Theoretical Model of the MUD Algorithm Based on the Diagram of States and Markov's Chains

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***Abstract* – The paper presents a development of a Markov's chains of the work of quasioptimal algorithm for multiuser detection. Through it identifies: functional accuracy of the algorithm-respectively the lower limit of the error probability in a channel with Rayleigh fading and its computational complexity. Program models are realized in MATLAB, in order to research the algorithm performance and their parameters. Graphic presentations from the measurements are given to prove the exactness of the analytical formulas.**

Keywords* – *MUD, CDMA, Markov chain, diagram of states.

I. INTRODUCTION

CDMA is an efficient method for sharing a mobile radio channel. The noise resistance of the system decreases due to internal system interference – MUI (multi user interference). The correlation receiver is optimal when no MUI exists. The multi user detection (MUD) is used to minimize the influence of the MUI [1].

The MUD receiver, based on the maximum likelihood criteria (ML) [1], gives an optimal solution and checks all possible combinations of transmitted symbols. The number of the calculations grows exponentially with the number of the active users, which is a disadvantage of ML MUD. There are many suggested methods and algorithms for suboptimal receiving, decreasing the needed number of detection calculations. They are compromise between calculation complexity and quality parameters of the receiver. [8,9].

The research is focused on the parameters and the possibilities of the quasioptimal algorithm of MUD [8,9], based on the serial search. The fast-response of the algorithm is due to strong criteria of search discontinuation and selection of start point of the optimization after the single correlation receiving.

A diagram of states and Markov's chains is used for modeling the process of the algorithm. The result of the analysis is the error probability of the algorithm, the dependency of its accuracy from the number of iterations and users.

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II. MUD MODEL IN SYNCRONIOUS CDMA SYSTEM

A block diagram of the system is shown on figure 1. The signal processing is made in baseband.

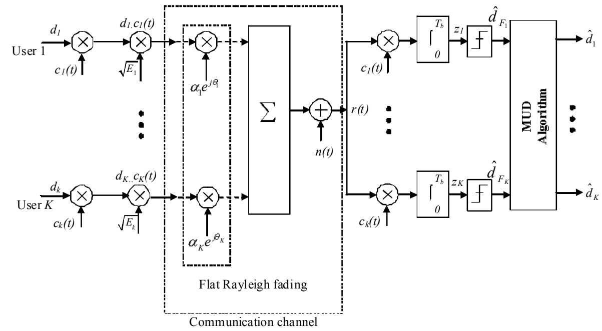

Fig. 1. Block diagram of MUD

The model is described in details in [8]. Here K users are using synchronous transition with direct spread spectrum – DSS and 2PSK modulation.

After applying criteria for maximum of aposterior probability (MAP), optimal MUI is achieved. A logarithmic function of likelihood is presented in matrix form [1]:

$$\Psi(\boldsymbol{d}) = \boldsymbol{d}^T \boldsymbol{EARA}^* \boldsymbol{Ed} - 2\Re(\boldsymbol{d}^T \boldsymbol{EA}^* \boldsymbol{z}) \quad (1).$$

Base components in (1) are:

$\boldsymbol{d} = [d_1, d_2, ..., d_K]^T$ - a matrix, containing data, transmitted from user k;

$\boldsymbol{A} = diag[\alpha_1 e^{j\theta_1}, \alpha_2 e^{j\theta_2}, ..., \alpha_K e^{j\theta_K}]$ - a diagonal matrix with complex coefficients of transmission channel for the corresponding user. The amplitudes are with Rayleigh distribution. The phase shifting is with normal distribution in [0, 2π]. The channels of all users are statistically independent;

$\boldsymbol{E} = diag\left[\sqrt{E_1}, \sqrt{E_2}, ..., \sqrt{E_K}\right]$ - a diagonal matrix. $\sqrt{E_k}$ is the symbol energy of user k;

$\boldsymbol{c} = [c_1(t), c_2(t), ..., c_K(t)]$ - a matrix, where every row is a pseudorandom binary sequence (PBS) for the corresponding user;

$\boldsymbol{R}$ - a cross-correlation KxK matrix, which coefficients are the values of the normalized cross-correlation functions of PBS.

