

A SIMULATION STUDY OF THE CONTROLLABILITY OF A CAR UNDER THE INFLUENCE OF A SUDDEN GUST OF CROSSWIND

Assoc. Prof. Dr. eng. Nikolov V.

Faculty of Mechanical Engineering – Technical University Sofia, Plovdiv Branch, Bulgaria

Abstract: The paper presents the development of a simulation model of a car for the research of its controllability under the influence of a sudden gust of crosswind, viewed as a mechanical system with three stages of freedom. The parameters of the mechanical system, consisting of ideal solid body are determined. A model of three differential equations is obtained, which describes the translational movement along the horizontal transversal axis, and the rotary motions respectively around the vertical and the longitudinal axes. The acceleration functions are obtained, the linear transversal velocity, the two angular velocities and the rotation angle are determined. The simulation model is created by MATLAB and its toolbox SIMULINK. Some results are given and analyzed for six locations of the mass centre, which show the influence of the mass centre location upon the car controllability.

Keywords: CAR, VEHICLE, CONTROLLABILITY, MATHEMATICAL MODELING AND COMPUTER SIMULATION

1. Introduction

Until the advent of computers, analysis can be divided into two areas - theoretically and experimentally. The numerical modelling is a separate direction, apart from the first two, though it has its aspects that rather complement them or even replace them. Numerical modelling does not have the roughness of the strict mathematical analysis, although it is based on the fundamental mathematical analysis in the construction of equations for numerical solution. This makes it closer to the experimental analysis, retaining many of its disadvantages. To research the controllability of a car through simulation models, the analyst at start-up of the computer, can monitor the processes the same way as with the experimental models. During the simulation, "experiments" are performed with a specific configuration of the machine with determining of the effects by changing various parameters on its work, and the advantage in relation to the experiments is that the system configuration is changing without additional manufacturing or processing. With the simulation models it is not necessary to watch for the installation and connection of measuring instruments, since there is constant access to all places in the machine.

On the other hand, the model will always be an approximation of reality, and can not completely replace physical experiments and theoretical analysis. Therefore, the simulation results [6, 7] require confirmation of practical results, to which are dedicated a number of issues about controllability of cars [1, 2, 3, 4, 5].

The main objective of this work is to investigate the influence of the mass centre location in a vertical longitudinal plane on the parameters of controllability of a car with impact of a sudden gust of crosswind, examined with the established simulation model as a mechanical system with three degrees of freedom [7].

2. Formulation of the problem

When building a mechano-mathematical model to analyze controllability of a car, as in most cases, the following assumptions are taken [2, 7]: the movement is through a horizontal surface with a specified speed, the steering wheels are set for a rectilinear motion, the vertical displacements and rotations of the chassis in relation to the transverse axis are absent; the redistribution of the normal load on the wheels is ignored, the resulting motion consists of rotation around the vertical axis, lateral movement and heel of the masses located above the springs.

For small deviations of the car from its rectilinear motion is assumed that the external forces are proportional to the linear and angular displacements and their derivatives.

Instead of the heel axis, joining the centres of the front and rear suspension heel, another axis is selected that is parallel to the supporting surface and lying in the longitudinal vertical plane of symmetry. The new axis passes through the intersection of the

vertical line that runs through the mass centre, with the axis of the heel [2]. The intersection O is considered to be the beginning of the coordinate system and the basic kinematic, geometric and mass parameters of the mechanical system; these are shown in Fig. 1.

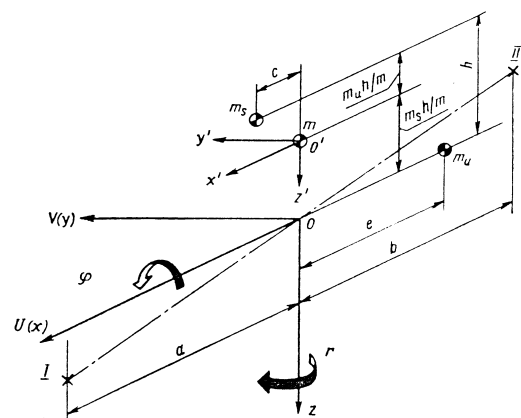


Fig. 1. Coordinate system of a car with three degrees of freedom: I - front and rear heel centers; m - mass of the car; m_s - suspended masses; m_u - unsprung masses

