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Analysis of Fizeau wedge with a non-air gap by plane wave expansion

Elena Stoykova*[a], Margarita Deneva[b], Marin Nenchev[b]
[a]Institute of Optical Materials and Technologies, Bulgarian Academy of Sciences, Acad. Georgi Bonchev Str., Bl.109, 1113 Sofia, Bulgaria; [b]Technical University - Sofia, Plovdiv Branch, Optoelectronics and Laser Eng. Department, 25 Tsanko Diustabanov Str. Plovdiv, 4000, Bulgaria

ABSTRACT

The optical interferential wedge or Fizeau wedge (FW) is a useful optical element with various applications in optical metrology, spectroscopy and laser technique. Various FW applications require knowledge of its response to illumination by a laser beam with an arbitrary wavefront. Recently, we applied the plane wave expansion method to study transmission and reflection of an air-gap FW under illumination with a Gaussian beam. The approach is based on the angular spectrum of the beam and the known FW response to illumination with a plane wave. In this study, we adapt this approach for the more general and more frequently encountered case of a FW with a non-air gap. We developed an approximate algorithm, which is applicable at small incidence angles to wedges with refractive indices different from 1 and illuminating beams with arbitrary amplitude and phase distributions. Comparison to the experiment is also provided.

Keywords: Fizeau wedge, plane wave expansion, fringe pattern, transmission

1. INTRODUCTION

The optical interferential wedge or Fizeau wedge (FW)[1] finds application in optical metrology[2,3], spectroscopy[4,5] and laser technology[6]. A conventional FW consists of two flat mirrors divided by a gap with linearly increasing thickness. A plane monochromatic wave falling on the FW generates an infinite number of plane waves both in transmission and in reflection. The angular separation between these waves is given by twice the angle at the wedge apex. The amplitudes of the waves form a geometric progression with a common ratio given by two reflections inside the FW. As a result, the apex angle, reflection coefficients of the wedge coatings and the refractive index of the gap determine a unique interference pattern on both FW's sides. A high-reflectivity coating FW with an apex angle of 5–100 μrad and thickness of 5–1000 μm has linear dispersion for a spatially expanded and spectrally wide light beam and makes possible tuning of the transmitted light wavelength for a narrow light beam.

Various FW applications require knowledge of its response to illumination by a laser beam with an arbitrary wavefront. Calculation of the response at low reflectivity coatings can be done by two beams interference. Higher reflectivity requires taking in account multiple beam interference. A popular approach in this case is the plane wave approximation which relies on summation of the optical path differences between the multiple plane waves generated inside the FW[1]. The plane-wave approximation gives the fringe pattern at any distance from the wedge but fails in describing the change of this pattern and decrease in transmitted intensity with the wavelength. To avoid this drawback, we have proposed an approximate ray-tracing approach which calculates the response at arbitrary amplitude distribution in the falling beam under restriction of a constant phase distribution[7.8]. Because of this restriction, the fringe pattern is calculated only on the front or rear wedge surfaces. Both plane-wave approximation and ray-tracing approach enable calculations for a nor-air gap wedge. Recently, we applied the plane wave expansion method to study transmission and reflection of an air-gap FW under illumination with a Gaussian beam[9,10]. The essence of this approach is usage of the angular spectrum of the beam and the known FW response to illumination with a plane wave. In this study, we adapt this approach for the more general and frequently encountered case of a FW with a non-air gap. The developed algorithm is applicable to wedges with refractive indices different from 1 and illuminating beams with arbitrary amplitude and phase distributions. Comparison to the experiment is also provided.

*elena.stoykova@gmail.com

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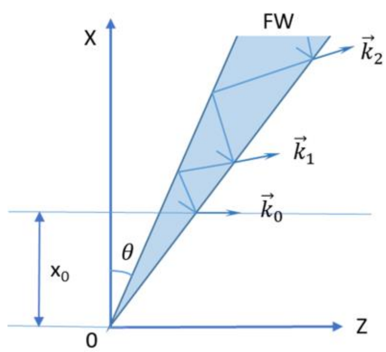
2. TRANSMISSION OF A NON-AIR GAP FIZEAU WEDGE

Let us consider a coherent monochromatic light beam with a wavelength λ falling normally to the FW's ridge. This illumination geometry corresponds to two-dimensional calculation. The FW has an apex angle, α_W , reflectivity of the coatings *R* and index of refraction of the gap between the reflecting surfaces, *n*. A plane wave falling on the wedge undergoes multiple reflections inside the wedge gap and produces an infinite number of plane waves both in transmission and in reflection. These plane waves have a zero phase difference at the wedge apex. For this reason, we choose the wedge apex for an origin of the coordinate system (X,Z). The axis Z is parallel to the propagation direction of the plane wave (Fig.1 left). The X axis subtends angle θ with the front wedge surface, where θ is the angle of incidence of the plane wave with respect to the wedge normal.