$\boldsymbol{n} = [n_1, n_2, ..., n_K]^T$ - Gaussian noise after the correlator, with the covariance matrix $\boldsymbol{R}_n = 0.5 N_o \boldsymbol{R}$.

III. Theoretical Model of Work of the MUD Algorithm Based on the Diagram of States and Markov's Chains

For simplicity, we will consider the following conditions:

1. The communication channels are without additive white Gaussian noise. This assumption is made in order to calculate the error probability rate of the algorithm caused by its quasioptimal solution;
2. The power of the received signals from all users is equal;
3. Bipolar code sequences with uniform distribution of the values ±1 are used for DSS;
4. The process gain is equal to the length of the random sequence N, when it is sufficiently long.

The work of the algorithm could be described with a cell diagram, where the value of the target function $\Psi(\boldsymbol{d})$ is included.

Figure 2 shows a possible cell diagram for K=3. Each junction represents a data vector $\boldsymbol{d}$ located away from the optimal point $\boldsymbol{d_0}$ with certain code distance. The problem is to find the minimum of $\Psi(\boldsymbol{d})$.

For a given initial vector (point) $\boldsymbol{d_{l-1}}$ the algorithm goes through the vector space with Hamming distance H_d=1 with consecutive changes of each element of the vector [8]. For each vector from that space, $\Psi(\boldsymbol{d})$ is calculated. This represents a step of the algorithm to find the next point which is closer to transmitted vector $\boldsymbol{d_0}$.

The solution of the algorithm for the $\boldsymbol{l}$-th iteration is calculated with:

$$\boldsymbol{d}_l = \arg\left\{\min_{d\in M_d}\left[\Psi(\boldsymbol{d})\right]\right\} \text{ и } \quad M_d=\left\{d: H_d\left(\boldsymbol{d}_{l-1},\boldsymbol{d}\right)=1\right\} \qquad (2).$$

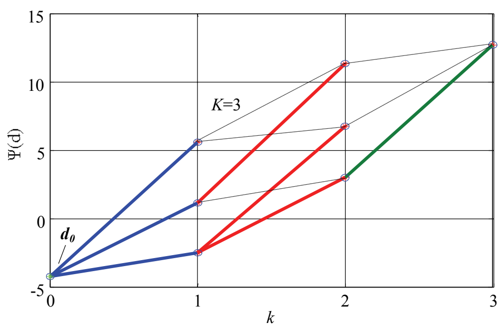

Fig. 2. Cell diagram for K=3

One step of iteration is inverting the value of one element of vector $\boldsymbol{d_{l-1}}$. The new vector $\boldsymbol{d_l}$ is defined. The attempt is successful when:

$$\max_{d\in M_d}\left\{\left[\Psi(\boldsymbol{d_{l-1}})-\Psi(\boldsymbol{d_l})\right]>0\right\} \text{ и } H_d(d_{l-1},d_0)>H_d(d_l,d_0) \qquad (3).$$

If the algorithm is in point $\boldsymbol{d_{l-1}}$, during the iteration, the following hypotheses are possible:

1. H_{+1} - **step forward** – A vector $\boldsymbol{d_l}$ is found, which code distance to the optimal point $\boldsymbol{d_0}$ is less than the one to $\boldsymbol{d_{l-1}}$. The hypothesis is true when the below conditions are satisfied:
$$\max_{d\in M_d}\left\{\left[\Psi(\boldsymbol{d_{l-1}})-\Psi(\boldsymbol{d_l})\right]>0\right\} \text{ и } H_d(d_{l-1},d_0)>H_d(d_l,d_0);$$
2. H_{-1} - **step backward** – The vector $\boldsymbol{d_l}$ has longer code distance than $\boldsymbol{d_{l-1}}$. The necessary conditions are:
$$\max_{d\in M_d}\left\{\left[\Psi(\boldsymbol{d_{l-1}})-\Psi(\boldsymbol{d_l})\right]>0\right\} \text{ и } H_d(d_{l-1},d_0)<H_d(d_l,d_0);$$
3. H_0 - **an extremum is reached** – $\boldsymbol{d_l}=\boldsymbol{d_{l-1}}$, $[\Psi(\boldsymbol{d_{l-1}})-\Psi(\boldsymbol{d_l})]\le 0$ for $\boldsymbol{d}\in \boldsymbol{M_d}$.