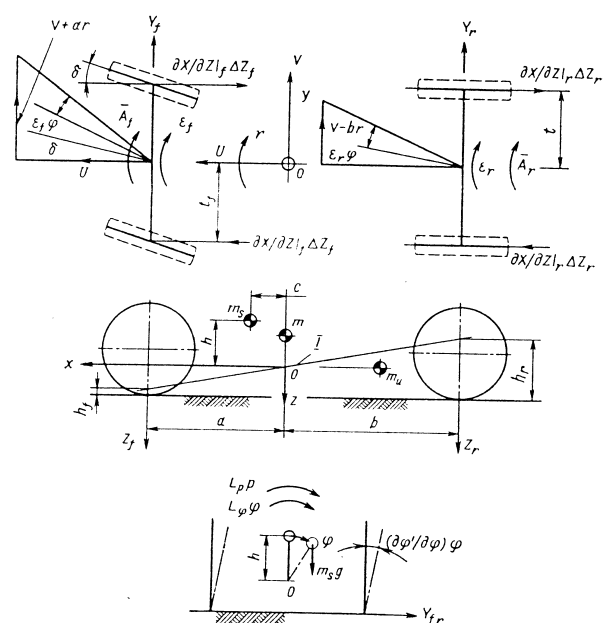


Fig. 2. Forces and moments acting on the car with three degrees of freedom: I - axis of the heel

With the thus chosen coordinate of origin, just below the mass centre of the car, the latter is exemplified by two concentrated masses m_s and m_u , arranged so that the sum of their moments in relation to the mass centre of the car is equal to zero. The system in hand has only three degrees of freedom and for independent coordinates are taken the transverse velocity V , [m / s], the angular velocity along the vertical axis r , [rad / s] and the heeling angle φ , [rad].

The forces and moments acting on the car, viewed as a mechanical system with three degrees of freedom, and their indications are shown in Fig. 2.

Lateral force Y , [N] and moments N and L in [Nm], respectively, around the vertical and longitudinal axes caused by the action of crosswind on the car are presented in Fig. 3.

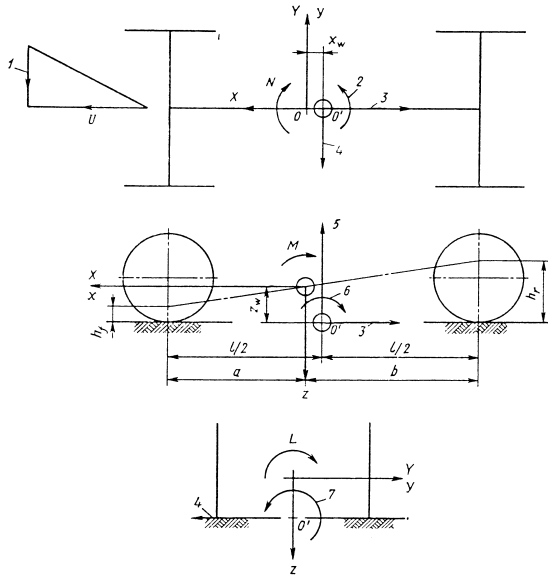


Fig. 3. Forces and moments generated by the action of crosswind on the car

To determine the aerodynamic forces and moments, the results of the coordinate system used for measurements in wind tunnels are brought forth to the adopted coordinate system. The lateral force and moments around the vertical and longitudinal axes caused by the action of the crosswind, are determined by the formulas

$$Y = -\frac{1}{2}\rho\bar{U}^2 AC_Y, \quad (1)$$

$$N = -\frac{1}{2}\rho\bar{U}^2 A(C_\psi l - x_w C_Y), \quad (2)$$

$$L = -\frac{1}{2}\rho\bar{U}^2 A(C_\phi t - z_w C_Y), \quad (3)$$

where ρ is the air density, [kg/m³];

$\bar{U} = \sqrt{U^2 + V_w^2}$ - the relative speed of the air flow [m/s];

U - the velocity of the coordinate origin in the direction of the axis Ox , [m/s];

V_w - the lateral speed of the air flow, [m/s];

A - the cross sectional area of the car, [m²];

C_Y , C_ψ , C_ϕ - the aerodynamic coefficients, respectively of the lateral force, the steering moment and the moment of heel;

l , t - distances between axles and wheels of the car, [m];

x_w , z_w - the distances in the longitudinal and vertical directions of the adopted the coordinates of the origin and the point O' (Figure 3), which is the origin of the coordinate system for the measurements in wind tunnel [m].

