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Resonance and Dispersion Characteristics of Microstrip Slow-Wave Open-Loop Resonator

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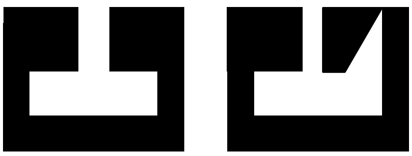
Abstract: **This paper represents a theory of admittance loaded slow-wave open loop resonators. The main resonator is extended to periodically loaded line. A closed form formulas for the fundamental and the first spurious resonance frequency are derived. Based on the Floquet's theorem, a dispersion equation of the slow-wave resonator is derived. The resonance characteristics are studied for different impedances of the loading stubs. Dispersion characteristics, defining the filtering function of the periodically loaded line, are also studied for different impedances and electrical length of the loading stubs.**

Keywords**: Dispersion equation, resonance characteristics, slow-wave open loop resonator.**

I. Introduction

Radio frequency and microwave planar bandpass filters are presently required in wide variety of applications of wireless communication systems, WLAN, software radio etc.

The compact sizes of the filters, combined with wider upper stopband are crucial factors for some wireless applications. However most of the microstrip bandpass filters are large in size and their first spurious frequencies appear at $2f_0$ or $3f_0$, where f_0 is the filter central frequency. Many authors propose using stepped impedance resonators [1,2] to shift the first spurious resonance frequency not on a multiple of the fundamental resonance. In [3] is presented a theory and experiment of slow-wave open loop resonators (Fig.1a) and their extension for capacitively loaded transmission line. This theory is a special case of a periodically admittance loaded transmission line. In [4] is proposed a slow-wave resonator loaded with different loading capacities (Fig.1b). [4] claims for higher unloaded quality factor than the slow-wave resonator presented in [3].



(a) (b)

Fig.1 Topologies of slow-wave open loop resonators

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This paper presents a general theory of slow-wave open loop resonator. Based on the circuit theory, closed form formulas for the fundamental resonance frequency and the first spurious resonance frequency are derived. Using the electrical parameters of the transmission lines, a general dispersion equation is derived.

The dispersion effect conducts the filtering function of the slow-wave resonator. It is studied four cases for slow-wave resonators. The obtained results are graphically shown and are applicable for practical design.

II. Resonance characteristics of slow-wave open loop resonator

The slow-wave open-loop resonator consists of a transmission line loaded on both sides with admittances Y_1 and Y_2. The main parameters of the transmission line are the characteristic impedance Z_c, length of the line l, and the propagation constant k. The resonator is shown on Fig.2.

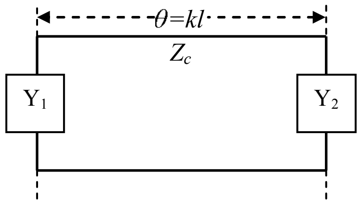

Fig.2 Admittance loaded slow-wave resonator

The electric length of the unloaded line becomes $\theta = \gamma l$, where $\gamma = jk$. The electrical characteristics of the resonator may be described by the ABCD matrix. The overall matrix of the resonator is a product of the three ABCD matrices - admittances Y_1, Y_2 and the unloaded transmission line.

$$[ABCD] = [ABCD]_1 [ABCD]_2 [ABCD]_3 \quad (1),$$

where

$$[ABCD]_1 = \begin{bmatrix} 1 & 0 \\ Y_1 & 1 \end{bmatrix},\ [ABCD]_3 = \begin{bmatrix} \cosh\theta & Z_c \sinh\theta \\ \frac{1}{Z_c}\sinh\theta & \cosh\theta \end{bmatrix},$$

$$[ABCD]_3 = \begin{bmatrix} 1 & 0 \\ Y_2 & 1 \end{bmatrix}.$$

After the multiplication, the elements of the ABCD matrix are derived as:

$$A = \cosh\theta + Y_2 Z_c \sinh\theta \quad (2a)$$

$$B = Z_c \sinh\theta \quad (2b)$$

$$C = \left(Y_1 + Y_2\right)\cosh\theta + \left(Y_1 Y_2 Z_c + \frac{1}{Z_c}\right)\sinh\theta \qquad (2c)$$

$$D = Z_c Y_1 \sinh\theta + \cosh\theta \qquad (2d)$$

It is easily checked out the main property of the ABCD matrix: $AD - BC = 1$.

