

# OPTIMAL CONTROL FOR DAILY SCHEDULING OF COMBINED TURBO AND HYDRO POWER GENERATION

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**Abstract:** In this paper a cost model for optimal combined operation of turbo and hydro generators is considered. The resulting problem is a mixed integer linear programming problem which is solved with Matlab. The results analysis shows the usefulness of building such model and user interface in optimising power generation operation from turbo generators by combining them with hydro generators as reserved power and/or additional power resources. The model allows for including different demand levels variations and optimising the structure of the installation.

**Key words:** optimal combined generation scheduling, mixed-integer linear programming

## INTRODUCTION

The smart energy grids require an optimal combined operation of different types of generating units - thermal power plants steam turbines, diesel engines, gas turbines, hydro power plants and renewable energy sources in a power production system. This goal can be achieved by means of optimization approach with taking into account specific physical and technical restrictions. The problem becomes more complex with the increasing implementation of renewable power plants (wind turbine generators, photovoltaic generators etc.). The problem importance is related to its environmental, economical and social impact. Wind power generation is highly influenced from the wind characteristic due the diurnal and seasonal patterns of the wind behavior. This requires that a large-scale integration of wind power in electric power systems needs to be carefully developed, together with the implementation of new operational concepts and management tools. In [1] a concept of the combined use of wind power production and hydro storage/production is exploited, through the development of an operational optimization approach applied to a wind generator park with water storage ability. The solution of the optimization problem is obtained in Matlab environment using a primal-dual interior point method. It is mentioned that proposed model can be used to assist in the hydraulic design of the wind-hydro plant, calculating the optimal equipment specifications. The introduction of pumped storage systems in the isolated power production systems, in order to reduce the energy production annual cost and maximize the wind energy penetration, is investigated in many papers [1, 2, 3, 4, 5]. The pumping station design and performance of a hybrid wind-hydro power plant is analyzed in detail in [2] in order to optimize its configuration. The simulation results showed that among the three different pumping scenarios examined, the pumping station equipped with a variable-speed pump is the most advantageous configuration, and it can substantially increase the amount of the wind power energy that is transformed and stored as hydraulic energy. The improved pumping efficiency results also in remarkable economic benefits, enhancing the overall financial prospects of the investment. The introduction of pumped storage systems (PSS) in isolated power production systems with high thermoelectric production and wind energy rejection is investigated in [3]. The introduced PSS aims at the maximization of the wind energy penetration and the

minimization of the energy production cost. An iterative procedure is performed, in order to calculate the optimum pumps and hydro turbines nominal powers in both islands Crete and Rhodes in Greece. The optimization criterion is the energy production specific cost minimization. Similar investigations are carried out in [5]. In [4] a discrete optimization model for determining the best pump and turbine hourly operation for 1 day is developed. This model is applied to the "Multi-purposes Socorridos" system located in Madeira Island, Portugal, which is a pumped storage system with water consumption and hydropower production. An integrated software tool has been developed together with EPANET model [6] to evaluate the results of the optimization model, in order to verify if the behavior of the hydraulic components of the system (reservoir levels, flow) is maintained between the desired maximum and minimum limits. To solve the problem several variables are identified, as well as the objective function, which represents the quantity that should be minimized or maximized, the problem constraints associated to the physical capacities of the hydraulic system, and the water demands. The model is very flexible in terms of input data: wind speed, water consumption, reservoirs volume, maximum flow and electricity tariff.

In this paper a model for optimal turbo and hydro generating units is considered. The optimization problem is formulated and solved by mixed-integer programming method. A graphical user interface (GUI) application in Matlab is designed for flexible input and visualization of the model parameters, variables and optimal solution.

## PROBLEM FORMULATION

A company disposes of different types of turbo generators and a reservoir that powers a set of hydro generators each operating at a fixed output level. The company wants to use in an optimal way the equipment in disposal in order to fulfill the levels in its contracts and achieve minimal production costs. Each type of turbo generator has to work between a minimal and maximal output power levels where the costs for operating at minimal level are fixed and the operation at levels above minimal involves costs per hour per MWatt. There is a certain availability of units of each type and starting up a unit also

involves start-up costs. The power generated from the turbo generators can be used to pump water in the reservoir while the hydro generators also include start up costs but they can be used to improve whole operation of the installation – both hydro and turbo. Because the hydro generators operate at fixed operating levels the cost of the produced power depends on the number of hours that the unit is operational. While producing electrical power, a hydro generator consumes water from the reservoir so its level decreases but it has to be maintained daily over certain level. The operation of the installation must meet the load demand levels and assure some extra output guarantee at a minimal price. Such increase in the load level can be guaranteed by switching on a hydro generator and/or by using this combination of turbo units, whose maximal levels assure the required extra load.

