Comparative analysis of tunable kinetic MEMS energy harvesters

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Comparative analysis of tunable kinetic MEMS energy harvesters

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***Abstract* — The paper presents piezoelectric vibrating micro electromechanical system (MEMS) harvesters with variable effective spring stiffness. The structure of the harvesters consists of proof mass suspended by an elastic beam. A viscous damper represents the energy dissipation of the system. Piezoelectric layers, disposed on the maximum stress zone of the beam surface, are used as generators of electric energy. The stiffness of the beam is varied by capacitors, which are positioned at the maximum deflection zone of the beam. Different types of feedback circuits are considered in order to investigate their influence on the resonant frequency of the system. Furthermore, two types of capacitors with transverse or lateral action are applied. According to the type of these capacitors and to the feedback circuits twelve different systems are introduced. The dynamic models of those systems have been investigated to compare their tuning capability. The results, shown in the paper, could be useful for designers of tunable kinetic MEMS harvesters.**

Keywords-kinetic energy harvester; electrostatic stiffness; frequency tuninig; resonance.

I. INTRODUCTION

The most frequently referred techniques for MEMS energy harvesting that applies electrical means of affecting the mechanical elements are based on capacitive, magnetic, piezoelectric and thermal forces [1]. The resonant frequency tuning method to soften the spring stiffness is based on application of a “negative” spring in parallel to the mechanical spring. Therefore, the effective spring constant of the harvesting MEMS device, decreases and can be adjusted in the desired range.

Scheibner et al. [2, 3] reported an oscillating system consisting of an array of eight comb resonators each with a different natural resonant frequency. Each resonator comb is tuned by electrostatically softening the structure by applying a tuning voltage to the fixed electrodes. The device was designed so that the resonator array had overlapping tuning ranges which allowed continuous performance in the frequency range of the device from 1 to 10 kHz.

Adams et al. [4] realized a tuning range from 7.7 to 146% of the central frequency of 25 kHz of a resonator with a single comb structure. The tuning of two of their devices requires driving voltage between 0 and 50 V.

Lee et al. [5] presented a frequency-tunable comb resonator with curved comb fingers. Fingers of the tuning comb were designed (curved) to generate a constant electrostatic stiffness or linear electrostatic force that is independent of the displacement of the resonator under a control voltage.

Yao and MacDonald [6] compared frequency tuning by applying either axial force or transverse force on the resonator electrostatically. Frequency tuning by applying transverse force was tested experimentally. It was found that the resonant frequency may increase or decrease with the applied tuning voltage depending on the position of the tuning electrode with respect to the excitation electrode and the resonating rod.

The aim of this paper is to investigate the influence of different feedback circuits on the natural frequency of the piezoelectric (MEMS) energy harvester. The feedback input is the piezoelectric voltage and the output is the electrostatic force which will be applied by either comb or parallel plate capacitors.

II. GENERAL DESCRIPTION OF THE COMPARED ENARGY HARVESTERS

The equivalent kinetic energy harvester model considered in the paper is described in Fig.1. A proof mass m is suspended by an elastic beam with stiffness k. The losses are presented by a damper with viscous coefficient b. Piezoelectric elements with generalized voltage coefficient d_y are used as voltage sources of the harvester [7]. The natural frequency of the system can by varied by different passive feedback circuits which supply the tunable part of the system.

The body of the harvester oscillates with respect to the ground and its absolute reference coordinate is denoted as X. The relative motion of the mass with respect to the body is denoted by y. The sum of both coordinates

$$Z = X + y \tag{1}$$

is the absolute motion of the mass with respect to the ground.