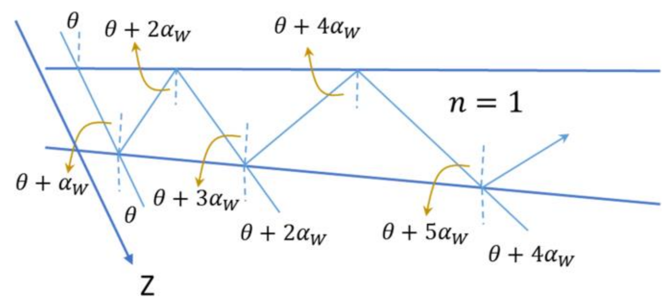


Figure 1. Left: geometry for calculation the FW's transmission. Right: ray-tracing for an air-gap wedge.

Calculation of the transmitted fringe pattern at plane wave illumination requires summation of the complex amplitudes of the plane waves formed due to multiple reflection from the front and rear wedge surfaces. For an air-gap FW, a ray falling on the front surface at angle, θ, with respect to its normal produces infinite number of rays. The directly transmitted ray leaves the wedge at angle, θ, to the front surface normal. The ray reflected once from the front surface inside the gap leaves the wedge at angle, $\theta + 2\alpha_W$, to this normal respectively. The ray reflected twice from the front surface inside the gap leaves the wedge at angle, $\theta + 4\alpha_W$, to the front surface normal etc. In the coordinate system (X,Z), the wave-vectors of the generated plane waves are given by:

$$\vec{k}_p = \left(k_{px}, k_{pz}\right), \quad k_{px} = k\sin\left(2p\alpha_W\right), \quad k_{pz} = k\cos\left(2p\alpha_W\right), \quad k = \frac{2\pi}{\lambda},\ p = 0,1,2... \tag{1}$$

When the index of refraction, *n*, inside the gap is larger than 1, the generated plane waves emerge at different angles, as is seen in Fig.2 left. The angles are given by:

$$\theta_{tp} = \arcsin\left\{n \times \sin\left[\arcsin\left(\frac{\sin\theta}{n}\right) + \left(2p+1\right)\alpha_W\right]\right\} - \alpha_W\ ,\ p = 0,1,2... \tag{2}$$

In (X,Z),the wave vectors of the of the produced multiple waves are

$$\vec{k}'_p = \left(k'_{px}, k'_{pz}\right), \quad k'_{px} = k\sin\left(\theta_{tp} - \theta\right), \quad k'_{pz} = k\cos\left(\theta_{tp} - \theta\right), \quad k = \frac{2\pi}{\lambda},\ p = 0,1,2... \tag{3}$$

The angles are determined with respect to the front surface normal, which subtends angle θ with the propagation direction of the falling plane wave. As a rule, a stable resonant fringe pattern is required in most of the FW's applications. This entails working at small angle of incidence (θ≤5°). The wedge apex angle, α_W , is of the order of tens of micro-radians. The maximum value of the parameter p, which gives the number of the multiple waves to be taken into account, depends on the reflectivity of the coatings. At R = 0.96 (high reflectivity) it does not exceed 150, and the largest value of $(2p+1)\alpha_W$ is less than 1 degree. Therefore, for many of the FW applications it is possible to accept the approximation $\sin\theta \approx \theta$. The approximate angles at which the multiple rays leave the wedge in this case are given in Fig.2 right. The wave vectors of the generated plane waves in the coordinate system (X,Z) are as follows:

$$\vec{k}'_p = (k'_{px}, k'_{pz}),\ k'_{px} = k\sin[(2p+1)n\alpha_W - \alpha_W],\quad k'_{pz} = k\cos[(2p+1)n\alpha_W - \alpha_W],\quad k = \frac{2\pi}{\lambda},\ p = 0,1,2... \tag{4}$$

