

# Conditional Feedback Control of Second-Order Process

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**Abstract** — Conditional feedback control systems reduce the sensitivity to changes in the parameters of the control plant in terms of control transient performance. They are known as passive adaptation systems. The main difference with other control systems with an internal model control systems is the presence of a forming element in their structural implementation. The application of conditional feedback control for the control of thermal process leads to energy savings as the parameters of the PID (proportional integral derivative) controller algorithm are tuned for "worst case scenario" control plant.

**Keywords** — Conditional Feedback, Forming Element, Robust and Transient Performance, Insensitivity

## I. INTRODUCTION

Often thermal processes in indoor air quality control systems as well as in industrial process control systems are described by second-order transfer functions (TF). The systems for their control are subject to a number of requirements related to the performance of the ongoing processes and energy savings. The availability of 'a priori' information about eventual change in the parameters of the control plant makes the design of a controller with conditional feedback possible.

The article presents a method for synthesis of a second-order control system, which uses conditional feedback and a PID controller. The control of thermal plants with PID controller is associated with energy saving [1], and the combination with the method of conditional feedback results in great control performance when taking into an account the uncertainty in the parameters of the control plant [2-6].

## II. CONDITIONAL FEEDBACK CONTROLLER DESIGN

### A. Block diagram of a control system with conditional feedback

The control systems with conditional feedback are assigned to the class of control systems, in the structure of which an internal model of the control plant is included, fig.1.

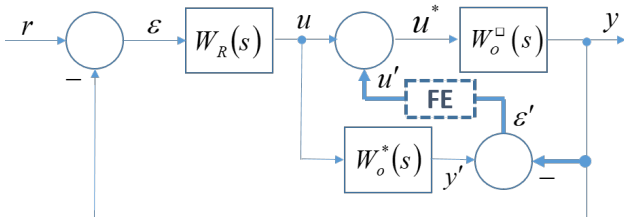


Fig.1 Block diagram of a control system with conditional feedback

The following notations are used:

$u^*$  - output signal of controller with conditional feedback;

$u'$  - output signal of forming element;

$u$  - output signal of nominal controller;

$y'$  - output signal of the nominal control system;

$y$  - output signal of the conditional feedback control system;

$\varepsilon'$  - error, difference between the output signal of the conditional feedback control system and the output of the nominal control system;

$\varepsilon$  - error;

$W_o^*(s)$  - TF of nominal control plant;

$W_o^\square(s)$  - TF of "worst case scenario" control plant;

$W_R(s)$  - TF of nominal controller;

$r$  - set point;

FE – forming element.

A change in the control signal  $u$  is achieved that reduces the impact of the change in the parameters of the plant on the performance of the control process.

In the system with conditional feedback, Fig. 1, the difference between the output of the system  $y$  and the model of the plant  $y'$  is fed to the input of the control plant, aiming to reduce the influence of change in the control plant's parameters and is performed by the so-called forming element FE, Fig.1.

### B. Structural synthesis of a conditional feedback controller

A necessary condition is availability of 'a priori' information about the range of variation of the control plant's parameters. The transfer function of the control plant  $W_o^\square(s)$  represents the "worst case scenario" transfer function fig.1.

Conditional feedback results in a correction signal  $u'$  on the control signal  $u$  when there is a difference  $\varepsilon'$  between the output of the system with a nominal plant  $y'$  and the output of the system  $y$  with the actual control plant.

The block diagram on Fig.1 is converted into a block diagram that is shown in Fig. 2.

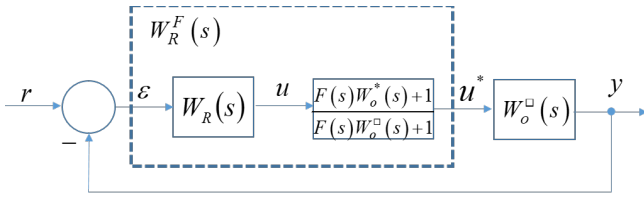


Fig.2. Equivalent block diagram of a system with conditional feedback

In the shown block diagram, the control part of the system with conditional feedback is indicated with a dotted line.

The transfer function of the controller with conditional feedback is described with the following equation (1):

$$W_R^F(s) = W_R(s) \frac{W_o^*(s) + F^{-1}(s)}{W_o^□(s) + F^{-1}(s)}. \quad (1)$$

In order for the conditional feedback to be effective in correcting the uncertainties of the plant through the signal  $u^*$  of great importance is the variation of the control plant's parameters and the configuration of the FE noted as  $F(s)$  in the transfer function on Fig.2.

### C. Tuning of the forming element

There is no unambiguous solution in the literature for choosing the forming element.

