Dynamic Studies on Blade of Wind Turbine Generator, Taking into Account Aerodynamic Interaction with Non-Homogeneous, Non-Stationary Wind Field

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DYNAMIC STUDIES ON BLADE OF WIND TURBINE GENERATOR, TAKING INTO ACCOUNT AERODYNAMIC INTERACTION WITH NON-HOMOGENEOUS, NON-STATIONARY WIND FIELD

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Abstract. The purpose of the current study is to evaluate the dynamic behavior of the blade of a high class wind turbine generator taking into account aerodynamic interaction with non-homogeneous, non-stationary wind field. In order to accomplish this task, classical Blade Element Momentum (BEM) theory has been adapted, taking into consideration the effects of the vertical wind speed gradient. A non-homogeneous, non-stationary wind field has been modeled, taking the wind velocity as a function of time and altitude. Based on the modified BEM theory and the wind model, a numerical algorithm has been developed for evaluating the parameters of the aerodynamic interaction of the non-homogeneous, non-stationary fluid field with the turbine disk of NREL 5 MW wind turbine generator. Mechanical-mathematical model of the wind turbine blade has been developed in order to analyze the dynamics of the structures, subjected to external influence by the non-homogeneous, non-stationary fluid field. The obtained results show the significance of the accounting of the non-stationary, non-homogeneous distribution.

Keywords: wind turbine generator, finite element method, BEM theory.

1. INTRODUCTION

One of the most popular methods of analyzing the aerodynamics of wind turbines is the Blade Element Momentum (BEM) theory, [1–3]. The theory is consisted of two major parts, 1D Betz theory, Fig. 1, [4]. It couples the momentum equation and the Bernoulli equation:
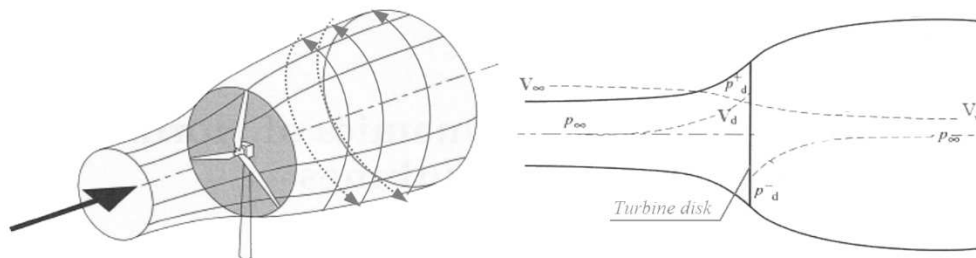
1D Betz theory. Flow distribution over a wind turbine

$$F_{Thrust} = Q_m(V_\infty - V_w) = \rho A_d V_d(V_\infty - V_w) = A_d \Delta p_d = A_d(p_d^+ - p_d^-), \quad (1)$$

$$F_{Thrust} = 0.5\rho A_d \left(V_\infty^2 - V_w^2\right) = 0.5\rho A_d \left(V_\infty + V_w\right)\left(V_\infty - V_w\right). \quad (2)$$

The second part is the classic BEM theory. The theory transforms the complex 3D problem of the aerodynamic interaction between wind and the turbine to a sequence of two dimensional problems, Fig. 2.
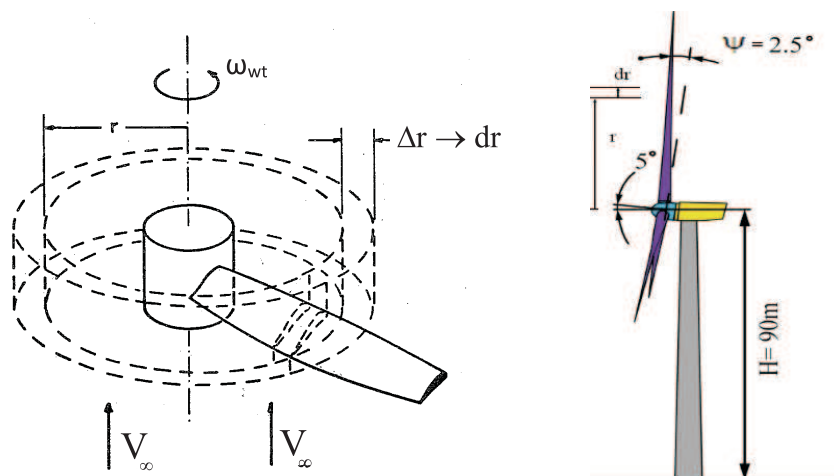