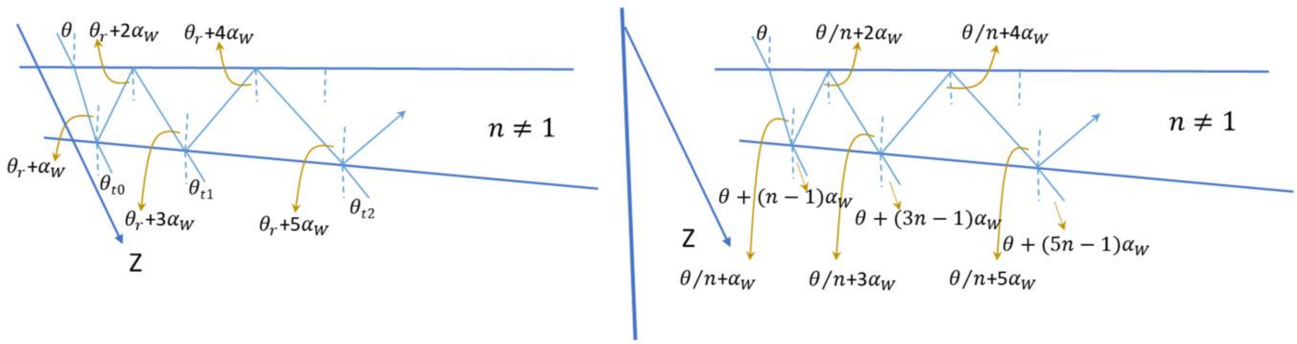


Figure 2. Ray tracing for a non-air gap wedge (left) and the approximate geometrical relations at small incidence angles.

To find the reflected or transmitted fringe pattern for a monochromatic light beam with an arbitrary wavefront, we use the plane-wave expansion of the complex amplitude

$$g(x) = g(x, z_0) = \int_{-\infty}^{\infty} G_0(\alpha)\exp(i2\pi\alpha x)d\alpha \tag{5}$$

of the beam at a generic plane z = z_0 , where $G_0(\alpha)$ is the angular spectrum of the light field $g(x, z_0)$. The complex amplitude of the light field in any other plane can be found from

$$g(x,z) = \int_{-\infty}^{\infty} G_0(\alpha)\exp\{i2\pi[\alpha x + \gamma(z - z_0)]\}d\alpha \tag{6}$$

The unit-amplitude plane wave $\exp[i2\pi(\alpha x + \gamma z)]$ has directional cosines $\lambda\alpha$ and $\lambda\gamma = [1 - (\lambda\alpha)^2]^{1/2}$ with respect to the X and Z axes. This wave falls on the front FW's surface at angle $\theta + \eta$, where $\eta = \arcsin(\lambda\alpha)$. The beam axis coincides with $\lambda\alpha = 0$, and the angle η takes positive and negative values. The plane wave $\exp[i2\pi(\alpha x + \gamma z)] = \exp[ik(x\sin\eta + z\cos\eta)]$ creates a transmitted wave behind the FW with the complex amplitude

$$A_\eta(x,z) = T\sum_{p=0}^{\infty} R^p \exp[ip(\phi + \phi')]\exp\{ik[x\sin(\xi_p + \eta) + z\cos(\xi_p + \eta)]\} \tag{7},$$

where $R = rr'$ and $T = tt'$ with $r(r')$ and $t(t)$ being the reflection and transmission coefficients of the front and rear wedge surface, ϕ, ϕ' are the phase shifts at reflection from the front and rear wedge surfaces inside the gap, $\xi_p = (2p+1)n\alpha_W - \alpha_W$. We assume $r = r'$, $t = t'$, $\phi = \phi' = \pi$ and zero phase shifts for external reflections. Therefore, the FW's response to $\exp[ik(x\sin\eta + z\cos\eta)]$ is given by

$$\tau_\eta(x,z) = \frac{A_\eta(x,z)}{\exp[ik(x\sin\eta + z\cos\eta)]} = T\sum_{p=1}^{\infty} R^{p-1}\exp[i2(p-1)\pi]\exp[ik(x\psi'_p + z\psi_p)] \tag{8}$$

where summation starts at p = 1 and $\psi_p = \cos(\xi_{p-1} + \eta) - \cos(\eta)$ and $\psi'_p = \sin(\xi_{p-1} + \eta) - \sin(\eta)$. The FW response to an arbitrary wavefront can be found by summation of responses to all plane waves in the angular spectrum of the complex amplitude describing this wavefront:

$$A(x,z)=T\exp\left[ik(z-z_0)\sum_{p=1}^{\infty}R^{p-1}\int_{-\infty}^{\infty}G_0(\alpha)\exp\left[-i\pi\lambda(z-z_0)\alpha^2\right]\exp(i2\pi\alpha x)\exp\left[ik\left(x\psi'_p+z\psi_p\right)\right]\right] \tag{9}$$