In [7] the following type of modeling transfer function of the forming element is proposed (2):

$$F(s) = -\frac{W_R^□(s) - W_R(s)}{1 + W_R(s)W_o^*(s)}. \quad (2)$$

In (1) a fictitious transfer function  $W_R^□(s)$  is used, which is a model of a controller set for the „worst case scenario“ of combination of the parameters of the control plant.

It is of interest to consider the case where a coefficient much larger than 1 is chosen for the transfer function of the forming element. ( $F(s) = k, k \rightarrow \infty$ ).

In that case for the transfer function of the controller (1) with conditional feedback is obtained (3):

$$W_R^F(s) \approx W_R(s) \frac{W_o^*(s)}{W_o^□(s)} \quad (3)$$

Expression (3) shows that in addition to the properties of the control plant, for the successful application of the conditional feedback method of great importance is also the range in which the parameters of the plant change [1-7].

It turns out that for plant described with second-order transfer function this restriction is not valid because (3) is converted into a PID controller tuned for 'a priori' known „worst case scenario“ control plant.

### D. Design procedure of a controller with conditional feedback for control plant with second order transfer function

The design procedure of the controller with conditional feedback requires the implementation of four steps, graphically represented by the algorithm of Fig.3.

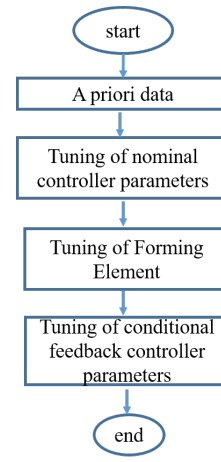


Fig.3 Algorithm

*Step 1.* A priori information about the plant model is required. Two transfer functions (4) and (5) are defined, which describe the dynamic behavior of the plant and take into account the uncertainty in its parameters:

$$W_o^*(s) = \frac{k^*}{(T_1^*s + 1)(T_2^*s + 1)} \quad (4)$$

$$W_o^□(s) = \frac{k^□}{(T_1^□s + 1)(T_2^□s + 1)} \quad (5)$$

In this step the criterion of the control performance is also defined.

Since (4) and (5) usually describe thermal plants, as control variable is considered temperature. It is known that the aperiodic nature of the transient response of systems with control variable temperature is associated with energy savings. The local criteria is overshooting  $\sigma \approx 0\%$  and settling time  $t_s \approx const, s$

*Step 2.* The next step is tuning the parameters of the nominal controller for the presented in step 1 criteria. The transfer function of the nominal controller has the form (6), as the PID controller provides quasi-optimal control in terms of energy criteria [1].

$$W_R(s) = k_t \frac{(T_{1R}s + 1)(T_{2R}s + 1)}{k_{RS}} \quad (6)$$

The required controller settings are (7):

$$T_{1R} = T_1^*, T_{2R} = T_2^*, k_R = k^* \quad (7)$$

The coefficient is an adjustable parameter that provides the necessary time-response of the control system.

*Step 3.* It is chosen  $F(s) = k, k \rightarrow \infty$ .

*Step 4.* For the transfer function of the controller with conditional feedback is obtained (8):

$$W_R^F(s) = k_t \frac{(T_1^□s + 1)(T_2^□s + 1)}{k^□s}. \quad (8)$$

The expression (8) shows that the controller with conditional feedback will guarantee robustness of the system, as it provides a complete match of the real transient response

process of the controlled variable, Fig. 2 with the nominal case (the system designed in step 2).

### III. NUMERICAL EXAMPLE

The following thermal plant is considered, represented by a nominal control model (9) and a control model at „worst case scenario“ (10).

$$W_o^*(s) = \frac{1}{(s+1)(2s+1)} \quad (9)$$

$$W_o^\square(s) = \frac{2}{(2s+1)(3s+1)} \quad (10)$$

The control system should satisfy the following criteria: overshooting  $\sigma \approx 0\%$  and settling time  $t_s \approx 10, s$ .

According to the design procedure described in II for the nominal PID controller and the given criteria, the following equation is obtained (11):

$$W_R(s) = 0.4 \frac{(s+1)(2s+1)}{s} \quad (11)$$

The coefficient  $k_t$  is set by the root locus method, Fig.4.

The distance to the imaginary axis is related to the time-response of the system through the equation (12):

$$t_s \approx 4T \approx \frac{4}{p} \approx \frac{4}{0.4} = 10s \quad (12)$$

It is chosen  $k_t = 0.4$ , Fig. 4.