If a standing wave is excited in the slow-wave resonator and taking into account the boundary conditions $I_1 = 0$ and $I_2 = 0$, consequently for non-zero voltages V_1 and V_2 it is required that:

$$\frac{C}{A} = \left.\frac{I_1}{V_1}\right|_{I_2=0} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = 0\,. \qquad (3)$$

It is clear that for the fundamental resonance:

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = -1\,. \qquad (4)$$

Solving the Eqs (3) and (4) it is derived the following quadratic equation:

$$\left(Y_1 + Y_2\right)\tan h^2\frac{\gamma l}{2} - \frac{2}{Z_c}\tan h\frac{\gamma l}{2} + Y_1 - Y_2 = 0 \qquad (5)$$

The solutions for the electrical length of unloaded transmission line are derived from the Eq.(5).

$$\tan h\frac{\gamma_0 l}{2} = \frac{1/Z_c \pm \sqrt{1/Z_c^2 - \left(Y_1^2 - Y_2^2\right)}}{Y_1 + Y_2} \qquad (6).$$

For the first spurious resonance we have:

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = 1 \qquad (7).$$

Then solving the system of Eqs. (3) and (7) leads to the following quadratic equation:

$$\left(Y_2 - Y_1\right)\tan h^2\frac{\gamma l}{2} + \frac{2}{Z_c}\tan h\frac{\gamma l}{2} + Y_1 + Y_2 = 0 \qquad (8)$$

The solutions for the electrical length of the unloaded transmission line of the slow-wave resonator ($Y_1 \neq Y_2$) are:

$$\tan h\frac{\gamma_1 l}{2} = \frac{-1/Z_c \pm \sqrt{1/Z_c^2 - \left(Y_1^2 - Y_2^2\right)}}{Y_2 - Y_1} \qquad (9).$$

In the most practical cases the loading admittances Y_1 and Y_2 are equal ($Y_1 = Y_2 = Y$) [1,3]. Then the transmission line electrical length for the fundamental resonance is derived from Eq.(6) as:

$$\tan h\frac{\gamma_0 l}{2} = \frac{1}{Z_c Y} \qquad (10)$$

For the spurious resonance the equation is:

$$\tan h\frac{\gamma_1 l}{2} = -Z_c Y \qquad (11)$$

For microstrip slow wave resonators often the loading admittances are open circuited stubs. In the case of open stub the input admittance is $Y_{in} = Y_{opst}\tan h\theta_{opst}$. Assuming a lossless lines ($\alpha = 0$) the propagation constant is a pure imaginary value. Then the fundamental resonance condition becomes:

$$\theta_0 = 2arc\tan\left(Y_{opst} Z_c tg\theta_{opst}\right) \qquad (12)$$

The electrical length of the unloaded transmission line for the first spurious resonance is found as:

$$\theta_1 = 2\pi - 2arc\tan\left(Y_{opst} Z_c tg\theta_{opst}\right) \qquad (13)$$

It is clearly seen that for unloaded resonator, the resonator electrical length for the fundamental resonance is $\theta_0 = \pi$, and for the first spurious resonance- $\theta_1 = 2\pi$.

It is shown on Fig.3 the dependence of the overall electrical length for the fundamental resonance of the slow-wave resonator ($\theta_{t0} = 2\theta_{opst} + \theta_0$) as a function of the open stub length (θ_{opst}) for a constant impedance ratio $K = Z_c / Y_{opst}$.

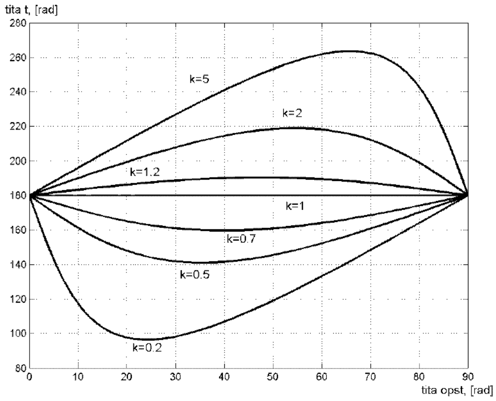

Fig.3 Electrical length of the resonator for the fundamental frequency

If the impedance ratio $K < 1$ the resonator is shorter than the halfwave unloaded resonator. This fact is applicable for small sized resonators used in mobile communication systems. But for $K > 1$, the resonator is longer than a halfwave resonator. It is also seen that for big or small values of the impedance ratio K, it is well pronounced a maximum of the resonator length θ_t.

Fig.3 shows the dependence of the slow-wave resonator electrical length for the first spurious resonance as a function of as a function of the open stub length θ_{opst} for a constant impedance ratio K.

