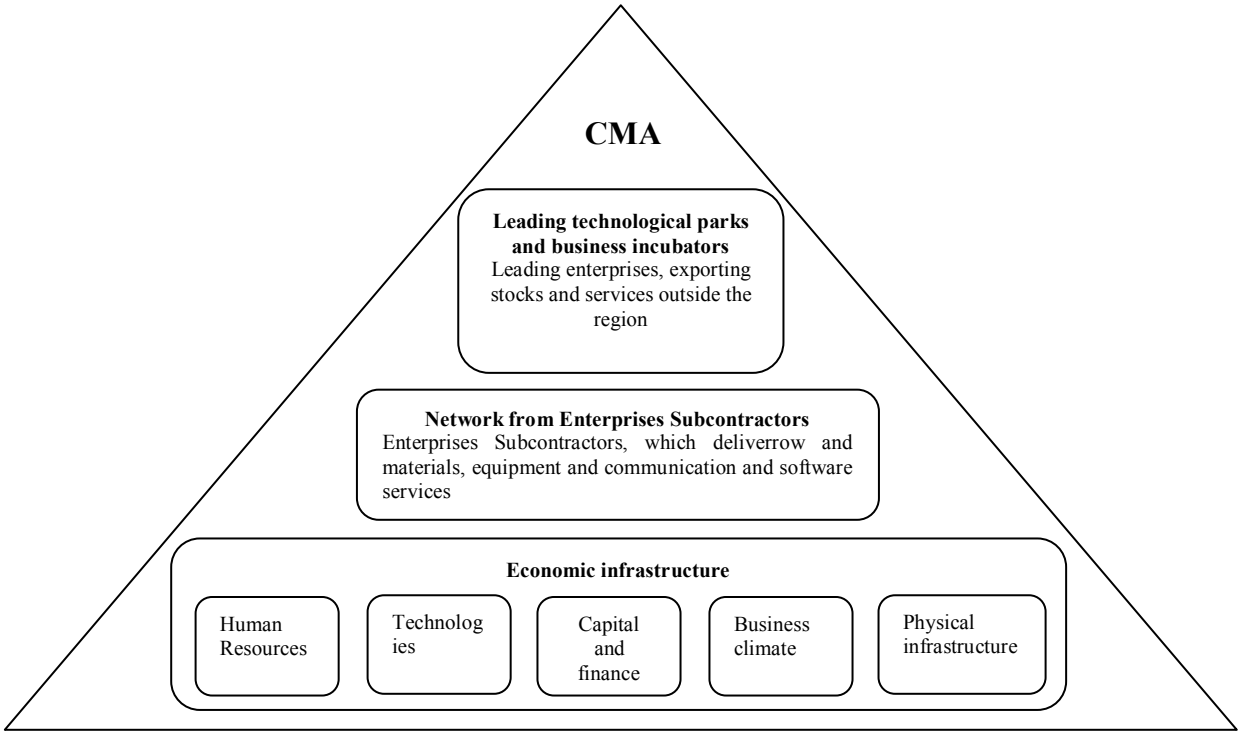


**FIGURE 1.** Project of the structure Cluster "Mechatronics and Automation"

- **First problem of the mathematical model**

Data are taken from studied enterprises, member of Cluster "Mechatronics and Automation". Studied enterprises are in the field of manufacture.

**Studied leading enterprises are following:**

$К_1$ – RITTBUL Ltd.

$К_2$ – Joint Stock Company (JSC) SAMEL 90

$К_3$ – SPESIMA Ltd.

**Studied enterprises subcontractors of leading enterprises are following:**

$F_1$–ULTRAFLEKS KORPOREYSHAN Ltd.

$F_2$ – General partnership "SEMIS – CANEV i SIE"

$F_3$ – JSC MEHATRONIKA

**Scheme of leading enterprises and their enterprises subcontractors, i. e. the leading enterprise and its enterprise subcontractor, is shown of Fig. 2.**

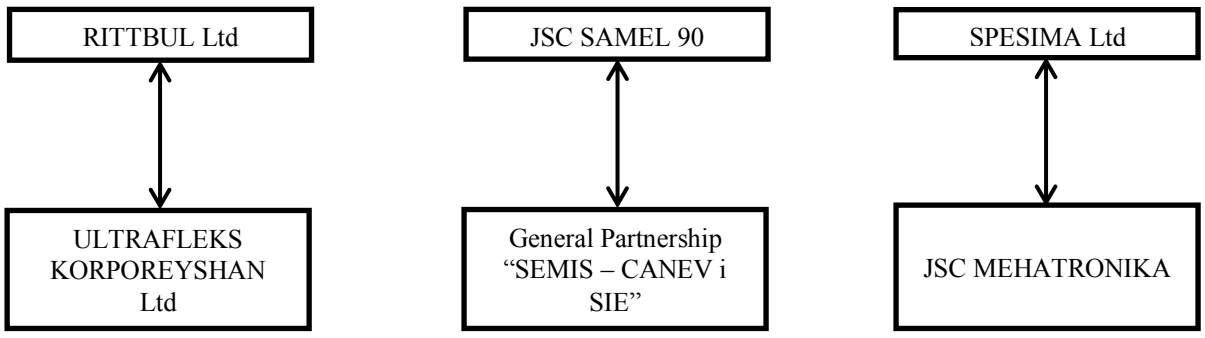


**FIGURE. 2.** Scheme of leading enterprises ant their enterprises subcontractors in Cluster "Mechatronics and Automation"

The collected data are indicator $A_i, i = 1 \div 7$ each leading high-tech enterprise, member of Cluster "Mechatronics and Automation". Assessments made by experts in potential fields in every single one, member of the cluster, and they are shown in the Table 7:

**TABLE 7**. Assessment of the enterprise $K_m, m = 1 \div 3$, for his proposed goods and services $S_n, n = 1 \div 8$

| Goods and services / Enterprise | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ | $S_6$ | $S_7$ | $S_8$ |
|---|---|---|---|---|---|---|---|---|
| $K_1$ | $O_{11} = 35$ | $O_{12} = 42$ | $O_{13} = 35$ | $O_{14} = 41$ | $O_{15} = 37$ | $O_{16} = 32$ | $O_{17} = 46$ | $O_{18} = 31$ |
| $K_2$ | $O_{21} = 39$ | $O_{22} = 45$ | $O_{23} = 33$ | $O_{24} = 36$ | $O_{25} = 31$ | $O_{26} = 36$ | $O_{27} = 34$ | $O_{28} = 43$ |
| $K_3$ | $O_{31} = 34$ | $O_{32} = 31$ | $O_{33} = 34$ | $O_{34} = 30$ | $O_{35} = 45$ | $O_{36} = 37$ | $O_{37} = 36$ | $O_{38} = 28$ |

For the distribution of goods and services between leading high-tech enterprises are used Annual Profit and Loss Accounts of the relevant enterprise.

Let potential forecast and plan costs for each leading high-tech enterprise for each proposed good and service are $R_{ij}, i = 1 \div m, j = 1 \div l$ – table 8: [thousand. lev / year] (for 1 year):

**TABLE 8**. Potential forecast and plan costs for each leading high-tech enterprise for each proposed good and service (for 1 year)

| Goods and services / Enterprise | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|
| $K_1$ | $R_{11} = 18\,236$ | $R_{12} = 29\,365$ | $R_{13} = 56\,135$ | $R_{14} = 28\,393$ |
| $K_2$ | $R_{21} = 984\,521$ | $R_{22} = 851\,230$ | $R_{23} = 632\,458$ | $R_{24} = 1\,132\,487$ |
| $K_3$ | $R_{31} = 351\,242$ | $R_{32} = 643\,252$ | $R_{33} = 139\,245$ | $R_{34} = 231\,065$ |
| **Goods and services / Enterprise** | $S_5$ | $S_6$ | $S_7$ | $S_8$ |
| $K_1$ | $R_{15} = 32\,846$ | $R_{16} = 11\,138$ | $R_{17} = 44\,369$ | $R_{18} = 38\,518$ |
| $K_2$ | $R_{25} = 0$ | $R_{26} = 343\,739$ | $R_{27} = 3\,745\,673$ | $R_{28} = 1\,598\,066$ |
| $K_3$ | $R_{35} = 225\,116$ | $R_{36} = 132\,895$ | $R_{37} = 0$ | $R_{38} = 68\,185$ |

$R_{ij}$– forecast and plan costs of the leading high-tech enterprise $K_i$ for single amount of his proposed $S_j$ good or service (for 1 year)

The size of the Costs of raw materials and external services is as follows:

For $K_1$ – RITTBUL Ltd: 259 000 lv.

For $K_2$ – Stock company SAMEL 90: 12 288 000 lv.

For $K_3$–SPESIMA Ltd: 1 791 000 lv.

Let potential forecast and plan profits for each leading high-tech enterprise for each proposed good and service are $\boldsymbol{P_{ij}, i = 1 \div m, j = 1 \div l}$ – table 9: [thousand. lev / year] (for 1 year):

**TABLE 9.** Potential forecast and plan profits for each leading high-tech enterprise for each proposed good and service (for 1 year)

| Goods and services / Enterprise | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|
| $K_1$ | $P_{11} = 25\,185$ | $P_{12} = 38\,139$ | $P_{13} = 59\,343$ | $P_{14} = 41\,139$ |
| $K_2$ | $P_{21} = 3\,435\,297$ | $P_{22} = 8\,939\,745$ | $P_{23} = 2\,143\,595$ | $P_{24} = 6\,227\,898$ |
| $K_3$ | $P_{31} = 28\,475$ | $P_{32} = 39\,145$ | $P_{33} = 48\,745$ | $P_{34} = 22\,764$ |
| **Goods and services / Enterprise** | $S_5$ | $S_6$ | $S_7$ | $S_8$ |
| $K_1$ | $P_{15} = 23\,148$ | $P_{16} = 18\,936$ | $P_{17} = 29\,365$ | $P_{18} = 28\,745$ |
| $K_2$ | $P_{25} = 0$ | $P_{26} = 3\,321\,116$ | $P_{27} = 995\,138$ | $P_{28} = 924\,211$ |
| $K_3$ | $P_{35} = 53\,147$ | $P_{36} = 65\,234$ | $P_{37} = 0$ | $P_{38} = 56\,490$ |

$\boldsymbol{P_{ij}}$ – forecast and planning profit of the leading high-tech enterprise $\boldsymbol{K_i}$ for single amount of his proposed $\boldsymbol{S_j}$ good or service (for 1 year)

The size of Accounting profit is as follows:
For $K_1$ – RITTBUL Ltd: 264 000 lv.
For $K_2$ – Stock company SAMEL 90: 25 987 000lv.
For $K_3$–SPESIMA Ltd: 314 000 lv.

- **Second problem of the mathematical model**

The collected data for relevant activities, implemented from enterprises subcontractors for economic infrastructure factors $\boldsymbol{f_1 \div f_5}$ and elements subset of individual indicator $\boldsymbol{a_{ij}, j = 1 \div 7}$. Data are processed and received efficiencies of activities of enterprises subcontractors ant they are systematized in Table 10.