Fig. 2. BEM theory approach

The theory divides the blade on 2D cross-sections and applies the change of moment of momentum equation on every distinct element. The torque acting on the turbine is defined as a function of the radius of the blade:

$$\Delta M_{wt}(r) = \frac{d\Delta L(r)}{dt} = \frac{d\left(\Delta I(r)\omega_d(r)\right)}{dt} = \frac{d\Delta I(r)}{dt}\omega_d(r) =$$

$$\rho \underbrace{\underbrace{2\pi r \Delta r}_{\Delta A_d(r)} V_d(r)}_{d\Delta m(r)/dt} r^2 \omega_d(r) = 4\rho\pi\omega_{wt}V_\infty a'(r)(1-a(r))r^3\Delta r. \tag{3}$$

$$\underbrace{}_{d\Delta I(r)/dt}$$

The BEM theory establishes induction factors. Using them in iterative cycles, one can determine the parameters of aerodynamic interaction. The two induction factors are α – axial and α' – tangential induction factor. They are defined as follows:

$$a = \frac{V_\infty - V_d}{V_\infty},$$
$$a'(r) = \frac{\omega_d(r)}{2\omega_{wt}}. \tag{4}$$

The output of the theory are the axial-thrust force and the tangential torque force acting on the turbine of the generator:

$$F_{Thrust} = \left\{\frac{8}{\lambda(R)^2}\int_0^\lambda a(r)(1-a(r))\lambda(r)d\lambda(r)\right\}\frac{\rho A_d V_\infty^2}{2}$$

$$= C_{Thrust}\frac{\rho A_d V_\infty^2}{2} = C_{Thrust}\frac{P_{wt}^i}{V_\infty}, \tag{5}$$

$$M_{wt} = \frac{P_{wt}}{\omega_{wt}} = \frac{C_p}{\omega_{wt}}\frac{\rho A V_\infty^3}{2} = C_p\frac{P_{wt}^i}{\omega_{wtr}}, \tag{6}$$

where

$$P_{wt} = \int_0^R \omega_{wt}dM_{wt} \underset{r=\frac{V_\infty}{\omega_{wt}}\lambda}{=} \frac{8}{\lambda(R)^2}\int_0^{\lambda(R)} a'(r)(1-a(r))\lambda(r)^3 d\lambda(r)\frac{\rho A_d V_\infty^3}{2},$$

$$P_{wtr} = C_p\frac{\rho A_d V_\infty^3}{2} = C_p P_{wt}^i. \tag{7}$$

Major disadvantage of this theory is that it assumes the wind flow as stationary and homogeneous. As it can be seen in equations (3)–(7), the only independent variable is the distance from the axis of rotation to the considered cross-section r. In the current study the BEM theory is analytically modified, accounting both the vertical wind velocity gradient and the exact position of the blade in the turbine blade.

2. WIND MODEL

In order to evaluate the aerodynamic interaction in a non-homogeneous, non-stationary wind field, an appropriate wind model is required, Fig. 3. To satisfy the purpose of the study, the wind velocity function U is designed as a sum of two articles [5]:

$$U(z,t) \approx u_u(z,t) = u_0(z) + u'_u(z,t), \tag{8}$$

where $u_0(z)$ is the stationary vertical distributed component and $u'_u(z,t)$ is a turbulent, time dependent component of the wind, showing change of the wind velocity in time.
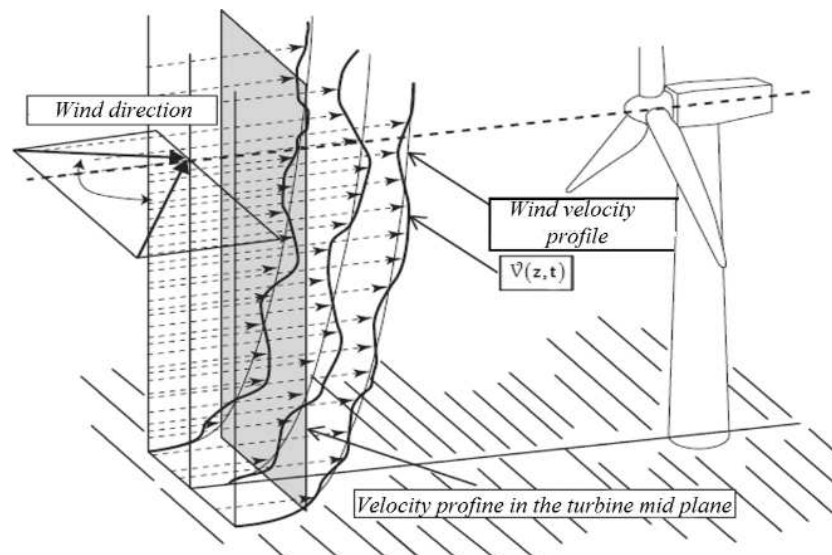