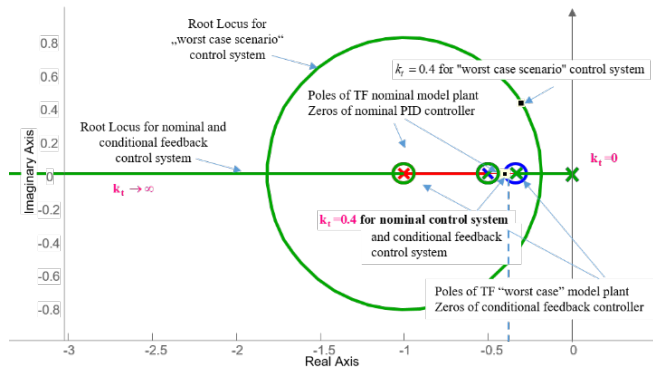


Fig.4 Root Locus method for tuning  $k_t = 0.4$

When the parameters of the control plant change, the quality criterion is not satisfied.

The transient response of the closed system will be characterized by overshooting, as the dominant roots are complexly conjugated, for the same  $k_t = 0.4$ , Fig.4.

The next step is to fix the forming element  $F(s) = 10000$

According to the procedure for the controller with conditional feedback is derived (13):

$$W_R^F(s) = 0.4 \frac{(2s+1)(3s+1)}{2s} \quad (13)$$

Fig. 5 and Fig. 6 show simulation studies that present the behavior of the control systems in conditions close to the real-life application. A control signal limitation in the range  $[0, 10]$  V is also included.

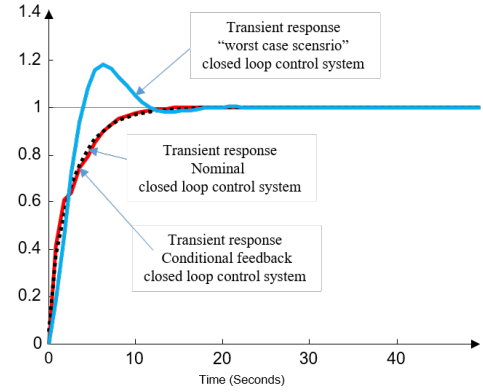


Fig.5 Transient responses of the control systems

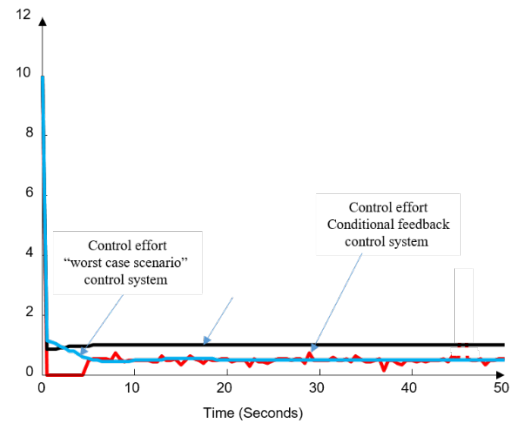


Fig.6 Control effort of control systems

The studies presented on Fig.5 and Fig.6 show a complete match between the responses of the nominal control system and the conditional feedback control systems.

The obtained result corresponds to insensitivity of the controlled variable to changes in the control plant's parameters. In regard with the control signal, it can be seen that the control with conditional feedback requires less energy, Fig.7.

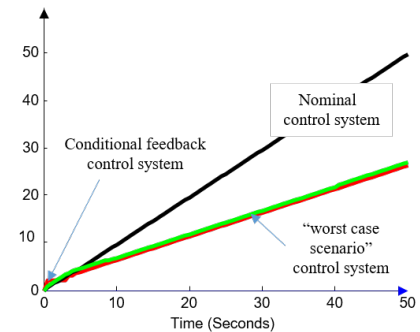


Fig.7 Energy consumption

In relative units, the energy consumption during the transition period for the nominal, „worst case scenario“ and the conditional feedback systems respectively are (14):

$$E_{NOM} = 49.5, E_{\text{worst case}} = 26.87, E_{CF} = 26.24 \quad (14)$$

The following facts could be stated: In conditional feedback control, energy savings are greatest.

When implementing a nominal PID controller and „worst case scenario“ of the plant, an increased energy consumption is achieved, as the transient process is characterized by overshooting, Fig.5.

#### IV. ROBUST ANALYSIS

On Fig.8 and Fig.9 the modus of the sensitivity functions and the complementary sensitivity functions for the nominal control system, for the „worst case scenario“ control system and for the conditional feedback control system are shown.

It can be seen that the introduction of a forming element reduces the functions of sensitivity and complementary sensitivity.