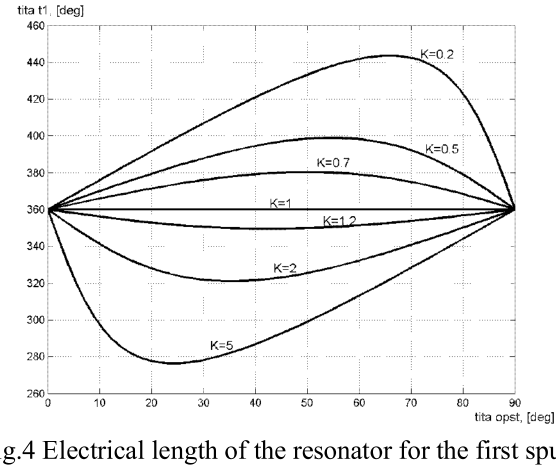

Fig.4 Electrical length of the resonator for the first spurious frequency

It is clearly seen form Fig.4 that for unloaded halflength resonator, the overall electrical length is $2\pi\, rad$ for the first spurious resonance. For impedance ratio $K<1$, the resonator electrical length is larger than halfwave resonator. It is obvious that for larger impedance ratio the resonator length for the first spurious resonance is shorter.

Fig.4 shows the dependence of the ratio of the resonator overall lengths θ_{t1}/θ_{t0} as a function of the electrical length of the open stub for various impedance ratios K. For $K=1$ (halfwave resonator) it is seen that $\theta_{t1}/\theta_{t0}=2$ i.e. the first spurious frequency is twice the fundamental resonance frequency.

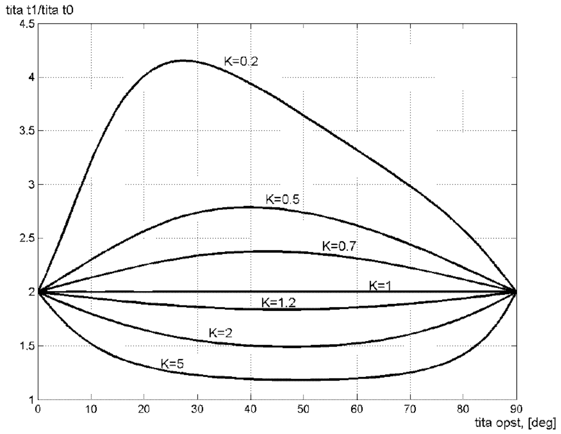

Fig.5. Ratio of the resonator lengths θ_{t1}/θ_{t0} as a function of the electrical length of the open stub.

To control the first spurious frequency, it is necessary to choose a proper impedance ratio. In the most common cases the first spurious resonance is not a multiple of the fundamental resonance and it is greater than twice the fundamental. From Fig.5 it is clear that in these cases the impedance ratio should be less than 1. Then the length of the resonator is less than the halfwave resonator.

III. Dispersion Equation

We may consider the slow-wave resonator from Fig.1 for a unit cell of as periodically loaded transmission line. This assumption allows explaining the physical mechanism of the filtering characteristics of the periodically loaded transmission line. Let's β is the propagation constant in the periodic admittance loaded transmission line. Then applying the Floquet's theorem [5] i.e.:

$$V_2 = e^{-j\beta d}V_1$$
$$-I_2 = e^{-j\beta d}I_1 \qquad (14)$$

to the [ABCD] matrix of an unit cell results in:

$$\begin{bmatrix} A-e^{-j\beta d} & B \\ C & D-e^{-j\beta d} \end{bmatrix} \cdot \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad (15)$$

The non-zero solution for the output voltage V_2 and current $-I_2$ exists only if the determinant is zero. This leads to the following equation:

$$\left(A-e^{-j\beta d}\right)\left(D-e^{-j\beta d}\right)-BC=0 \qquad (16)$$

Substituting Eq.(2a-2d) in (16) and taking into account the [ABCD] matrix property $AD-BC=1$ we derive the dispersion equation of the slow wave open loop resonator.

$$\cosh(\beta d)=\cosh(\theta)+Z_c\frac{(Y_1+Y_2)}{2}\sinh(\theta)$$
(17)

If the transmission line is loaded with equal open stubs $Y_1=Y_2=Y$, the dispersion equation (17) becomes:

$$\cosh(\beta d)=\cosh(\theta)+Z_cY\sinh(\theta) \qquad (18)$$

For the fundamental frequency substituting (12) in (18) it is derived $\cosh(\beta_0 d)=-1$. For the first spurious frequency it is seen that $\cosh(\beta_1 d)=1$. Here the propagation constants are $\beta_0=\omega_0/v_{p0}$ and $\beta_1=\omega_0/v_{p1}$, where v_{p0} and v_{p1} are the phase velocities for the fundamental and the first spurious resonance. If there is no dispersion, the phase velocity is a constant. This is true for unloaded transmission line. The phase velocity is frequency dependant for periodically admittance loaded transmission line. This fact determines the filtering function of the periodically loaded transmission lines.