Fig. 3. Desired wind velocity distribution

A logarithmic law is chosen for the vertical distribution of the flow velocity [6]:

$$u_0(z) = \frac{u^*}{k} \left[\ln \left(\frac{z-d}{z_0} \right) + \psi(z, z_0, L) \right] \quad \text{[m/s]}. \tag{9}$$

A graphical visualization of the wind velocity vertical distribution is shown in Fig. 4.

The time dependent wind velocity function is derived by decomposing von Karman turbulence Power Spectral Density (PSD) function [7] in Shinozuka
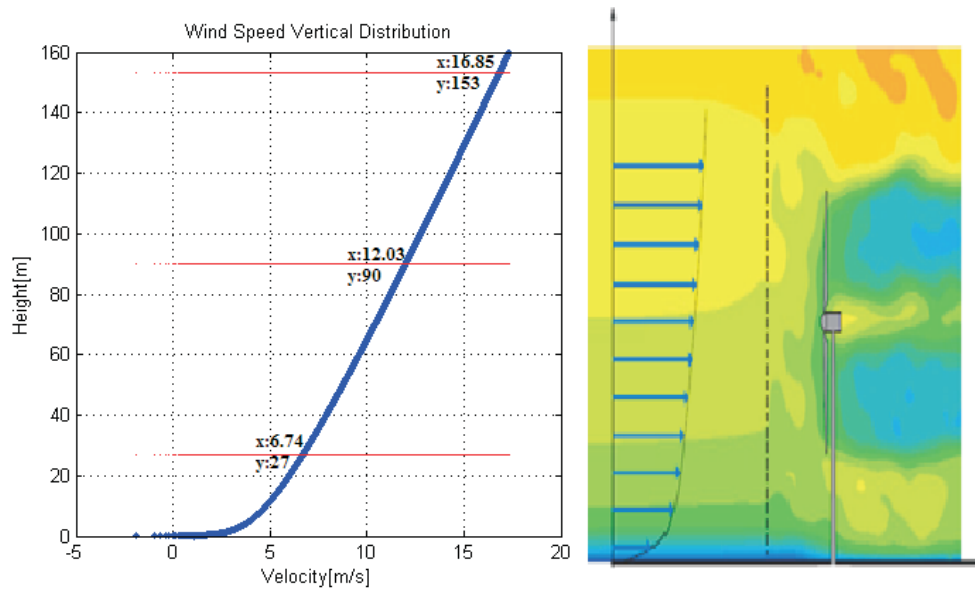
Fig. 4. Wind velocity vertical distribution

series [8, 9]:

$$u'(t) = \sum_{i=0}^{N-1} \sqrt{S_{u'}(f)\Delta f} \cos(2\pi f_i t + \phi_i). \qquad (10)$$

A 300 seconds time dependent wind velocity function is derived, shown in Fig. 5.
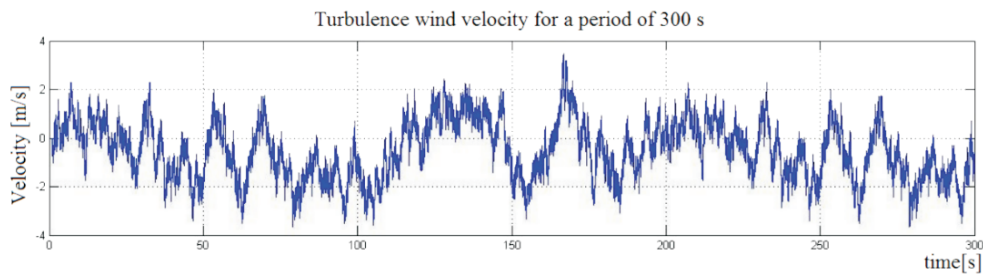

Fig. 5. Turbulent component of wind speed

In order to evaluate the truthfulness of the generated signal, the following mathematical processings are done. Autocorrelation function is generated to evaluate the absence of periodicity of the turbulent signal. Frequency composition of the same turbulent signal is derived, as well PSD function. Once

deriving PSD function of the newly generated signal, one can compare it with the original von Karman PSD. Considering the purpose of the current study, turbulence is taken into account only in horizontal direction.