In this paper a model is constructed to obtain an optimal unit commitment for combined turbo and hydro power generation and the problem is solved using custom GUI in Matlab. The GUI provides for analyzing the benefits of investments in equipment or modernization, the advantages of enlarging or narrowing the number and the types of the units in operation. The GUI is designed to meet up to 6 different types of turbo generators, up to 6 different hydro generators and up to 24 time periods. The model implementation provides also for building objective financial strategies to an energy producer because it gives the value of costs involved in its business activity.

$C_{ij} = c_i T_j$  - costs for operating a turbo generator of type  $i$  per MWatt above its minimal output power level in a period  $j$

$F_i$  - start-up costs for a turbo generator of type  $i$

$L_k$  - fixed output power level of hydro of type  $k$  n MWatts

$R_k$  - water level reduction per hour of hydro of type  $k$  operation in m/h

$K_k$  - costs per hour for operating a hydro of type  $k$

$G_k$  - start-up costs for a hydro of type  $k$

$l_{\min}$  - minimal level of water in the reservoir in meters

$l_{\max}$  - maximal level of water in the reservoir in meters

$l_0$  - initial value of the level of water in the reservoir, this is a level that has to be maintained at the beginning of each first time period in meters

$p_N$  - the energy that is required to increase the level of water in the reservoir with 1 meter; rainfall and evaporation do not affect the water level

$\sigma$  - extra load guarantee in percentage [%];

The variables used in the problem are:

$x_{ij}$  - total output power of the turbo generators of type  $i$  operating in a time period  $j$

$n_{ij}$  - number of operating generators of type  $i$  in a time period  $j$  (integer variables)

The screenshot shows the 'Combined\_Generation' software interface. It is divided into several sections:

- Demand data:** A table with columns 'Duration, [hrs]' and 'Demand, [MW]'. It contains 24 rows of data.
- Turbo generators costs structure:** A table with columns for Type 1 through Type 6. Rows include 'Available units', 'Min power level, [MW]', 'Max power level, [MW]', 'Cost at min power level, [\$/h]', 'Cost above min power level, [\$/h.MW]', and 'Startup cost, [\$/h]'.
 

	Type 1	Type 2	Type 3	Type 4	Type 5	Type 6
Available units	8	10	5	0	0	0
Min power level, [MW]	820	1300	1700	0	0	0
Max power level, [MW]	1850	1900	4100	0	0	0
Cost at min power level, [\$/h]	1100	2800	3100	0	0	0
Cost above min power level, [\$/h.MW]	2.4000	1.2000	2.3000	0	0	0
Startup cost, [\$/h]	1800	1100	1400	0	0	0
- Hydro generators cost structure:** A table with columns for Hydro 1 through Hydro 6. Rows include 'Operating power level, [MW]', 'Cost/hour, [\$/h]', 'Water level reduction per hour, [m]', and 'Start-up cost, [\$/h]'.
 

	Hydro 1	Hydro 2	Hydro 3	Hydro 4	Hydro 5	Hydro 6
Operating power level, [MW]	300	320	340	0	0	0
Cost/hour, [\$/h]	80	86	90	0	0	0
Water level reduction per hour, [m]	0.1500	0.1800	0.2200	0	0	0
Start-up cost, [\$/h]	250	260	275	0	0	0
- Parameters:** Input fields for 'Extra power guaranteed: 15 %', 'Reservoir min level: 10', 'Reservoir max level: 25', 'Electricity required for 1 m increase in water level: 920 MWh', and 'Initial reservoir level to be maintained at midnight: 15'.
- Buttons:** A 'Solution ..' button is located at the bottom right.