**TABLE 10**. Efficiencies of enterprises subcontractors for proposed of them activities

| Activities / Enterprises Subcontractors | $d_1$ | $d_2$ | $d_3$ | $d_4$ | $d_5$ | $d_6$ | $d_7$ | $d_8$ |
|---|---|---|---|---|---|---|---|---|
| $F_1$ | $Q_{11} = 0$ | $Q_{12} = 0$ | $Q_{13} = 0$ | $Q_{14} = 116{,}04$ | $Q_{15} = 77{,}36$ | $Q_{16} = 13{,}38$ | $Q_{17} = 0$ | $Q_{18} = 58{,}02$ |
| $F_2$ | $Q_{21} = 118{,}$ | $Q_{22} = 59{,}4$ | $Q_{23} = 198$ | $Q_{24} = 138{,}6$ | $Q_{25} = 158{,}4$ | $Q_{26} = 178{,}2$ | $Q_{27} = 39{,}6$ | $Q_{28} = 79{,}2$ |
| $F_3$ | $Q_{31} = 19{,}4$ | $Q_{32} = 77{,}8$ | $Q_{33} = 97{,}3$ | $Q_{34} = 58{,}38$ | $Q_{35} = 116{,}76$ | $Q_{36} = 19{,}46$ | $Q_{37} = 136{,}22$ | $Q_{38} = 175{,}44$ |

Distribution activities between enterprises subcontractors are used Annual reports on income and expenses in relevant enterprise subcontractor.

Forecast and plan costs for each activity of given enterprise subcontractor with specific data - $\boldsymbol{r_{bc,}\ b = 1 \div m, c = 1 \div k}$ for 1 year, are shown in Table 11. Forecast and plan profits for each activity of given enterprise subcontractor with specific data - $\boldsymbol{p_{bc,}\ b = 1 \div m, c = 1 \div k}$ , for 1 year, are shown in Table 12.

**TABLE 11**. Forecast and plan costs for each activity of given enterprise subcontractor with specific data - $\boldsymbol{r_{bc,}\ b = 1 \div m, c = 1 \div k}$ (for 1 year)

| Activities / Enterprise-Subcontractor | $d_1$ | $d_2$ | $d_3$ | $d_4$ |
|---|---|---|---|---|
| $F_1$ | $r_{11} = 0$ | $r_{12} = 0$ | $r_{13} = 0$ | $r_{14} = 258\,359$ |
| $F_2$ | $r_{21} = 11\,586$ | $r_{22} = 10\,578$ | $r_{23} = 13\,432$ | $r_{24} = 10\,342$ |
| $F_3$ | $r_{31} = 795\,856$ | $r_{32} = 1\,125\,433$ | $r_{33} = 956\,793$ | $r_{34} = 899\,135$ |
| **Activities / Enterprise-Subcontractor** | $d_5$ | $d_6$ | $d_7$ | $d_8$ |
| $F_1$ | $r_{15} = 455\,786$ | $r_{16} = 525\,643$ | $r_{17} = 343\,672$ | $r_{18} = 157\,540$ |
| $F_2$ | $r_{25} = 15\,785$ | $r_{26} = 9\,581$ | $r_{27} = 11\,158$ | $r_{28} = 10\,538$ |
| $F_3$ | $r_{35} = 536\,458$ | $r_{36} = 129\,333$ | $r_{37} = 110\ 311$ | $r_{38} = 133\,681$ |

**The size of Costs of raw materials and external services of studied enterprises subcontractor is as follows:**

For $F_1$ – ULTRAFLEKS KORPOREYSHAN Ltd: 1 741 000 lv.

For $F_2$ – General Partnership "SEMIS – CANEV i SIE": 93 000 lv.

For $F_3$ – JSC MEHATRONIKA: Costs for raw materials: 3 776 000 lv.; Costs for external services: 938 000 лв.

**Total Costs for raw materials, materials and external services for JSC MEHATRONIKA: 3 776 000 + 938 000 = 4 714 000 $lv$.**

**TABLE 12**. Forecast and plan profits for each activity of given enterprise subcontractor - $\boldsymbol{p_{bc,}\ b = 1 \div m, c = 1 \div k}$ (for 1 year)

| Activities / Enterprise-Subcontractor | $d_1$ | $d_2$ | $d_3$ | $d_4$ |
|---|---|---|---|---|
| $F_1$ | $p_{11} = 0$ | $p_{12} = 0$ | $p_{13} = 0$ | $p_{14} = 12\,586$ |
| $F_2$ | $p_{21} = 3\,143$ | $r_{22} = 4\,574$ | $p_{23} = 3\,256$ | $p_{24} = 5\,135$ |
| $F_3$ | $p_{31} = 343\,259$ | $r_{32} = 258\,963$ | $p_{33} = 425\,369$ | $p_{34} = 139\,456$ |
| **Activities / Enterprise-Subcontractor** | $d_5$ | $d_6$ | $d_7$ | $d_8$ |
| $F_1$ | $p_{15} = 13\,476$ | $p_{16} = 9\,799$ | $p_{17} = 14\,786$ | $p_{18} = 21\,353$ |
| $F_2$ | $p_{25} = 1\,795$ | $p_{26} = 3\,956$ | $p_{27} = 1\,955$ | $p_{28} = 1\,186$ |
| $F_3$ | $p_{35} = 236\,432$ | $p_{36} = 195\,347$ | $p_{37} = 310\,189$ | $p_{38} = 132\,985$ |

**The size of Accounting profit of studied enterprises subcontractor is as follows:**
For $F_1$ – ULTRAFLEKS KORPOREYSHAN Ltd: 72 000lv.
For $F_2$ – General Partnership "SEMIS – CANEV i SIE":25 000 lv.
For $F_3$ – Stock Company MEHATRONIKA: 2 069 000 lv.

## 3. NUMERICAL DECISION AND FINDINGS OF CONDITIONED PROBLEMS

A suitable software product LINDO &LINGO that has ready module for solve the Linear integer optimization problem with Boolean variables, for solving both parts of the mathematical model is used.