The extremum can be local, when the cost function is not unimodal. The global extremum corresponds to the transmitted vector. The local extremum may not be the same as the global extremum $\boldsymbol{d_0}$, because the algorithm is quasioptimal. If the impact of AWGN is ignored, the algorithm fully compensates the cross-channel interference if the point $\boldsymbol{d_{l-1}}$ has a code distance 1 to the global extremum $\boldsymbol{d_0}$, and then with the next iteration the global extremum would be found.

The algorithm performance, error decision probability and the minimal limit error probability with defined number of operations can be found with the help of the probability theory.

The probability to get to the global extremum can be specified by describing the algorithm by Markov chain. During iteration the probability for transition from one point to another can be described by directed graph shown on Figure 3.

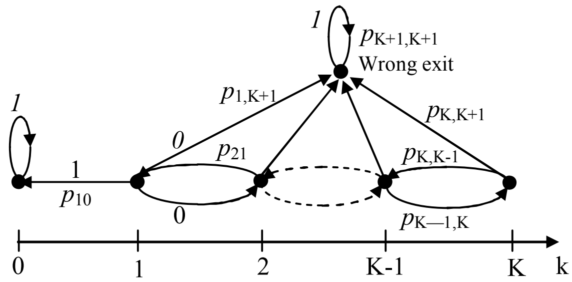

Fig. 3. Directed graph

Every point corresponds to a junction and is located on fixed code distance from the global extremum. The number of the junction is equal to the number of the users K+1. Junction 0 represents the transmitted vector, which is the global extremum $\boldsymbol{d_0}$. The probability p_{10} of passing through junction 1 to 0 is one. If there is no receiving error the algorithm will begin the search with 0 code distance and will remain in this junction – this is the probability p_{00}=1. Due to this condition the Markov chain is reducible. There is one more reducible state when the algorithm reaches a local extremum that is different from the global extremum. On figure 3 this state is marked with index K+1 – „Wrong exit”. The algorithm will cancel the search and the received vector is with errors.

$$[P_M]=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & p_{21} & 0 & p_{23} & 0 & \cdots & 0 & 0 & 0 & p_{2,K+1} \\ 0 & 0 & p_{32} & 0 & p_{34} & \cdots & 0 & 0 & 0 & p_{3,K+1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \cdots & p_{K-1,K-2} & 0 & p_{K-1,K} & p_{K-1,K+1} \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & p_{K,K+1} \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & p_{K,K-1} & 0 & 1 \end{bmatrix} \qquad (4).$$

The Markov transition matrix for iteration of the algorithm is described with (4).

The transition probabilities can be defined, if the distribution of the values of the target function is known. Due to the great number of random variables in the matrix it is suggested to operate with equivalent random variable y, which probability density is $f_y(y)$. It is the difference between the values of the target function $\boldsymbol{d}_{l-1}(k)$ and $\boldsymbol{d}_l(k-1)$, where the argument is the code distance to $\boldsymbol{d}_0$ and:

$$y_l(k) = \Psi(\boldsymbol{d}_{l-1}(k)) - \Psi(\boldsymbol{d}_l(k-1)) \quad (5).$$