There are studied two main types of slow wave resonators - with impedance ratios $K<1$ and $K>1$.

The first case is for slow wave open loop resonator with $\theta_{opst}=\pi/6\, rad, Z_{opst}=1/Y_{opst}=20\Omega$ and $Z_{opst}=1/Y_{opst}=40\Omega$. The characteristic impedance of the main transmission line is $Z_c=50\Omega$. The dispersion curves are shown on Fig.6.

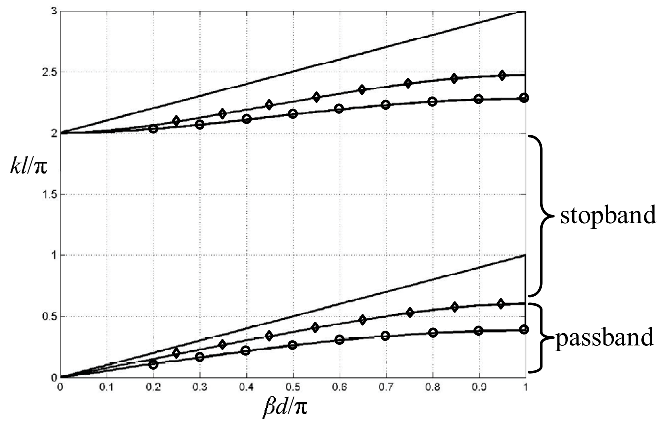

Fig.6 Dispersion curves for $\theta_{opst}=\pi/6\, rad, Z_{opst}=20\Omega$ (circles) and $Z_{opst}=40\Omega$ (diamonds), halfwave resonator-solid line

It is seen from Fig.5 that increasing the characteristic impedance of the loading stub leads to decreasing of the dispersion effect. For $Z_{opst}=40\Omega$, the passband is wider than the case with $Z_{opst}=20\Omega$. For unloaded transmission line resonator (halfwave resonator case) it is seen that the first spurious passband is placed on twice the fundamental resonance frequency.

The next studied resonator is with loading stubs with electrical length $\theta_{opst}=\pi/12\, rad$ and characteristic impedances $Z_{opst} = 20\Omega$ and $Z_{opst} = 40\Omega$. The results are shown on Fig.7.

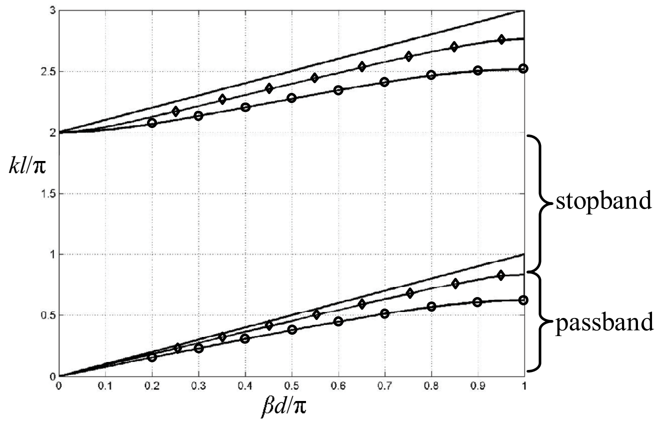

Fig.7 Dispersion curves for $\theta_{opst} = \pi/12\,rad, Z_{opst} = 20\Omega$ (circles) and $Z_{opst} = 40\Omega$ (diamonds), halfwave resonator-solid line

Comparing both Fig.6 and Fig.7, it is clearly seen that longer loading stubs lead to well pronounced dispersion effect. This effect results in better filtering functions and a possibility to control the passband and the stopband. Moreover choosing proper characteristic impedance of the loading stubs and their electrical length, it may control the first spurious passband.

The dispersion curves for $K < 1$ and $\theta_{opst} = \pi/6\,rad$ are shown on Fig.8.