3. Differential equations

Based on the adopted above assumptions and symbols, the differential equations system can be written in the following form

$$\begin{cases} m(\dot{V} + rU) + m_s h \dot{\varphi} = Y_v V + Y_r r + Y_\varphi \varphi + Y \\ I_z \dot{r} - I_{xz} \ddot{\varphi} = N_v V + N_r r + N_\varphi \varphi + N \\ I_x \ddot{\varphi} - I_{xz} \dot{r} + m_s h (\dot{V} + rU) = L_\varphi \varphi + L_p \dot{\varphi} + L, \end{cases} \quad (4)$$

where the right sides of the equations are the active forces and moments represented as linearly dependent on the coordinates and their derivatives, except those obtained by the action of crosswind shear Y , [N], the moment about the vertical axis N , [Nm] and time around the longitudinal axis L , [Nm].

From the differential equation system (4), after the transformation are obtained functions of the three accelerations

$$\begin{cases} \dot{V} = \frac{1}{m}(Y_v V + Y_r r + Y_\varphi \varphi + Y - m_s h \ddot{\varphi}) - rU \\ \dot{r} = \frac{1}{I_z}(N_v V + N_r r + N_\varphi \varphi + N + I_{xz} \ddot{\varphi}) \\ \ddot{\varphi} = \frac{1}{I_x}[L_\varphi \varphi + L_p \dot{\varphi} + L + I_{xz} \dot{r} - m_s h (\dot{V} + rU)]. \end{cases} \quad (5)$$

The coefficients of shear forces, representing their private derivatives in relation to the generalized coordinates are calculated by the formulas

$$Y_v = \frac{1}{U}(C_f + C_r), \quad (6)$$

$$Y_r = \frac{1}{U}(aC_f - bC_r), \quad (7)$$

$$Y_\varphi = \frac{\partial Y_f}{\partial \varphi_f} \frac{\partial \varphi_f}{\partial \varphi} - C_r \varepsilon_r, \quad (8)$$

where U is the velocity of the mass centre along the axis Ox , [m/s];

C_f , C_r - the coefficients of the lateral drag of the front and rear wheels [N/rad];

a , b - the distances from the mass centre to the axis of the front and rear axle [m];

$\frac{\partial Y_f}{\partial \varphi_f}$ - the coefficient of lateral force on the front wheels of the heel of the front suspension [N/rad];

$\frac{\partial \varphi_f}{\partial \varphi}$ - the coefficient of the front suspension heel along the chassis heel;

ε_r - the coefficient of the rear wheels steering due to the heel.

The coefficients of the moments around the vertical axis, representing their partial derivatives in relation to the established coordinates are

$$N_v = \frac{1}{U} \left[aC_f - bC_r + \frac{\partial(\overline{AT})}{\partial \alpha_f} + \frac{\partial(\overline{AT})}{\partial \alpha_r} \right], \quad (9)$$

$$N_r = \frac{1}{U} \left[a^2 C_f - b^2 C_r + \frac{a \partial(\overline{AT})}{\partial \alpha_f} + \frac{b \partial(\overline{AT})}{\partial \alpha_r} \right], \quad (10)$$

$$N_\varphi = b\varepsilon_r C_r - \varepsilon_r \frac{\partial(\overline{AT})}{\partial\alpha_r} + \frac{a\partial Y'_f}{\partial\varphi'_f} \frac{\partial\varphi'_f}{\partial\varphi}, \quad (11)$$

where $\frac{\partial(\overline{AT})}{\partial\alpha_f}$, $\frac{\partial(\overline{AT})}{\partial\alpha_r}$ are the coefficients of stabilization

moments, respectively of the front and rear wheels of the corners of their lateral drag [Nm/rad];

The coefficients of the moments around the longitudinal axis, representing partial derivatives in relation to the angular velocity and the angle of heel are

$$L_p = \frac{\partial L}{\partial p} \Big|_f + \frac{\partial L}{\partial p} \Big|_r, \quad (12)$$

$$L_\varphi = \frac{\partial L}{\partial\varphi} \Big|_f + \frac{\partial L}{\partial\varphi} \Big|_r + m_s g h, \quad (13)$$

where $\frac{\partial L}{\partial p} \Big|_f$, $\frac{\partial L}{\partial p} \Big|_r$ are coefficients of damping moments

respectively of the front and rear suspension [Nms/rad];

$\frac{\partial L}{\partial\varphi} \Big|_f$, $\frac{\partial L}{\partial\varphi} \Big|_r$ - the coefficients of the moments resulting in

deformation of the elastic elements, respectively on the front and rear suspension [Nm/rad].