### 3.1. Numerical decision of conditioned problems

**Decision of First problem of the mathematical model**

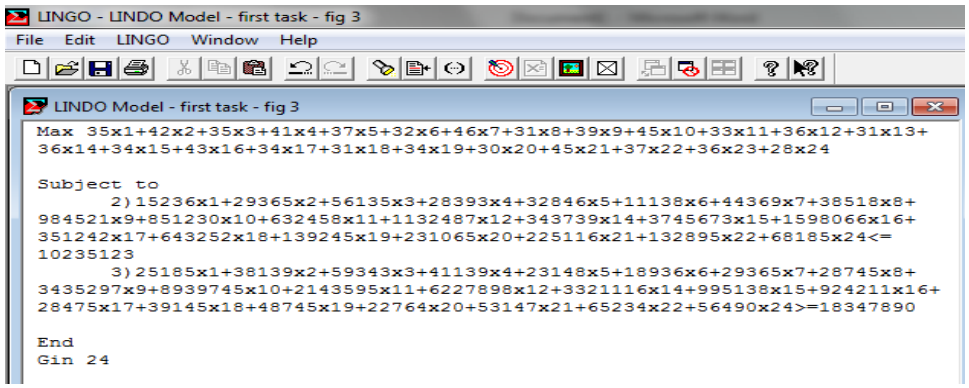


**FIGURE 3.** Module with data from the First problem of the model

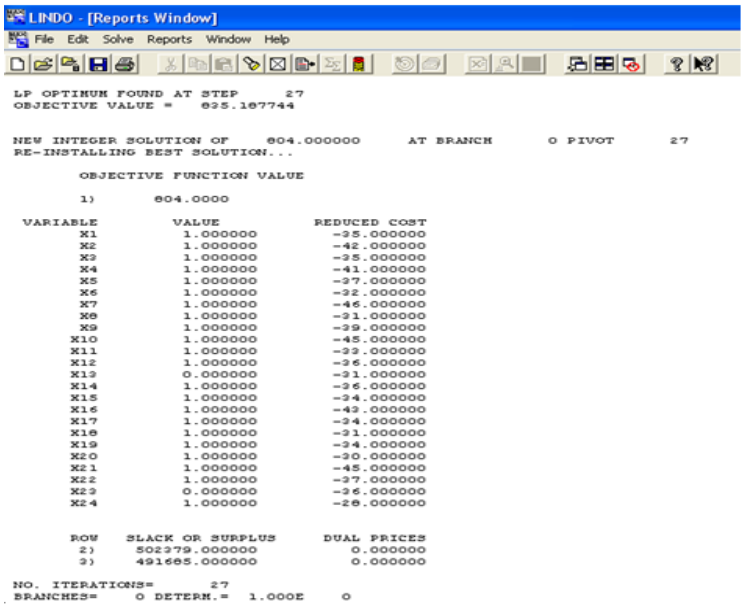


**FIGURE 4.** The decision with data from the First problem of the model

## The decision of the Second problem of the mathematical model

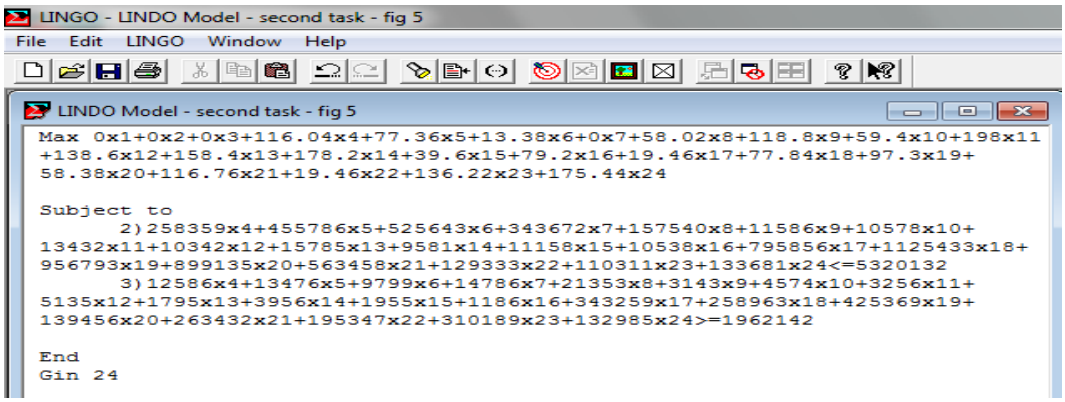


**FIGURE 5.** Module with data from the Second problem of the model

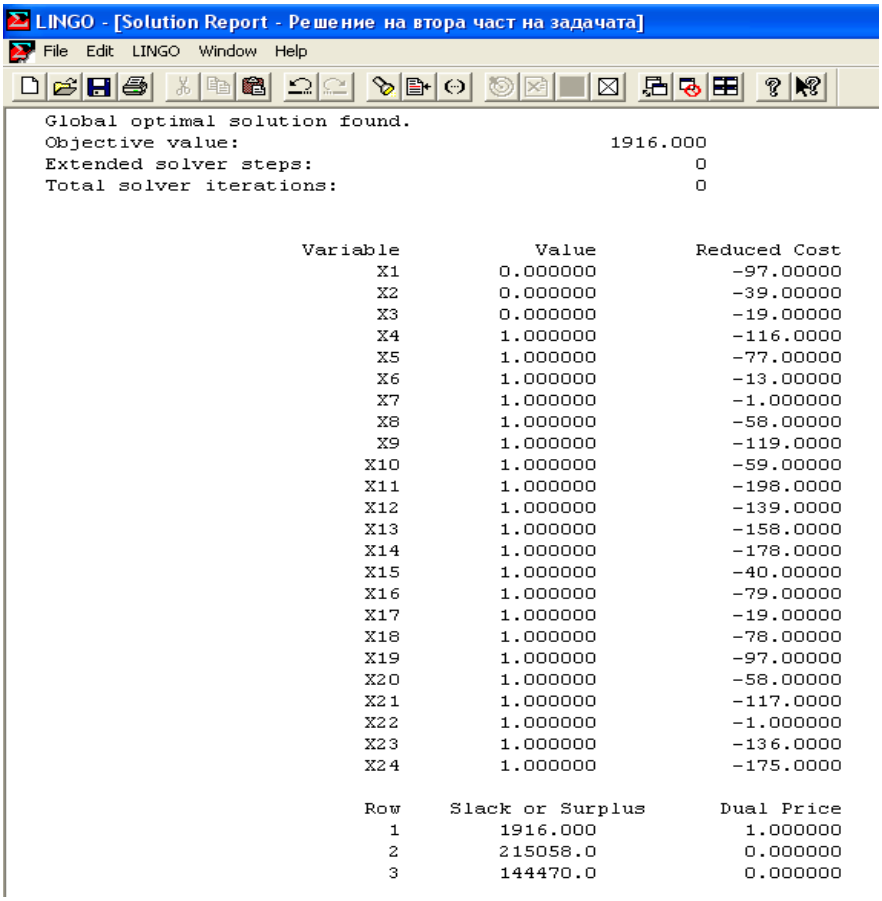


**FIGURE 6.** The decision with data from the Second problem of the mode

## 3.2. Findings from the application of the mathematical model in Cluster "Mechatronics and Automation"

**Decision of the First problem of the mathematical model**

**TABLE 13.