In paraxial approximation $\lambda\alpha << 1$, we have

$$\psi_p=\left[1-\frac{(\lambda\alpha)^2}{2}\right]\nu_p-(\lambda\alpha)\nu'_p,\ \psi'_p=\left[1-\frac{(\lambda\alpha)^2}{2}\right]\nu'_p+(\lambda\alpha)\nu_p \tag{10}$$

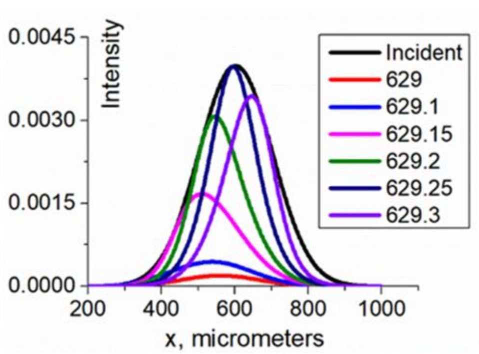
Both accepted approximations – a small incident angle and a light beam with low divergence – make possible obtaining an analytical expression for the complex amplitude of the transmitted field at Gaussin beam illumination. We consider a power-normalized Gaussian beam with a minimum beam waist, w_0, an optical axis at $x=x_0$ (Fig.1), and $G_0(\alpha)=(2\pi)^{1/4}\sqrt{w_0}\exp\left[-(\pi w_0\alpha)^2\right]\exp(i2\pi x_0)$. At the point $x=x_0$, the wedge thickness is $e=x_0/\alpha_W$. The complex amplitude of the transmitted field can be found analytically from

$$A(x,z)=(2\pi)^{1/4}\sqrt{w_0}(1-R)\exp[ik(z-z_0)]\sum_{p=1}^{\infty}R^{p-1}\int_{-\infty}^{\infty}\exp\left(-\pi^2w_0^2\alpha^2\right)\exp\left[i\left(p_p\alpha^2+q_p\alpha+r_p\right)\right]\times d\alpha \tag{11}$$

with $p_p=-\pi\left(z-z_0+x\nu'_p+z\nu_p\right)\lambda$, $q_p=2\pi\left(x+x\nu_p-z\nu'_p+x_0\right)$, $r_p=2\pi/\lambda\left(x\nu'_p+z\nu_p\right)$. The transmitted intensity distribution is $I_T(x,z)=|A(x,z)|^2$.

3. SIMULATION AND EXPERIMENT

As a numerical example, we calculated the FW transmission at quasi-normal incidence in the case of a narrow beam illumination and illumination with an expanded and collimated beam. For the first case of a narrow Gaussian beam, the wedge parameters were as follows: $\alpha_W =$ 50 μrad, initial thickness corresponding to the leftmost point of the beam cross-section 50 μm, R = 0.9 and n = 1.5. The beam parameter w_0 is equal to 200 μm. We accepted that the beam impact area was 6 w_0. The results of calculation for *n* = 1 (air gap) and *n* = 1.5 (non-air gap) are presented in Fig.3, which shows intensity distributions corresponding to different wavelengths. The transmitted intensity distribution varies with the wavelength. The intensity distributions for *n* = 1.5 have more asymmetrical profile.



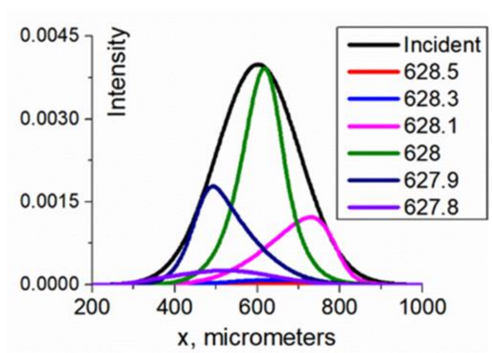


Figure 3. Transmitted intensity distributions at different wavelengths (given in nanometers) for n = 1 (left) and n = 1.5 (right); $\alpha_W =$ 50 μrad, initial wedge thickness 50 μm, R = 0.9.