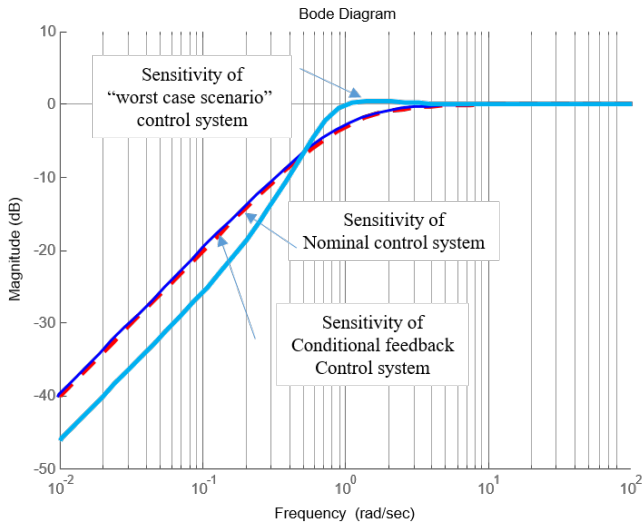


Fig.8 Bode plot of sensitivity of control systems

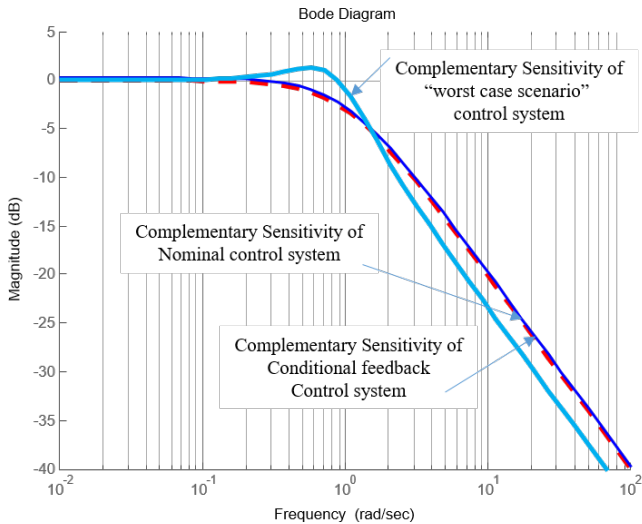


Fig.9 Bode plot of complementary sensitivity of control systems

The lack of a peak of the sensitivity and complementary sensitivity functions in the nominal system and in the conditional feedback system indicates a lack of over-shooting in the time domain (Fig.5).

The sensitivity function (15) is fundamental for the robust methods, namely zero error, in order to give priority to the complementary sensitivity function (16) in terms of tracking the set point for the system.

$$S(s) = \frac{1}{1 + W_R^F(s)W_o^*(s)} \quad (15)$$

$$T(s) = \frac{W_R^F(s)W_o^*(s)}{1 + W_R^F(s)W_o^*(s)} \quad (16)$$

Equation (17) is known, which is the basis of the mathematical formulation of the robust performance requirement (18).

$$S + T = 1 \quad (17)$$

$$|rS| + |w_M T| \leq 1 \quad (18)$$

$$\text{where } w_M = \frac{W_o^\square(s) - W_o^*(s)}{W_o^*(s)} \text{ и } r(s) = \frac{1}{s}.$$

Condition (18) is the sum of two components - the nominal performance condition (19) and the robust stability condition (20)

$$|rS| \leq 1 \quad (19)$$

$$|w_M T| \leq 1 \quad (20)$$

On Fig.10 it is shown a graphical representation of the conditions for nominal performance, robust stability and robust performance.

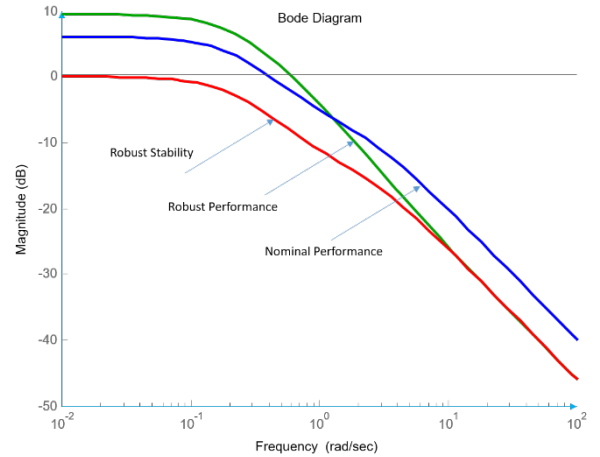


Fig.10 Nominal performance, robust stability and robust performance for conditional feedback control system

It could be seen that the condition for robust stability is fulfilled, which means that when the parameters of the control plant change, the closed system will always remain stable (fig.5). In regard with the robust performance, the stringency of the condition (18) is not satisfied, but in general the system with conditional feedback has robust properties, i.e. it retains the nominal performance of the controlled variable (Fig.5).

#### V. CONCLUSION

The presented control with conditional feedback allows to control thermal process, both in terms of robustness and in terms of energy efficiency.

However, the proposed control has limitations, as there is danger when there are great inaccuracies in the plant's model and a requirement for great time response control signal to

calculated such that could result in nonlinearities. In that case, including control limitation could lead to saturation.

In such cases, in case of significant uncertainty, it is necessary to apply another method to ensure robust properties, but in terms of energy efficiency there is no guarantee.

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