Once (5) is converted and the code distance between the starting point for an iteration and transmitted vector is $H_d(d_{l-1}, d_0)=k$, (6) is obtained:

$$y_l(k) = 2(\sum_{i=1}^{k-1} z_i - 2\chi^2) \quad (6),$$

where χ is a random variable with χ^2 distribution and variance $\sigma^2 = 1$ comes from the fact that the channel has Rayleigh distribution of transmission coefficient. Random variable z_i is: $z_i = \alpha_m \alpha_n \overline{r}_{mn}$, $\alpha_{m,n}$ takes values ±2 and discrete uniform distribution – depends on the transmitted symbols. Other random variable is:

$$\overline{r}_{mn} = R_{mn}[\mathrm{Re}(C_m)\mathrm{Re}(C_n) + \mathrm{Im}(C_m)\mathrm{Im}(C_n)],$$

where distribution of values of cross correlation function R_{mn} can be approximated with normal distribution with variance $\sigma^2=1/N$. The real part $\mathrm{Re}(C_m)$ is with normal distribution $\mu=0$ and mathematical expectation $\sigma^2=0.5$.

Analytical determination of the density distribution of $y_l(k)$ $f_y(y(k))$ is not an easy task, so computer modeling with method Monte Carlo is proposed. f_y distributions are independent and identical for each point, which is on the same code distance from $\boldsymbol{d}_0$.

Correct decision for the l-th iteration will occur (Hypothesis H_+), if both of the following conditions are observed:

1. At least one of the k points with code distance $H_d(d_l,d_0)<H_d(d_{l-1},d_0)$ satisfies $[\Psi(\boldsymbol{d}_{l-1}) - \Psi(\boldsymbol{d}_l)] > 0$;
2. At least one of total K-k points with $H_d(d_l, d_0) > H_d(d_{l-1}, d_0)$ satisfies the condition $[\Psi(\boldsymbol{d}_{l-1}) - \Psi(\boldsymbol{d}_l)] < 0$.

The probability of execution of the first and second condition and their independence for iteration (probability of correct decision (Hypothesis H_+)) is:

$$Pd_k = \left\{1 - \left[\int_{-\infty}^{0} f_y(y(k))dy\right]^k\right\}\left\{1 - \left[\int_{-\infty}^{0} f_y(y(k+1))dy\right]^{K-k}\right\} \quad (7).$$

The probability for not finding a better point during next iteration and the end of the search algorithm (Hypothesis H_0 (an extremum is reached)) depends on the distribution of the random variable $[\Psi(\boldsymbol{d}_{l-1}) - \Psi(\boldsymbol{d}_l)] \le 0$ where $\boldsymbol{d} \in \boldsymbol{M}_d$. In iteration, k attempts reduce the code distance and K-k attempts increase the code distance.

The probability algorithm remains at the same point and the search end is determined by:

$$Po_k = \left[\int_{-\infty}^{0} f_y(y(k))dy\right]^k \left[\int_{-\infty}^{0} f_y(y(k+1))dy\right]^{K-k} \text{ for } k=1...K \quad (8).$$

The algorithm's probability to make a mistake - to differ from the optimal point, hypothesis H_{-1}, is equal to:

$$Perr_k = 1 - Po_k - Pd_k \quad (9).$$

Once the transition probabilities in the diagram of figure 3 are known, the elements of the matrix [Pm] are determined by:

$p_{00} = p_{10} = 1$, $p_{k,k-1} = Pd_k$, $p_{k,K+1} = Po_k$, $p_{k,k+1} = Perr_k$ for $k = 2..K-1$, $p_{K,K-1} = Pd_K$, $p_{K,K+1} = Po_K$, за $k = K$, $p_{K+1,K+1} = 1$.

IV. Functional precision of the algorithm

The probability, after detection with hard decision, for the algorithm to begin a search from an initial vector with code distance k is given by [8]:

$$P_K(k) = \frac{K!}{(K-k)!\,k!}\left(0.5 - 0.5/\sqrt{1+(K-1)/N}\right)^k \cdot \left(0.5 + 0.5/\sqrt{1+(K-1)/N}\right)^{K-k} \quad (10).$$

The vector $[P_K]$, arranged by the $P_K(k)$ for k=0...K, is the initial vector of the initial states of the Markov Chain. The vector $[P_L]$, with probabilities of occurrence of k fold error (k=0...K) after L number of iterations, is determined by the matrix equation:

$$[P_L] = [P_K][P_M]^L \quad (11),$$

The probability of starting the algorithm from the k-th code distance is defined by (10). The error probability of the algorithm after iteration is determined by two hypotheses. The first hypothesis is that the algorithm finds a vector with smaller code distance. The second one is to end the search – Wrong exit.