** Goods and services assignment of leading high-tech enterprises

| Leading high-tech enterprises | Goods and services, assigned of leading high-tech enterprises | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ | $S_6$ | $S_7$ | $S_8$ |
| $K_1$ – RITTBUL Ltd | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $K_2$ – JSC SAMEL 90 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| $K_3$ – SPESIMA Ltd | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 |

**Maximum overall efficiency is 835.19, as total costs are 502 379, and the profit is 491 685.**

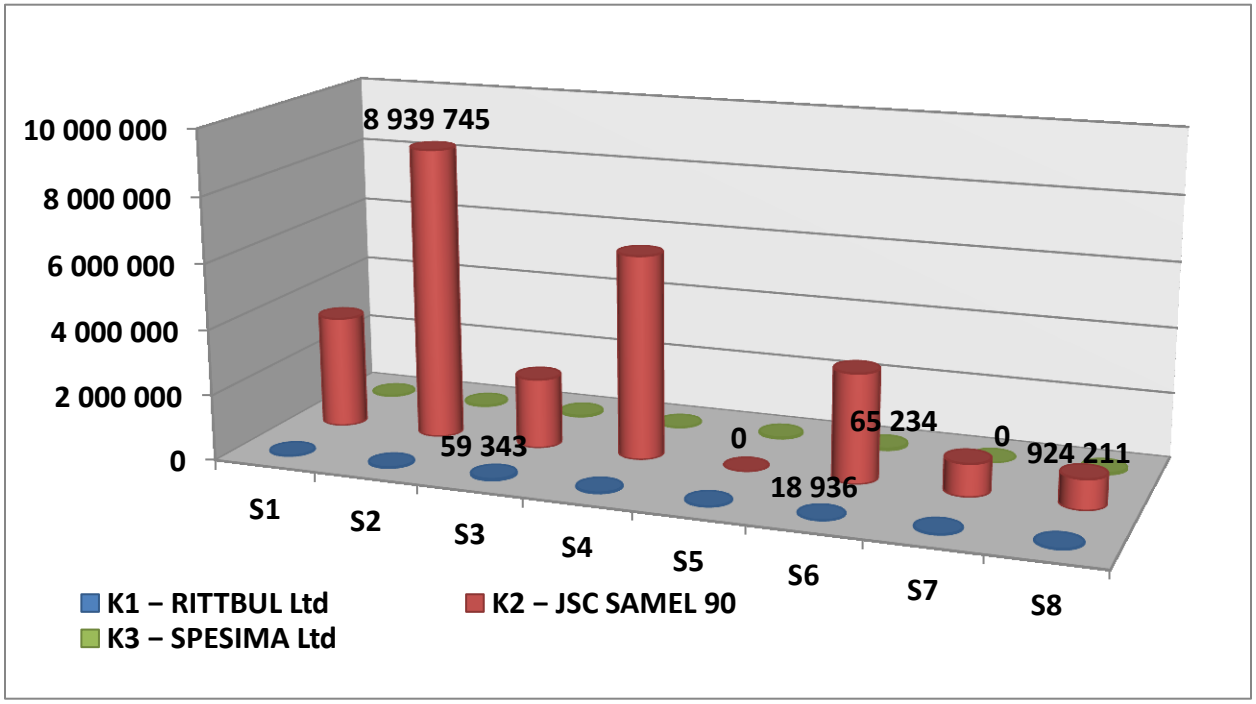


**FIGURE 7.** Distribution of cost for leading high-tech enterprises to the goods and services, which they offer (for 1 year)