4. Computer modeling

The integration of the system of the three functions of acceleration is performed with Simulink in MATLAB, by taking full advantage of formalization [6, 7].

For better efficiency the work is carried out by three windows:

1) MATLAB Editor/Debugger, where the values of the parameters are assigned and the preliminary estimates of the parameters and the coefficients involved in the functions of acceleration are calculated;

2) Simulink, where in parametric form the simulation model is built, presented in Fig. 4;

3) MATLAB Workspace, where the text file from MATLAB Editor/Debugger is launched and the results from the simulation are visualized, by using the options on the "plot".

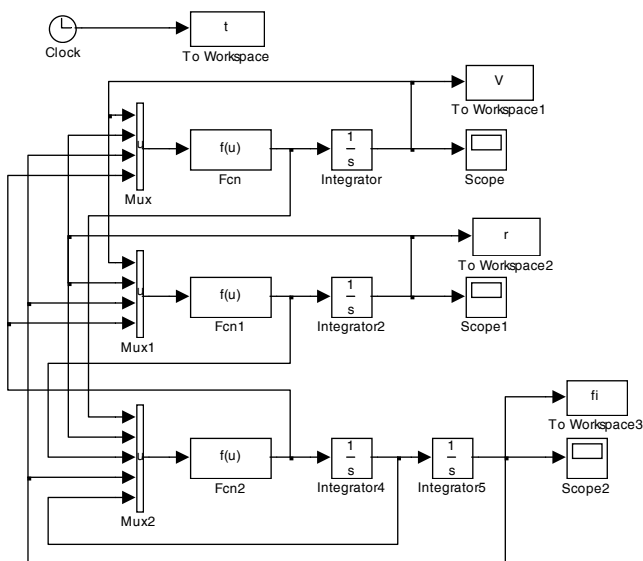


Fig. 4. Simulation model to study controllability of a car under the crosswind

The model provides visualization only of the state variables but practically it is possible for each point of the diagram.

5. Simulation results and analysis

With the thus created simulation model numerical experiments are performed for the six positions of the center of gravity of the car: at two heights and at three locations in the longitudinal direction, presented in Table 1:

Location №	h, [m]	a, [m]	b, [m]
1	0,3	0,8	1,3
2	0,3	1,05	1,05
3	0,3	1,3	0,8
4	0,5	0,8	1,3
5	0,5	1,05	1,05
6	0,5	1,3	0,8

The results of the numerical experiments for the transverse velocity V, the angular velocity r and the angular velocity $d\varphi / dt$, and the heeling angle φ , are presented graphically in Fig. 5, ..., 12.

From the graphically presented results for transverse velocity in Fig. 5 and 6 the following conclusions are drawn:

1) When the mass centre of the car is positioned behind the middle of the clearance (locations 3 and 6), a large negative value out of the transverse velocity is received, which increases in value by increasing of h (location 6), the transition process is with the longest duration and with aperiodic character.

2) When the mass centre of the car is positioned in the middle of the distance between the axes, a less negative set value of the transverse speed is obtained, and the transition process is of short duration, and with periodic characteristic (locations 2 and 5).

3) When the mass centre of the car is positioned in front midway between the axes, a small but established positive value of the transverse velocity is obtained, and the transition process is with the shortest duration and with periodic character (locations 1 and 4).

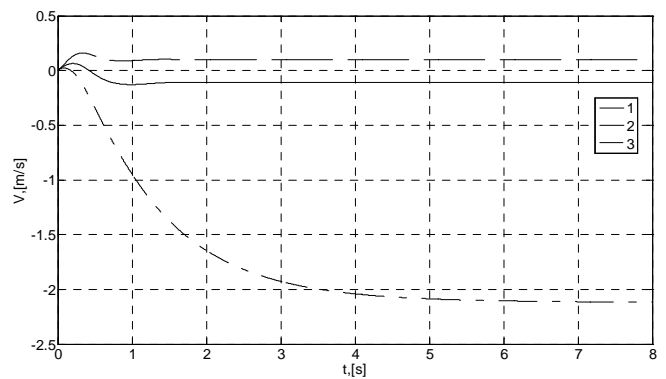


Fig. 5. Transverse vehicle speed with pulse effects of crosswinds for locations of mass centre 1, 2, 3