The matrix with error probability after the L number of iterations is:

$$[P_{err}]_L = [P_L] + [Po] \qquad [Po] = [Pu(:,K+2)']$$
$$[Pu] = [diag(P_K)][P_M]^L \quad (12).$$

The vector $[Po]$ is composed by elements of the last column of the matrix $[Pu]$. Due to the independence of the communication channels, the average probability of error per bit after the number L of iterations of the algorithm can be calculated with:

$$P_{e_L} = \sum_{k=1}^{L} \frac{k}{K} p_{err}(k) \quad (13).$$

V. Results of simulation

For the purposes of the research a model program in MATLAB, which finds the distribution of random variable *y* using the Monte Carlo method, is created.

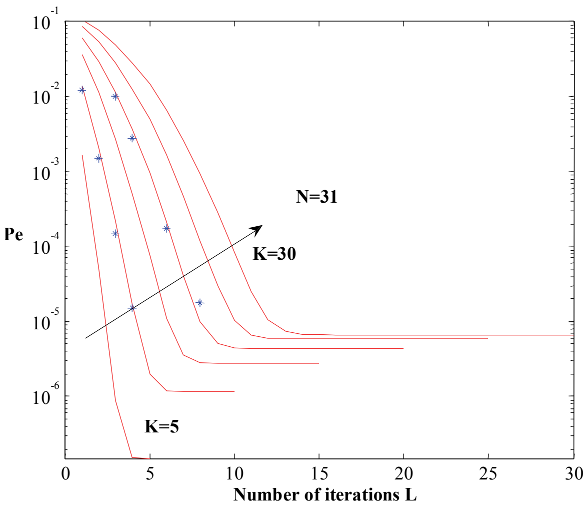

Fig. 4. P_e depending on *L* and *K*

Equation from formula (6) is used and with (4), (8), (9) and (12) the dependency of the probability of error per bit, depending on the number of iterations *L* with the number of users *K* as a parameter, is defined. The results are obtained for DS-CDMA system with transmission and reception of *K* number of users working simultaneously. The system operates in an environment with multipart slow Rayleigh fading. The results are shown on Figure 4. The red curves are theoretical estimates - from formula (6) and the blue dots (star *) are the results of the simulation.

The probability of error per bit, depending on the functional accuracy of the quasioptimal algorithm is shown in figure 5. The dependence is the number of active users *K*.

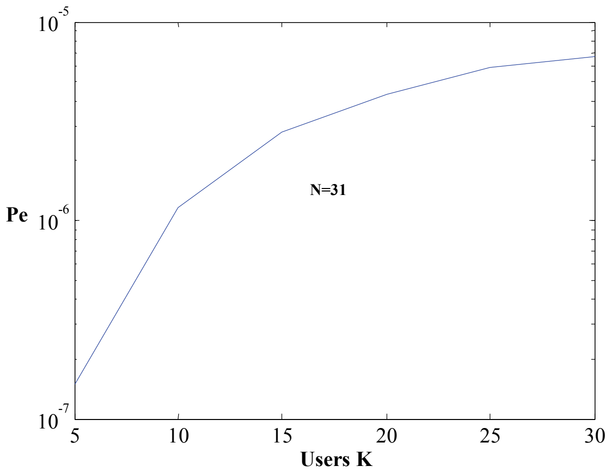

Fig. 5. P_e depending on *K*

VI. Conclusion

The paper presents a development of a Markov's chains of the work of quasioptimal algorithm for multiuser detection. Through it identifies: functional accuracy of the algorithm-respectively the lower limit of the error probability in a channel with Rayleigh fading and its computational complexity. Program models are realized in MATLAB, in order to research the algorithm performance and their parameters. Graphic presentations from the measurements are given to prove the exactness of the analytical formulas.

With this approach of modeling the algorithm of MUD with a Markov chains can be determined the average number of iterations needed to reach the extremum.

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