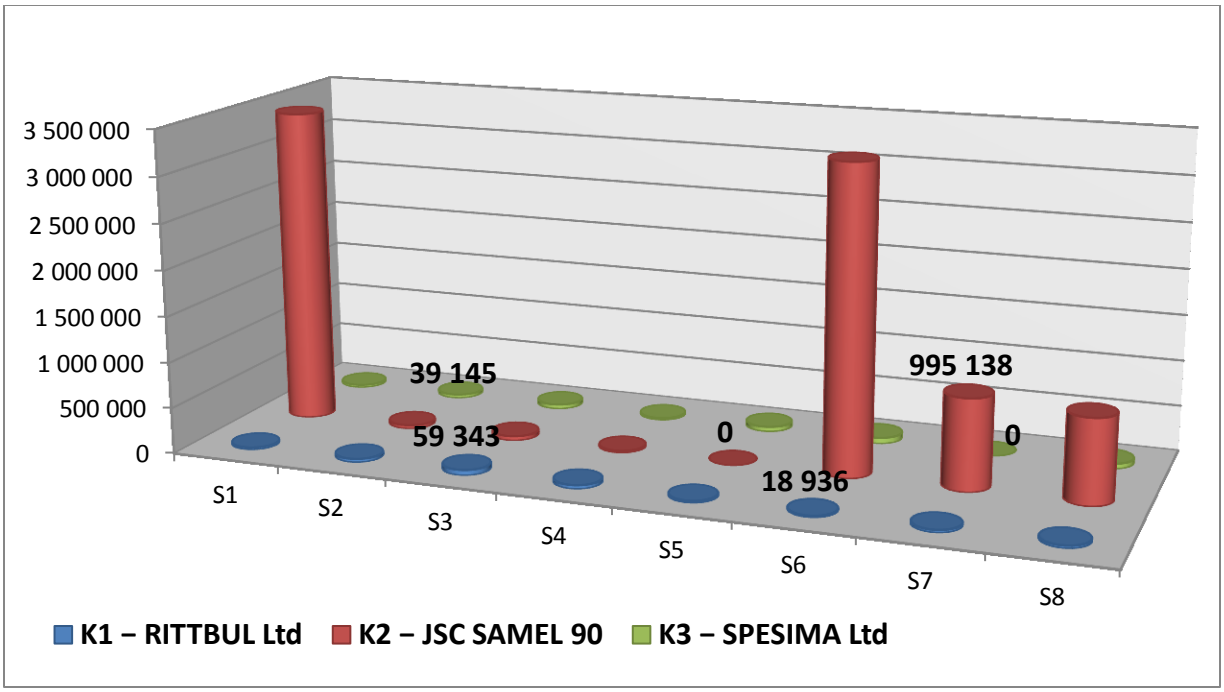


**FIGURE 8.** Distribution of profits for leading high-tech enterprises to the goods and services, which they offer (for 1 year)

Notice: All costs values and profits values of leading high-tech enterprises, shown in Fig. 7 (costs) and Fig. 8 (profits) are shown in table 8 (costs) and table 9 (profits). In Fig. 7 and Fig. 8 only the largest and smallest cost and profit figures of the leading high-tech enterprises are reflected, since if all values are shown, the chart will not be aesthetically.

**Conclusion of the decision of the First problem**

**1.** The $K_1$ **– RITTBUL Ltd** are assigned these goods and services: $S_1$ – Import and sale of Professional hand tools and electrical equipment; $S_2$ – Import and sale of instrumentation, suit Tools, LED lighting, software for design of electronic devices; $S_3$ − Production of LED lighting; $S_4$ – Production of transformers and coils; S_5 – Production of chokes, cable bundles, amplifiers, connectors; $S_6$ – Development and implementation of specialized systems for the production automation; $S_7$ – Production of specialized robots for automation of Horizontal machines for injection molding of zinc, magnesium, aluminum; $S_8$ –making on specialized Robotics systems (automated devices), specialized hardware and software.

The leading high-tech enterprise $K_1$ **– RITTBUL Ltd**, is assigned all goods and services $S_1 \div S_8$ , since they are with numerical value **"1".**

**.2.** The $K_2$ – **JSC SAMEL 90**, is assigned following goods and services: $S_1 \div S_8$ , since they are with numerical value **"1",** without good and service $S_5$ . The leading high-tech enterprise $K_2$ – **JSC SAMEL 90** is not assigned good and service $S_5$ , since has numerical value **"0".**

**3.** The $K_3$ **– SPESIMA LTD** is assigned following goods and services $S_1 \div S_8$ , since they have numerical value **"1",** without good and service $S_7$ . The leading high-tech enterprise $K_3$ – **SPESIMA LTD** is not assigned good and service $S_7$ , since it has numerical value **"0".**

**General conclusion from the decision of the First problem**

The leading high-tech enterprise **RITTBUL LTD** is assigned all goods and services $S_1 \div S_8$ . **The Joint Stock Company SAMEL 90** is assigned only seven from eight goods and services, without good and service $S_5$. The leading high-tech enterprise **SPESIMA LTD** is assigned just seven from eight goods and services, without good and service $S_7$.

The RITTBUL LTD is assigned all goods and services. The JSC SAMEL 90 and the SPESIMA LTD are assigned seven goods and services, as of the JSC SAMEL 90 is not assigned good and service $S_5$ , and of the SPESIMA LTD is not assigned good and service $S_7$.

**The decision of the Second problem of the mathematical model**

**TABLE 14**. Assignment activities for implementation from enterprises subcontractors

| Enterprises Subcontractors | Activities, assigned for implementation from enterprises subcontractors | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $d_1$ | $d_2$ | $d_3$ | $d_4$ | $d_5$ | $d_6$ | $d_7$ | $d_8$ |
| $F_1$ – ULTRAFLEKS KORPOREYSHAN Ltd | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| $F_2$ – General Partnership "SEMIS – CANEV i SIE" | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $F_3$ – JSC MEHATRONIKA | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