Coupling the steady vertical distribution and the turbulent, time dependent distribution, the following wind model is derived, Fig. 6.
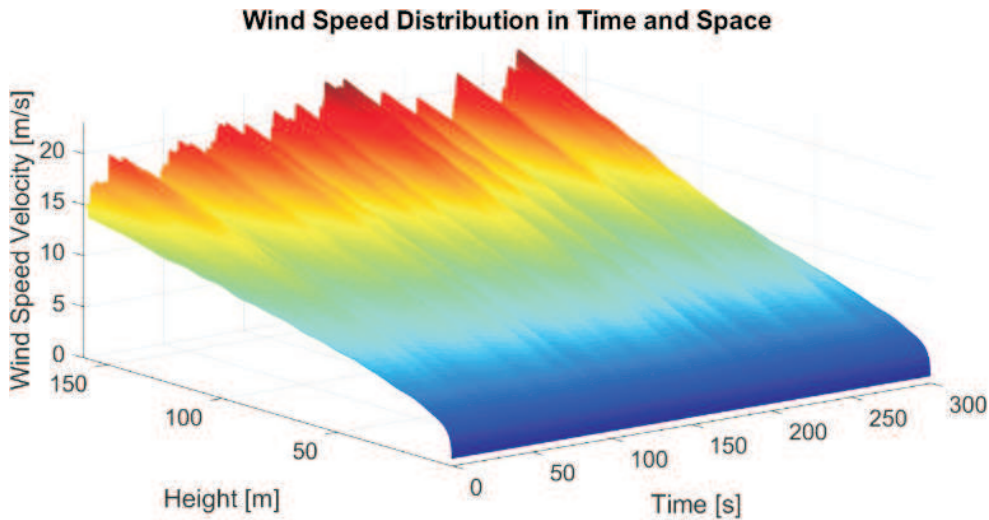
Wind Speed Distribution in Time and Space

Fig. 6. Wind model distribution as a function of fight and time

3. BEM THEORY MODIFICATION FOR A NON-HOMOGENEOUS, NON-STATIONARY WIND FIELD

Taking into account the vertical wind velocity gradient, the BEM theory dependences for a single element of the blade become a function of both the location of the element along the length of the blade r and the angle of rotation of the propeller γ, Fig. 7.

The element area described in polar coordinates is:

$$\Delta A_d \left(r, \gamma \right) = r \Delta r \Delta \gamma. \tag{11}$$
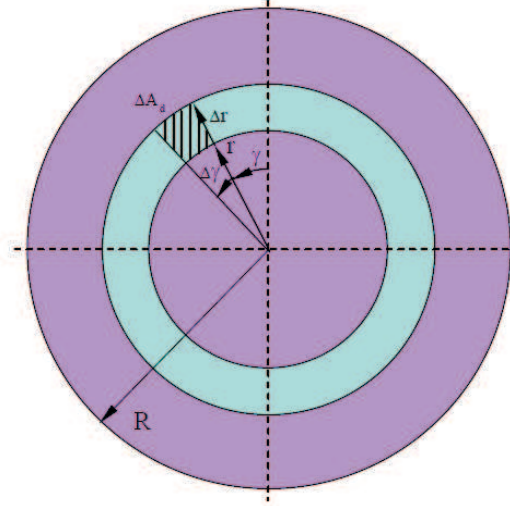
Fig. 7. Blade element in a polar frame of reference