Fig. 1: Input data User interface

The problem formulation uses the following symbols:

$i$  - turbo generators types number

$k$  - hydro generators number

$j$  - number of periods

$D_j$  - load level during a period  $j$  in MWatts

$T_j$  - duration of a period  $j$  in hours

$n_i$  - number of available units of type  $i$

$m_i$  - minimal output level of turbo generator of type  $i$

$M_i$  - maximal output level of turbo generator of type  $i$

$e_i$  - cost per hour for operating a turbo generator of type  $i$  at its minimal output power level

$E_{ij} = e_i T_j$  - costs for operating a turbo generator of type  $i$  at a minimal output power level during a period  $j$

$c_i$  - cost per hour per MWatt above minimal output power level for a turbo generator of type  $i$

$s_{ij}$  - number of generators of type  $i$  started up in a time period  $j$  (integer variables)

$h_{kj} = 1$  if a hydro generator of type  $k$  operates in a time period  $j$

$t_{kj} = 1$  if a hydro generator of type  $k$  is started up in a time period  $j$

$l_j$  - water level at the beginning of the period  $j$

$p_j$  - consumed power in pumping regime in a time period  $j$  in [MW].

So the problem consists of seven variables groups:

- three groups each containing total  $i.j$  number of variables,  $2.i.j$  is the number of general integer variables (for  $s_{ij}$  and  $n_{ij}$ ) and  $i.j$  is the number of continuous variables (for  $x_{ij}$ )

- two groups each containing  $k.j$  number of elements that are binary variables (for  $h_{kj}$  and  $t_{kj}$ )

- two groups each containing  $j$  number of continuous variables (for  $l_j$  and  $p_j$ )

The objective function to be minimized represents the total cost for power generation

$$\min J = \sum_{i,j} C_{ij} (x_{ij} - m_i n_{ij}) + \sum_{i,j} E_{ij} n_{ij} + \sum_{i,j} F_i s_{ij} + \sum_{i,j} K_{ih} T_j h_{ij} + \sum_{i,j} G_{ih} t_{ij} \quad (1)$$

$$l_j - l_{j-1} - \frac{T_j p_j}{p_N} + \sum_k T_j R_k h_{kj} = 0 \text{ for each } j \quad (7)$$

The variables bounds are:

$$x_{ij} \geq 0 \quad p_j \geq 0 \quad l_{\min} \leq l_j \leq l_{\max} \text{ and } l_1 = l_0$$

$0 \leq n_{ij} \leq n_i$ ,  $n_{ij}$  and  $s_{ij}$  are general integer variables

$h_{kj}$  and  $t_{kj}$  are binary integer variables

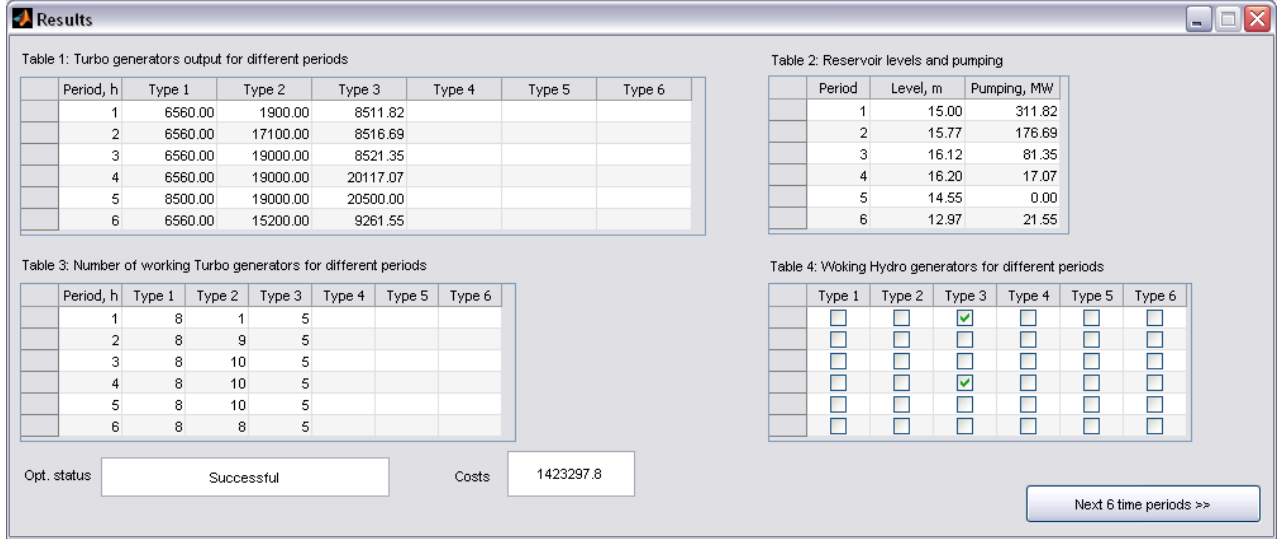


Fig. 2: Results user screen

The problems also considers  $j(3i+2)$  number of inequality constraints and  $j$  is the number of the equality constraints:

A *power balance constraint* – the total output power levels of turbo and hydro generators that operate in a time period  $j$  with taking into account the consumed power in a pumping regime, should be greater than the load demand level.

$$\sum_i x_{ij} + \sum_k L_k h_{kj} - p_j \geq D_j \text{ for all } j \quad (2)$$

An *extra load constraint* – the sum of the maximal output power levels of all operating in a time interval  $j$  turbo generators and the fixed generation of the operating hydro generators in the same period must ensure the extra power guarantee. In order to ensure the extra load starting-up turbo generators is not allowed but hydro generators may be started.

$$\sum_i M_i x_{ij} + \sum_k L_k t_{kj} \geq D_j (1 + \sigma) / 100 \text{ for each } j \quad (3)$$

The power output levels of all operating turbo generators in each time period must lie in their minimal and maximal output power levels:

$$x_{ij} \geq m_i n_{ij} \text{ for each } i \text{ and } j \quad (4)$$

$$x_{ij} \leq M_i n_{ij} \text{ for each } i \text{ and } j \quad (5)$$

The number of the units of type  $i$  that are started in the current period  $j$  must not be greater than the difference between the number of the units of the same type that are operating in the current period  $j$  and those operating in the previous interval  $j-1$ . If the current period is the 1<sup>st</sup> one then the previous period  $j-1$  is considered  $j$

$$s_{ij} \geq n_{ij} - n_{i,j-1} \text{ for each } i \text{ and } j \quad (6)$$

The difference in the levels between two successive time intervals must equal the reduction levels caused by operating hydro generators in the current time period  $j$ :

## CALCULATIONS AND RESULTS

The problem under consideration (1-7) is solved with Matlab via custom GUI for data input (Fig. 1) and results visualization (Fig. 2) according the input data listed in tables 1, 2 and 3. In this paper three different types of turbo and three types of hydro generators are a subject of an optimal combined operation in six different time intervals. The load demand levels (table 1) for the different time intervals with a given duration may be obtained from statistical and/or forecasted data.

Table 1: Load levels

Period $j$	$T_j$	$D_j$
	[hours]	[MWh]
1	6	17000
2	4	32000
3	4	34000
4	4	46000
5	3	48000
6	3	31000

Additional input parameters are the minimal and maximal reservoir level, the levels that has to be maintained daily at the beginning of the first time period, the extra power guarantee in percentage and the amount of required energy in MWh to increase the water level with 1 meter:

$$l_{\min} = 10 \text{ meters}$$

$$l_{\max} = 25 \text{ meters}$$

$$l_0 = 15 \text{ meters,}$$

$$p_N = 920 \text{ MWh}$$

$$\sigma = 15\%$$

Table 2 shows the cost structure and technological parameters of the three types of turbo generators under consideration. Table 3 shows the costs structure for the hydro units.

**Table 2: Turbo generators cost structure**

	Type 1	Type 2	Type 3
$m_i$ , [MW]	820	1300	1700
$M_i$ , [MW]	1850	1900	4100
$e_i$ , [\$/h/MWh]	1100	2800	3100
$c_i$ , [\$/h]	2,4	1,2	2,3
$F_i$ , [\$]	1800	1100	1400
$n_i$	8	10	5

**Table 3: Hydro generators cost structure**

	Hydro 1	Hydro 2	Hydro 3
$L_k$ , [MW]	300	320	340
$R_k$ , [m/h]	80	86	90
$K_k$ , [\$/h]	0,15	0,18	0,22
$G_k$ , [\$]	250	260	275

At the end of optimization procedures, the cost function value is \$1 423 297,80. The rest of the optimization values are listed in tables 4 and 5. Table 4 shows the total output level of all working turbo generators in the corresponding time intervals, and the number of the operating turbo.