The calculations for the case of an expanded collimated beam, which can be considered as a plane wave, are illustrated in Fig.4. Transmission of such a beam consists of the so called Fizeau lines separated by a distance, $\Delta X=\frac{1}{2}\frac{\lambda}{n\alpha_W}$. We used for the experiment a quartz FW with a rectangular form of length 3-5 cm and width 1-2 cm, index of refraction *n* = 1.42, *R* = 0.8 and thickness 3 μm. The experimentally observed Fizeau lines for this wedge are shown in Fig.5 at R = 0.8

and R = 0.95. We obtained a distance between the Fizeau lines equal to ΔX = 17.3 mm at 632.8 nm. This gave an apex angle equal to 12.88 μrad. We used this value for the calculations shown in Fig.4. For comparison, we calculated this intensity also at n = 1. As is seen, the developed plane waves expansion approach describes correctly the transmitted intensity.

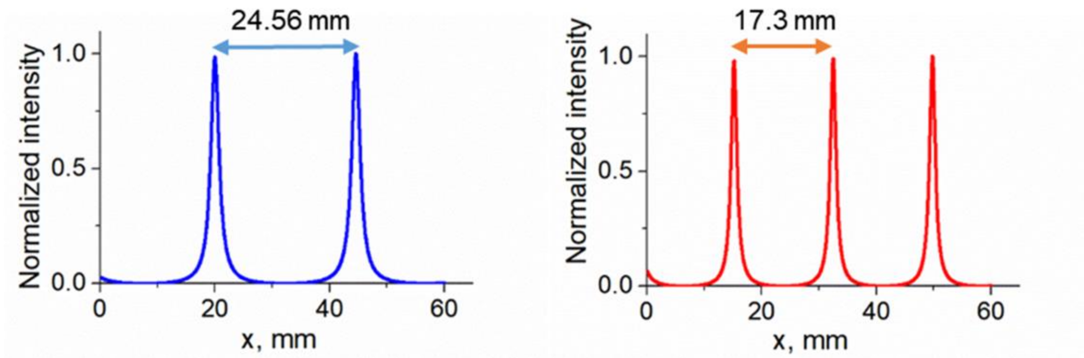


Figure 4. Transmitted intensity distributions for a FW under illumination with a large collimated beam from a He-Ne laser (λ = 632.8 nm) at n = 1 (left) and n = 1.42 (right); R = 0=9, α_W = 12.88 μrad, initial wedge thickness 3 μm.

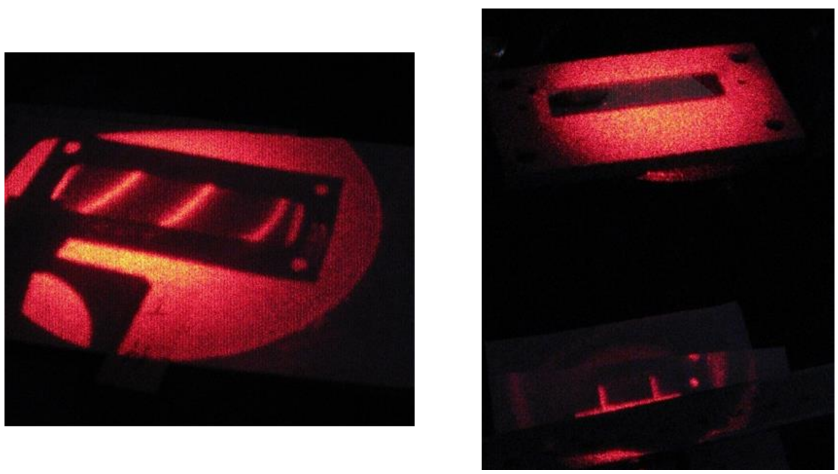


Figure 5. Photographs of the resonant peaks for a FW under illumination with a large collimated beam from a He-Ne laser (λ = 632.8 nm) at R = 0.8 (left) and R = 0.95 (right); n = 1.42, α_W = 12.88 μrad, initial wedge thickness 3 μm.

CONCLUSION

In summary, we analyzed the optical behavior of Fizeau wedge when the refractive index of its gap is different from 1. Increase of the refractive index changes the phase relations between the multiple plane waves generated inside the wedge gap and decreases the free spatial range between the Fizeau lines observed for a given wedge. We applied a plane wave expansion of the illuminating beam to find the FW response taking into account that the response to the plane wave illumination was known. In order to obtain analytical expression for the transmitted intensity, we considered illumination at small incidence angles. We analyzed only transmission, but the developed approach is readily adaptable for the case of reflection.

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