Thus, output axial-thrust force, torque force and power are:

$$
\begin{aligned}
\Delta F_{Thrust}(\varphi_{wt}, r, \gamma) &= \frac{d}{dt}\left[\Delta Q_m\left(V_\infty(r,\gamma) - V_w(\varphi_{wt}, r, \gamma)\right)\right] F_c(\varphi_{wt}, r) \\
&= \rho \Delta A_d(r,\gamma) V_d(\varphi_{wt}, r, \gamma)\left(V_\infty(r,\gamma) - V_w(\varphi_{wt}, r, \gamma)\right) F_c(\varphi_{wt}, r) = \\
&= 2\rho a(\varphi_{wt}, r)(1 - a(\varphi_{wt}, r)) F_c(\varphi_{wt}, r) r V_\infty^2(r, \gamma) \Delta\gamma \Delta r
\end{aligned}
$$

$$
\begin{aligned}
\Delta F_{Thrust}(\varphi_{wt}, r) &= \int\limits_{\gamma=0}^{2\pi} \Delta F_{Thrust}(\varphi_{wt}, r, \gamma) d\gamma \\
&= 2\rho a(\varphi_{wt}, r)(1 - a(\varphi_{wt}, r)) F_c(\varphi_{wt}, r) r \left[\int\limits_{\gamma=0}^{2\pi} V_\infty^2(r, \gamma) d\gamma\right] \Delta r
\end{aligned}
$$

$$
\begin{aligned}
F_{Thrust}(\varphi_{wt}) &= \int\limits_{r=0}^{R} \Delta F_{Thrust}(\varphi_{wt}, r) dr \\
&= 2\rho \int\limits_{r=0}^{R} \left\{ a(\varphi_{wt}, r)(1 - a(\varphi_{wt}, r)) F_c(\varphi_{wt}, r) r \left[\int\limits_{\gamma=0}^{2\pi} V_\infty^2(r, \gamma) d\gamma\right] \right\} dr,
\end{aligned}
$$

(12)

$$\left|\begin{array}{l} \Delta M_{wt}(r) = 2\rho\omega_{wt}a'(r)(1-a(r))r^3\left[\int\limits_{\varphi=0}^{2\pi} V_\infty(\varphi,r)F_c(r,\varphi)d\varphi\right]\Delta r \\ \\ M_{wt} = 2\rho\omega_{wt}\int\limits_{\varphi=0}^{2\pi}\int\limits_{r=0}^{R} a'(r)(1-a(r))V_\infty(\varphi,r)r^3F_c(r,\varphi)drd\varphi, \end{array}\right. \qquad (13)$$

$$\begin{aligned} P_{wt} &= \int\limits_0^R \omega_{wt}dM_{wt} \\ &= 2\rho\omega_{wt}^2\int\limits_{\varphi=0}^{2\pi}\int\limits_{r=0}^{R} a'(r)(1-a(r))V_\infty(\varphi,r)r^3F_c(r,\varphi)drd\varphi. \end{aligned} \qquad (14)$$

With the modified BEM theory and the derived wind model, a numerical algorithm was implemented in MATLAB in order to evaluate the aerodynamic characteristics of the NREL 5 MW wind turbine generator [10, 11]. Some of the results are shown in Fig. 8.
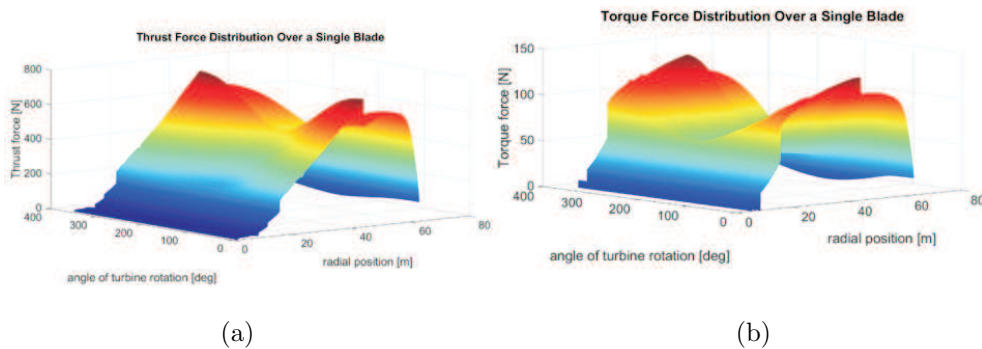

Fig. 8. Parameters of the aerodynamic interaction between a non-homogeneous, non-stationary wind field and NREL 5 MW wind turbine generator: (a) axial-thrust force; (b) torque force

4. DYNAMIC ANALYSIS ON THE BLADE OF NREL 5 MW WIND TURBINE GENERATOR, ACCOUNTING NON-HOMOGENEOUS, NON-STATIONARY WIND FIELD