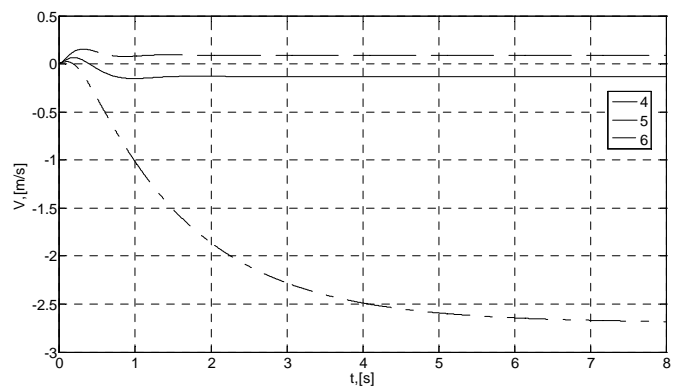


Fig. 6. Transverse vehicle speed with pulse effects of crosswinds for locations of mass centre 4, 5, 6

From the graphically presented results for the angular velocity around the vertical axis of the car in Fig. 7 and 8 following conclusions can be drawn:

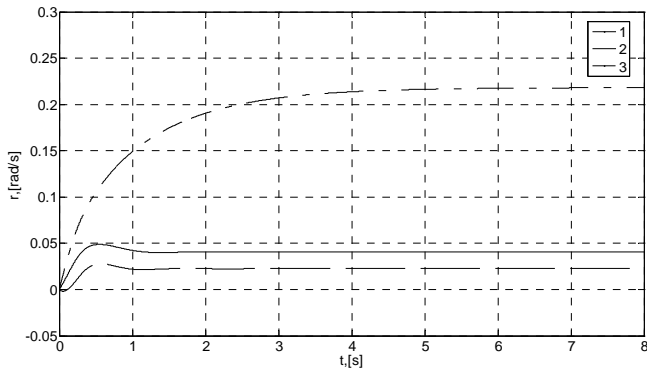


Fig. 7. Angular velocity relative to the vertical axis of the vehicle with pulse effects of crosswinds for locations of mass centre 1, 2, 3

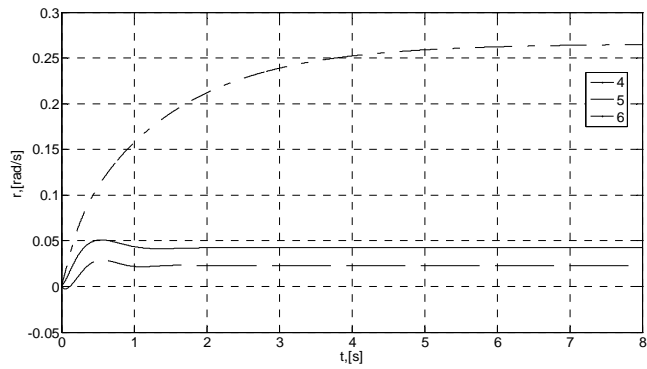


Fig. 8. Angular velocity relative to the vertical axis of the vehicle with pulse effects of crosswinds for locations of mass centre 4, 5, 6

1) With the mass centre of the car positioned behind midway between the axles (positions 3 and 6), the most positive set value of angular velocity around the vertical axis of the vehicle is obtained, which increases with increasing h (location 6), and the transition process is with the longest duration and with aperiodic character.

2) With the mass centre of the car positioned in the middle of the distance between the axles of a vehicle, a small positive set value of angular velocity around the vertical axis of the vehicle is obtained, and the transition process is with a shorter duration and with periodic character (locations 2 and 5).

3) With the mass centre of the car positioned in front midway between the axles of the car, the smallest positive set value of angular velocity about the vertical axis of the vehicle is obtained, the transition process is with the smallest duration and with periodic character (locations 1 and 4).

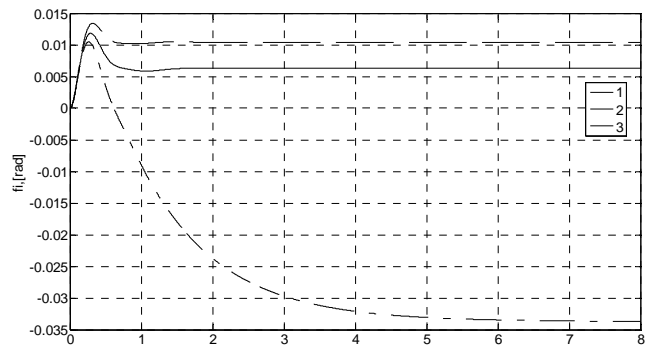


Fig. 9. Heel angle of the car with pulse effects of crosswinds for locations of mass centre 1, 2, 3