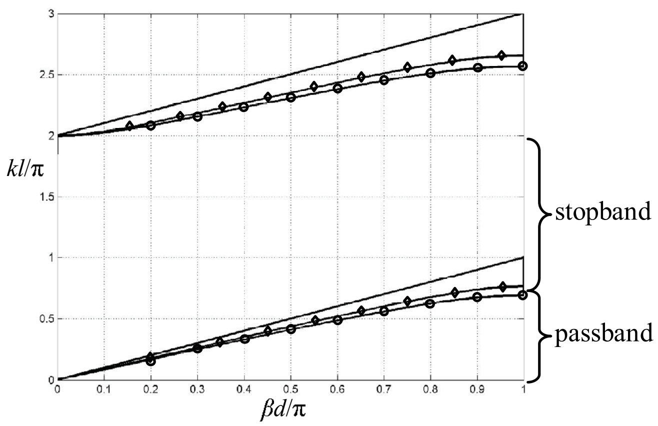

Fig.8 Dispersion curves for $\theta_{opst} = \pi/6\,rad, Z_{opst} = 55\Omega$ (circles) and $Z_{opst} = 75\Omega$ (diamonds), halfwave resonator-solid line

It can be found from Fig.8 that the dispersion effect is less pronounced than the case for $K > 1$ (Fig.6 and Fig.7). However the slope of the dispersion curve for the fundamental resonance is steeper than the slope for the first spurious resonance. This fact forms narrower spurious passband than the fundamental passband. If the characteristic impedance is bigger ($Z_{opst} = 75\Omega$), the dispersion effect is closer to the halfwave unloaded resonator.

The last studied case is for slowwave resonator loaded with open stubs, having characteristic impedance greater than the main transmission line and electrical length $\theta_{opst} = \pi/12\,rad$.

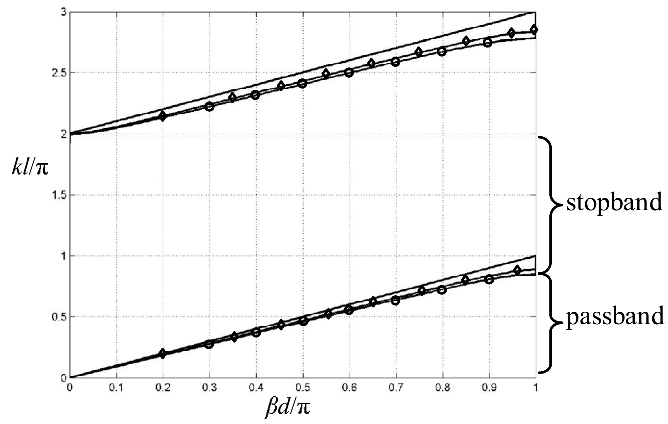

Fig.9 Dispersion curves for $\theta_{opst} = \pi/12\,rad, Z_{opst} = 55\Omega$ (circles) and $Z_{opst} = 75\Omega$ (diamonds), halfwave resonator-solid line

It is clearly seen from Fig.9 that the dispersion effect is less pronounced for short high impedance open stubs. The passband is almost equal to the passband of halfwave resonator.

V. CONCLUSION

This paper presents a general theory of slow-wave open loop resonator. Based on the circuit theory, closed form formulas for the fundamental resonance frequency and the first spurious resonance frequency are derived. Using the electrical parameters of the transmission lines, a general dispersion equation is derived. The theoretical view of the admittance loaded transmission line resonator gives insight for the main properties of this type of resonator. The filters designed with slow-wave resonators will have wider stopband characteristics, due to the dispersion effect.

REFERENCES

[1] Makimoto M. and S. Yamashita. "Bandpass Filters Using Parallel Coupled Stripline Stepped Impedance Resonators." 1980 Transactions on Microwave Theory and Techniques Dec. 1980 pp.1413-1417.

[2] Sheta A.F., J.P. Coupez, G. Tanne, S. Toutain and J.P. Blot. "Miniature Microstrip Stepped Impedance Resonator Bandpass Filters and Diplexers for Mobile Communications." 1996 MTT-S International Microwave Symposium Digest pp.607-610.

[3] Hong, Jia-Sheng and M.J. Lancaster. "Theory and experiment of novel microstrip slow-wave open-loop resonator filters." 1997 Transactions on Microwave Theory and Techniques 45.12 (Dec. 1997, Part II [T-MTT] (1997 Symposium Issue)): 2358-2365

[4] Akkaraekthalin, Jantree, Microstrip Slow-wave Open-Loop Resonator Filters with Reduced Size and Improved Stopband Characteristics, ETRI Journal, Vol.28, Number 5, Oct.2006, pp.604-617.

[5] Collin, Foundation for Microwave Engineerig, McGraw Hill, NY, 2002