In order to evaluate the dynamic behavior of the wind turbine blade under the influence of a non-stationary, non-homogeneous wind field, the axial-thrust force is derived as a function of the angular position of the blade on the turbine

plane $F_{Thrust}(x,\ \varphi(t))$, Fig. 8(a). It is converted into a function of time $F_{Thrust} = F_{Thrust}(x,\ t, \varphi(t))$, Fig. 9. So defined, the thrust force is applied as an external load to the blade of NREL 5 MW wind turbine. The following dynamic model is considered, Fig. 10.
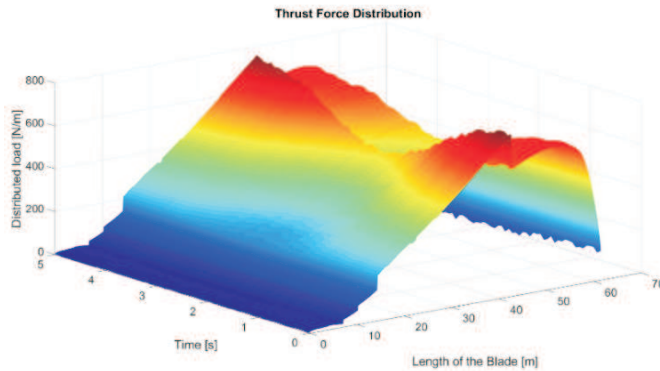

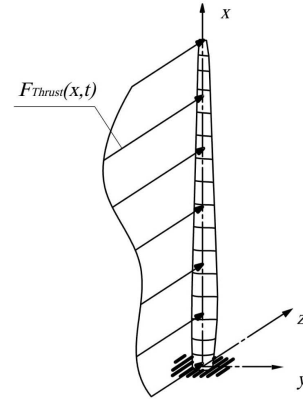**Fig. 9**. Axial thrust force as a function of time and length

Fig. 10. Blade dynamic model

The blade is considered as Euler-Bernoulli's beam structure [12, 13]:

$$\frac{\partial^2 \partial}{\partial x^2}\left[EI(x)\frac{\partial^2 w(x,t)}{\partial x^2}\right] + \rho(x)A(x)\frac{\partial^2 w(x,t)}{\partial t^2} + 2n(x)\rho(x)A(x)\frac{\partial w(x,t)}{\partial t}$$

$$= F_{Thrust}(x,t). \tag{15}$$

Since the blade of the wind turbine has a variable cross-section, i.e. the coefficients before the differentials in equation (15) are variables depending on the blade length, a direct analytical solution is not possible. The finite element method is used to solve the equation [14, 15]. The procedure that is used consists of:
• Discretization of the computational domain.
• Solution interpolation and derivation of the weak form of the differential equation.
• Derivation of the element matrices of the mass, stiffness, damping, internal and external forces.
• Derivation of a system of algebraic equations.

The numerical form for differential equation (15) is:

$$|\Phi_m^{gl}\ddot{q}_{gl}(t) + \Phi_c^{gl}\dot{q}_{gl}(t) + \Phi_k^{gl}q_{gl}(t) = \Phi_d^{gl}f_{gl}(t) = U^{gl}(t),$$
$$i = 1, 2, \ldots, N+1. \tag{16}$$

The obtained law of motion of the blade, Fig. 11, subjected to an external influence of the axial-thrust force, is derived as a result taking into account a non-homogeneous, non-stationary wind fluid. The oscillating motion leads to normal stresses with the following distribution, Fig. 12.
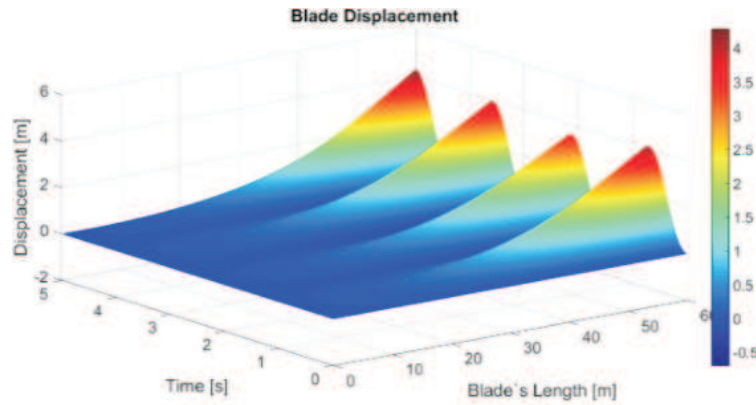