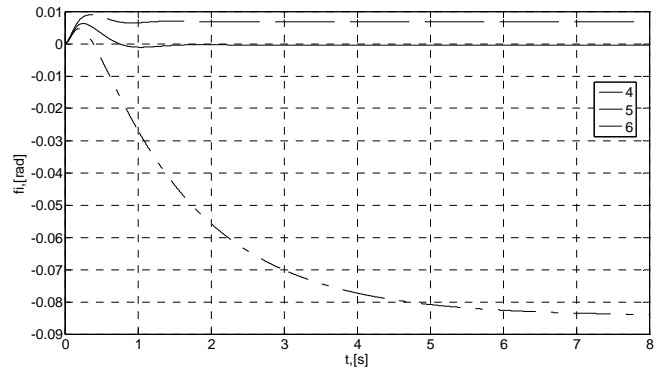


Fig. 10. Heel angle of the car with pulse effects of crosswinds for locations of mass centre 4, 5, 6

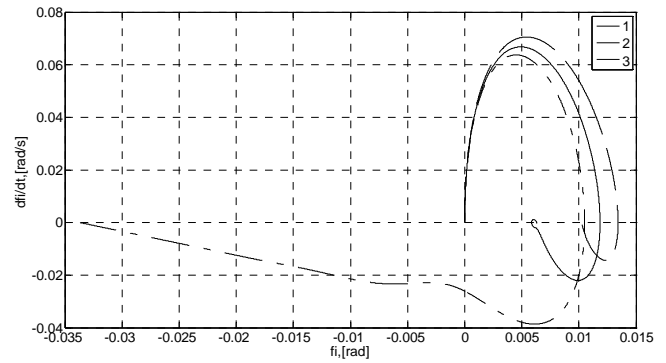


Fig. 11. Phase portraits of the movement of the heel of the car with pulse effects of crosswinds for locations of mass centre 1, 2, 3

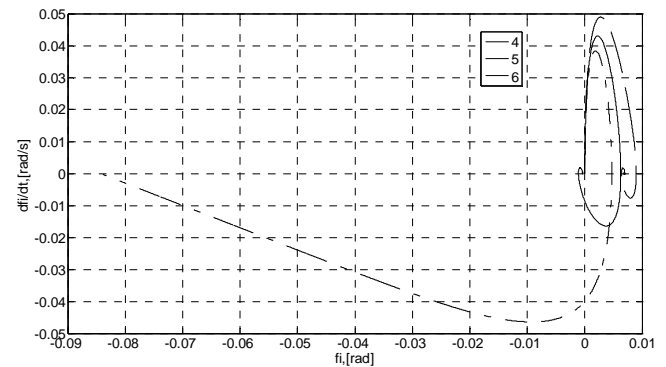


Fig. 12. Phase portraits of the movement of the heel of the car with pulse effects of crosswinds for locations of mass centre 4, 5, 6

From the graphically presented results for heeling angle in Fig. 9 and 10, and from the phase portrait of the movement of the heel - Fig. 11 and 12, the following conclusions can be drawn:

1) With mass centre of the car positioned behind midway between the axles of the car (positions 3 and 6), the highest negative set value of the heeling angle is received that increases in value with increasing h (location 6), the transition process is with the longest duration.

2) When the mass centre of the vehicle is positioned in the middle of the distance between the axles of a vehicle, a small positive value of the heeling angle is received (location 2), that decreases with the increasing of h (location 5), the transition process is of short duration and with periodic character (locations 2 and 5).

3) With mass centre of the car positioned in front midway between the axles of the car, the most positive set value of the heeling angle is received, the transition process is with the smallest length and with periodic character (locations 1 and 4).

4) The increasing of h leads to decrease in value of the maximum angular velocity of the heel, and increase of the

minimum angular velocity of the heel, and also leads to increase of the transition process duration.

6. Conclusion

With the here presented research with the established simulation model in MATLAB with Simulink environment of the parameters that characterize the controllability of a car, seen as a mechanical system with three degrees of freedom, under the impact of a sudden gust of crosswind the following conclusions can be drawn:

1) With pulse effect of a sudden gust of crosswind the parameters of controllability of the car significantly depend on the location of its mass centre in the longitudinal direction as well as in the vertical direction.

2) The simulation model for research of the controllability of car under the influence of a sudden gust of crosswind can be used both in designing new cars and in parameter analysis of controllability of the existing two-axle vehicles.

3) The simulation model for research of the controllability of car under the influence of a sudden gust of crosswind can be used for the training of students.

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