**Table 4: Turbo generators results**

Period	Total turbo generators power levels			Number of operating turbo generators		
	Type1	Type2	Type3	Type1	Type2	Type3
$j$	MW	MW	MW	$n_{1j}$	$n_{2j}$	$n_{3j}$
1	6560	1900	8511,82	8	1	5
2	6560	17100	8516,69	8	9	5
3	6560	19000	8521,35	8	10	5
4	6560	19000	20117,07	8	10	5
5	8500	19000	20500	8	10	5
6	6560	15200	9261,55	8	8	5

All 8 available turbo generators of type 1 operate at a minimal output power levels except in the period with the highest load level (period 5). This is because their cost per MW above minimal level is the highest of all available turbo generator types therefore they can be used to ensure the extra output guarantee. All 5 available units of type 3 also operate in all of the periods under consideration but their output levels vary according the demand load levels. In the 2<sup>nd</sup> time period 9 of all 10 units of type 2 operate at their maximal power output levels, while the 5 units of type 3 are still close to minimum i.e. despite the fact that the turbo generators of type 2 have the smallest operational range related to the other types, they carry a minimal cost for operating.

**Table 5**

Period	Level	Energy
	[m]	[MW]
1	15,00	311,82
2	15,77	176,69
3	16,12	81,35
4	16,20	17,07
5	14,55	0,00
6	12,97	21,55

In the 3<sup>rd</sup> period all available units operate to meet the demand levels – type 1 at minimal and type 3 near the minimal, while the 10 available units of type 2 are pushed to their maximal levels for periods 3, and 5. The increasing of the output levels of units of type 3 begins at the 3<sup>rd</sup> period and reaches its highest level at the 5<sup>th</sup> period (one with the highest load demand). All operating turbo generators in the previous periods power the pumps to accumulate energy in water level that is disposed in this 5<sup>th</sup> highest demand level period and no

pumping takes place. In this period the hydro generators and the maximal output power levels to assure the 15% extra output guarantee. Hydro 3 helps in the 1<sup>st</sup> and 4<sup>th</sup> period and this costs \$ 54 202,20 savings which is the cost of operating the same number and types of turbo generators in the same load levels but without the opportunity to store energy in the reservoir and reuse it.

## CONCLUSION

These results give two logical conclusions: First of all, an increasing in the number of units or their variety should be considered in such load levels in order to be able to meet these loads in case of unit failure and/or routine repairs. The GUI provides for evaluating the costs of such increasing of the turbo generators under disposal by implying simple operational coefficients. The conclusion of a higher importance however is that it is worth to work further in combined power generation as storing excess energy is mostly valuable in the case of non-controllable energy resources such as wind turbines and photovoltaics. This model uses deterministic data for the load levels and the amount of energy that can be stored in the reservoir. Future work will focus on customizing such model for storing and reusing energy from renewable resources with stochastic nature.

## REFERENCES

1. Castronuova E., J. Peca-s Lopes, Optimal operation and hydro storage sizing of a wind-hydro power plant, *Electrical Power and Energy Systems*, Vol. 26, 2004, pp.771-778.
2. Anagnostopoulos J., D. Papantonis, Pumping station design for a pumped-storage wind-hydro power plant, *Energy Conversion and Management* Vol. 48, 2007, pp. 3009–3017.
3. Katsaprakakis, D., et.al. Pumped storage systems introduction in isolated power production systems, *Renewable Energy* Vol. 33, 2008, pp. 467–490.
4. F. Vieira, H. Ramos, Hybrid solution and pump-storage optimization in water supply system efficiency: A case study, *Energy Policy* Vol. 36, 2008, pp. 4142–4148.
5. Kapsali, M., J. Kaldellis, Combining hydro and variable wind power generation by means of pumped-storage under economically viable terms, *Applied Energy*, Vol. 87, 2010, pp. 3475–3485.
6. Rossman, L.A., 2000. EPANET 2.0 User's manual. In: *Drinking Water Research Division* (Ed.), Risk Reduction Engineering Laboratory Environmental Protection.
7. Williams, H. P., *Model Building in Mathematical Programming*, 4th Edition, Willey 1999.
8. Matlab Optimization Toolbox Product Documentation available at <http://www.mathworks.com/help/toolbox/optim/>
9. Taha, H. A., *Operations Research: An Introduction to Operations Research*, 8<sup>th</sup> Edition, Prentice Hall, 2007.
10. Hillier, F. S., G. J. Lieberman, *Introduction to Operations Research*, 9<sup>th</sup> Edition, McGraw-Hill, 2009.
11. Sherif Tawfik, Mixed-Integer LP function description available at <http://www.mathworks.com/matlabcentral/fileexchange/6990>

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