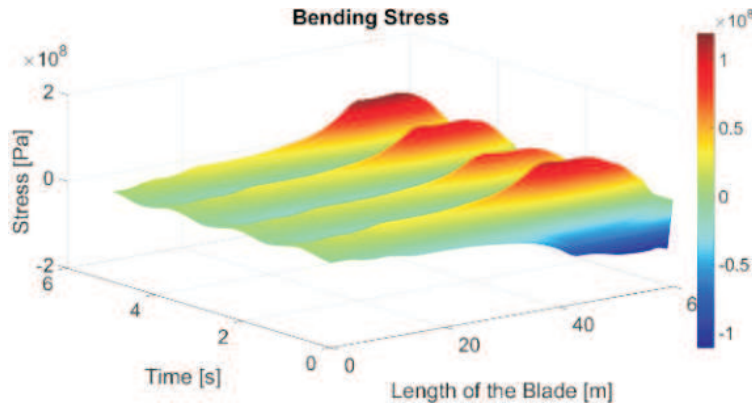**Fig. 11.** Blade law of motion, result of the axial thrust force

Fig. 12. Stress generated in the blade
due to the bending motion

It is interesting to mantion that the structure, subjected to such a polyharmonic excitation, vibrates with first natural frequency 1.25 Hz. The obtained

dynamic stresses, of order of 100 MPa, and deformations are similar to results, published by other authors [16, 17].

5. CONCLUSION

Aerodynamic loads, defined by the modified BEM theory, introduce oscillating motions of the NREL 5 MW blade. Those motions lead to alternating stress in the structure in the order of magnitude of around 100 MPa. Such alternating stress is the cause of material fatigue. The study suggests that turbulence and the vertical velocity gradient could have crucial effects on a wind turbine generator. Their effects should be considered in the wind turbine design process as well as in assessing reliability.

ACKNOWLEDGEMENTS

REFERENCES

[1] M. O. L. Hansen, *Aerodynamics of Wind Turbines*, London Sterling, VA (2008) 27–62, ISBN 978-1-84407-438-9.

[2] M. Ragheb, *Wind Energy Conversion Theory, Betz Equation* (2017).

[3] I. Grant, *Wind Turbine Blade Analysis Using the Blade Element Momentum Method* (2011).

[4] A. Betz, *Wind Energy and Its Extraction through Windmills* (1926).

[5] Xiaoyu Zhanh, *Crosswind Stability of Vehicles under Nonstationary Wind Excitation*, Karlsruher Institut für Technologie (2015), ISBN 978-3-7315-0376-7.

[6] L. Landberg, *Meteorology for Wind Energy – An Introduction*, John Wiley & Sons Ltd. (1997) 215.

[7] T. von Karman and L Howarth, *Theory of Isotropic Turbulence*, California Institute of Technology, Pasadena, November (1937).

[8] M. Shinozuka, *Simulation of Multivariate and Multidimensional Random Prosses, The Journal of the Acoustical Society of America* (1971) **49** 357.

[9] M. Shinozuka and C. I. L. Yun, *Stochastic Methods in Wind Engineering* (1990).

[10] J. Jonkman, S. Butterfield, W. Musial and G. Scott, *Definition of a 5-MW Reference Wind Turbine for Offshore System Development*, Technical Report NREL/TP-500-38060 (2009).

[11] DOWEC, *NREL 5 MW Blade Airfoil Data.*

[12] S. S. Rao, *Vibrations of Continuous Systems*, John Wiley & Sons, Inc. (2007), ISBN 978-0-471-77171-5.

[13] B. Belnikolovski, *Chosen Chapters of Dynamics*, Technical University of Sofia (2008), ISBN 954-438-419-7, (in Bulgarian).

[14] S. S. Rao, *The Finite Element Method in Engineering*, Sixth Edition (2018), ISBN 978-0-750-67828-3.

[15] K. Bathe, *Finite Element Procedures*, Prentice Hall, Pearson Education Inc. (2016), ISBN 978-0-9790049-5-7.

[16] S. Zuheir, O. Abdullah, and M. Al-Maliki, Stress and Vibration Analysis of the Wind Turbine Blade, *JMERD* (2019).

[17] P. Capponi, T. Ashuri, G. van Bussel, and B. Kallesoe, A Non-linear Upscaling Approach for Wind Turbines Blades Based on Stresses, *Journal of Mechanical Engineering* (2019).