**Maximum overall efficiency is 1916.00, as total costs are 215058.0, and the profit is 144470.00.**

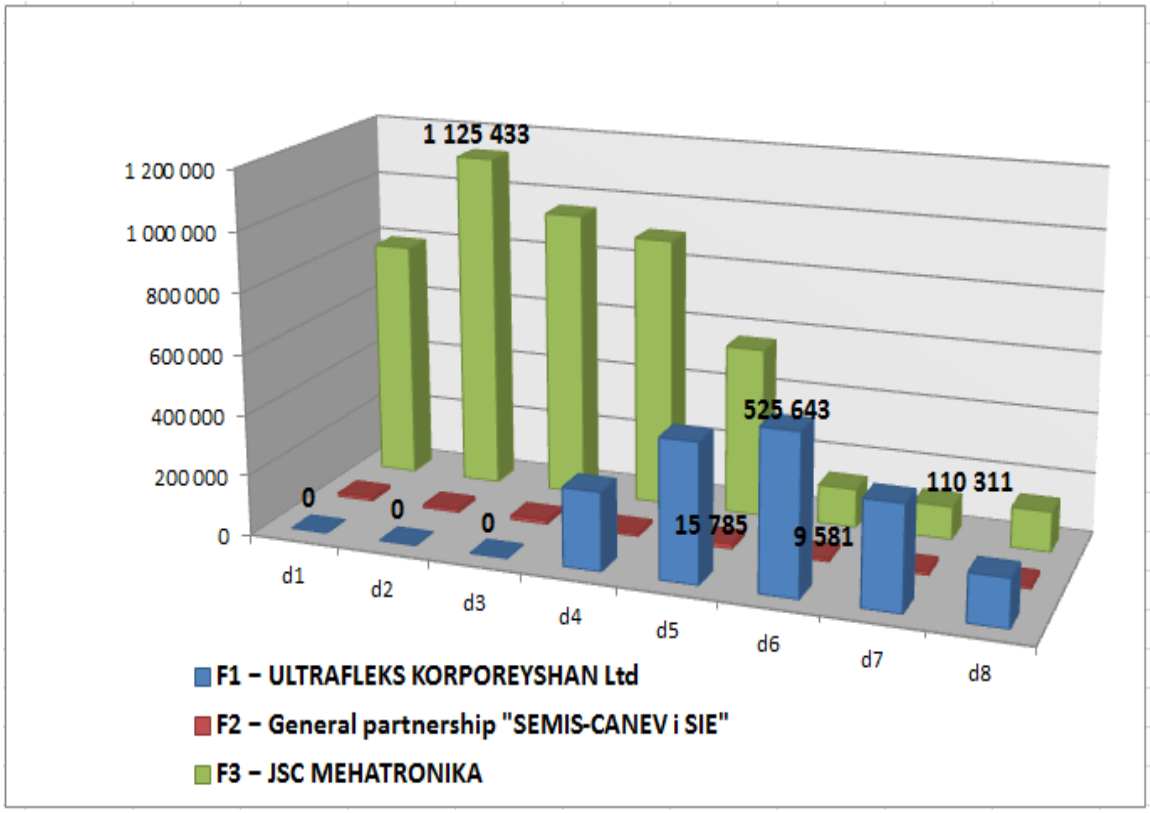


**FIGURE 9.** Cost allocation for enterprises subcontractors to the activities, which they assigned (for 1 year)

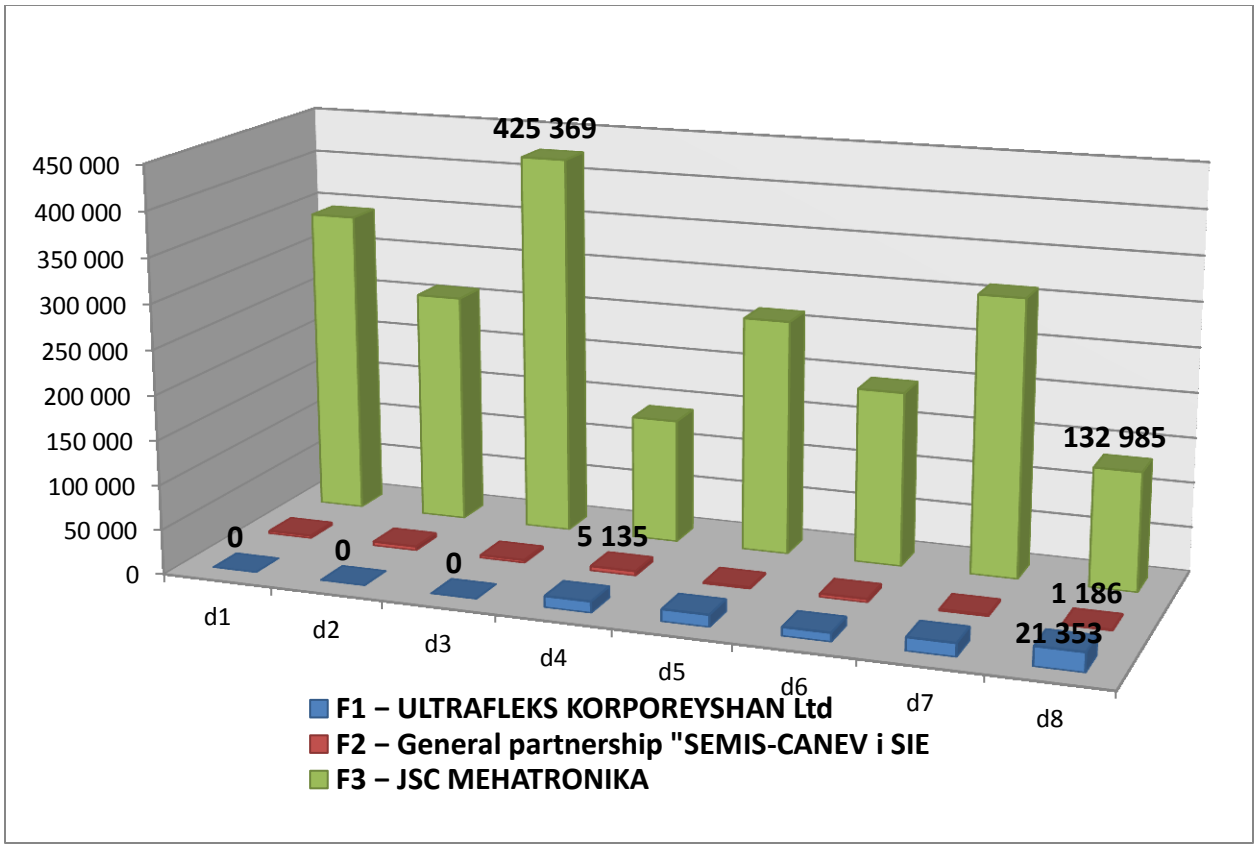


**FIGURE 10.** Distribution profits of enterprises subcontractors to activities, which they assigned (for 1 year)

**Conclusion of the decision of the Second problem**

**1.** The $F_1$ – **ULTRAFLEKS KORPOREYSHAN Ltd** is assigned these activities:$d_4$ − Services in engineering - manufacturing of electronic components and finished products, prototypes, complete solutions, specialized software and hardware; $d_5$ − Production of industrial electronics;$d_6$ − Repair of metal cutting machineries; $d_7$− Design of machines for production and filling of flexible packaging; $d_8$ − Productionandsale of machineryforproductionandfilling of flexiblepackaging.

The $F_1$ – **ULTRAFLEKS KORPOREYSHAN Ltd** is not assigned these activities: $d_1$ – Production of induction generators for industrial applications to heat treatment, soldering, welding, melting and casting of various metals and alloys; $d_2$ − Production of induction generators used in dentistry and jewelry casting dentures and jewelry; $d_3$ − Services in engineering - the development of electronic components and finished products, prototypes, complete solutions, specialized software and hardware.

The enterprise subcontractor $F_1$ – **ULTRAFLEKS KORPOREYSHAN Ltd** is assigned with the implementation of activities $d_4 \div d_8$. The $F_1$ – ULTRAFLEKS KORPOREYSHAN Ltd is not assigned for with the implementation of activities $d_1 \div d_3$.

**2.** The Enterprise Subcontractor $F_2$ – **General Partnership "SEMIS – CANEV i SIE"** is assigned with the implementation of all activities $d_1 \div d_8$ . Don't have activities, that are not assigned of the $F_2$ – **General Partnership "SEMIS – CANEV i SIE".**

**3.** The $F_3$ – **JSC MEHATRONIKA** is assigned with the implementation of all activities $d_1 \div d_8$. Don't have activities, that are not assigned with the implementation of the $F_3$ **– JSC MEHATRONIKA**.

**General conclusion from the decision of the Second problem**

The enterprise subcontractor **ULTRAFLEKS KORPOREYSHAN Ltd** is assigned with implementation of five of the activities: $d_4 \div d_8$. The **ULTRAFLEKS KORPOREYSHAN Ltd** is not assigned with the implementation of these activities: $d_1 \div d_3$.

The **General Partnership "SEMIS – CANEV i SIE"** is assigned with the implementation of all activities $\boldsymbol{d_1 \div d_8}$. **JSC MEHATRONIKA** is assigned with the implementation of all activities $\boldsymbol{d_1 \div d_8}$.

The **ULTRAFLEKS KORPOREYSHAN Ltd** is assigned with the implementation of only five activities, and three activities $\boldsymbol{d_1 \div d_3}$ are not assigned with the implementation. The General Partnership **"SEMIS – CANEV i SIE"** and the **JSC MEHATRONIKA** are assigned with the implementation of all activities $\boldsymbol{d_1 \div d_8}$.

## 4. CONCLUSION

From the conducted research and approbation of adapted managerial mathematical model to study the functions and interactions between enterprises in high-tech cluster, are made the following major conclusions:

1. The adapted managerial mathematical model allows studying the functions and interactions between enterprises not only in high-tech cluster, but also in other cluster. This fact on one hand forms preconditions for choice of cluster, where to be proposed this model for improvement of the interaction between enterprises, members of cluster.
2. Ready software product LINDO & LINGO, with that are performed calculations, has development ready module for calculate of this type mathematical model.
3. The adapted managerial mathematical model is approbated in high-tech cluster – Cluster "Mechatronics and Automation", where it becomes clear that the interaction between leading high-tech enterprises ant their enterprises subcontractors is carried out as efficiently as possible.
4. On the bases of the adapted managerial mathematical model is developed expert methodology for practical application, which includes description and characterization of its stages.

## REFERENCES


1. Gorbunov, V. M.,Decision theory, , Tomsk, 2010. (in Russian)
2. Cheshankov, B., Operation Research, part 1, Bulgaria, Sofia, 2004. (in Bulgarian)
3. Dantzig, G. B., Thapa, M. N., Linear Programing 2: Introduction and Extension, Germany, Berlin, 2003. (in Bulgarian)
4. Dimitrov, M., Study of operations, University press "Stopanstvo", ISBN 954-494-672-1. (in Bulgarian)
5. Fiedler, M., Nedoma, J., Ramik, J., Rohn, K., Zimmermann, Linear Optimization Problem with Inexact Data, Berlin, Germany, 2006.
6. Gartner, B., Matonsek, J., Understanding and Using Linear Programing, Berlin, 2006.
7. Porter, M., Competitive advantage of nations","Klas i kaistil", S., 2004. (in Bulgarian)
8. Velev, Ml., Cluster approach for increasing competitiveness, "Softtreid, S., 2007.(in Bulgarian)
9. How do industry clusters start? http://photonicsclusters.com/howstart.html, Accessed to 08.2015.
10. http://www.cluster-mechatronics.eu/index.php/bg_BG/about, Cluster "Mechatronics and Automation".