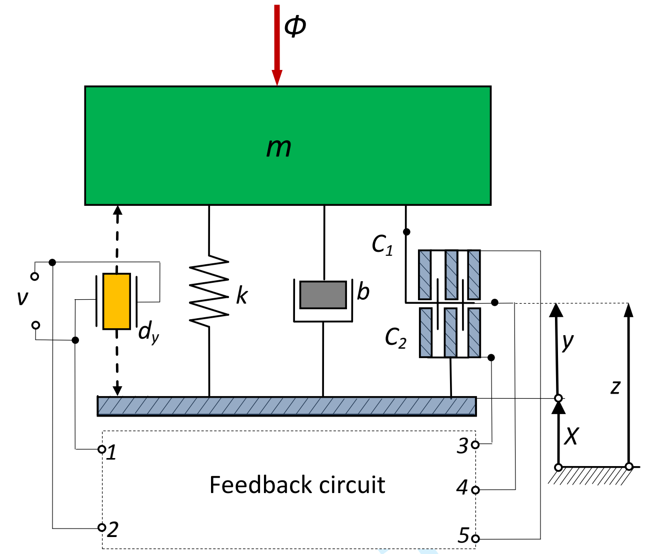

Figure 1. Common structure of the tunable MEMS energy harvesters

The operation of the described kinetic energy harvester is based on the generation of electricity by the piezoelectric elements when the body is subjected to ambient vibrations. In order to improve the efficiency of the harvester, the electrostatic force of a capacitive circuit is used to tune the system's natural frequency according to the ambient vibrations. The main consideration for tuning is to match the harvester's natural frequency with the frequency of the ambient vibrating sources.

The applied feedback circuits are shown in Fig.2. Six passive feedback circuit types are used. The first one shown in Fig.2a provides parallel connection of the variable capacitors. By the design given in Fig.2b a serial connection is achieved. The next two circuits shown in Fig.2c and Fig.2d represent the rectified parallel and serial feedback respectively. The two last circuits shown in Fig.2e and Fig.2f ensure a unidirectional feedback to the capacitors. Depending on the polarity of the diodes two cases are possible: a synchronous, i.e. positive feedback and an inverted phase feedback.

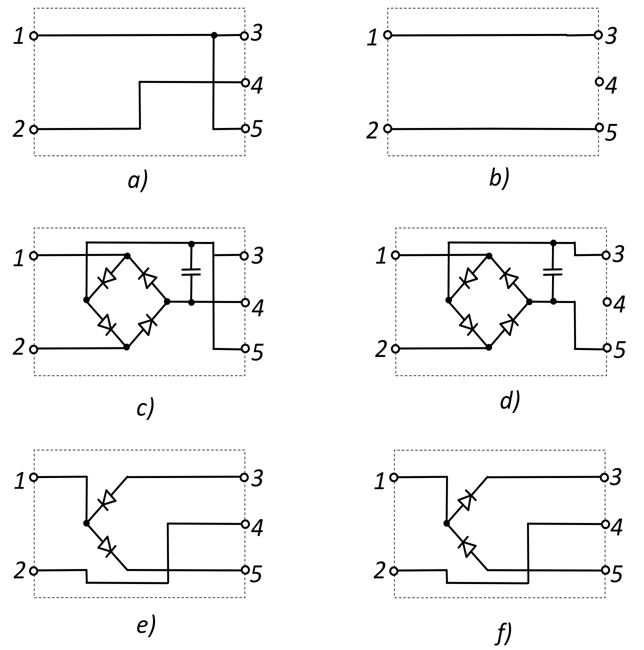

Figure 2. Feedback circuits types: a) direct parallel; b) direct serial; c) rectified parallel; d) rectified serial; e) in-phase (synchronous); f) inverted phase

The investigated harvester's structures are described in Fig.3. Both structures are based on double clamped elastic beams. In both structures: *a* and *b*, the top piezoelectric layers 1 and the bottom piezoelectric layers 1' are allocated near the fixed end of the beam.

The structure shown in Fig.3a consists of two arrays of comb capacitors. The first array is fixed at the top surface and the second one is underneath the bottom surface of the beam. These comb arrays form 2n capacitors with longitudinal action, where n is the number of the teeth of the body. The total capacitance of the top array is denoted by C_1 and for the bottom capacitance the notation C_2 is used.

The structure shown in Fig.3b has the same piezoelectric layers as those of the previous beam, but both type capacitors have parallel plates. Here the capacitance of the top capacitor is C_1 and the bottom capacitor has a capacitance C_2.

The equivalent electric elements of the device are described in Fig.4. The piezoelectric element is presented as a voltage source, a capacitor C_p and a series resistance R [8]. The capacitors for both structures shown in Fig.3 have an equal schematic representation. In Fig.4b the variable capacitors C_1 and C_2 form a network with a common point 4 and free ends 3

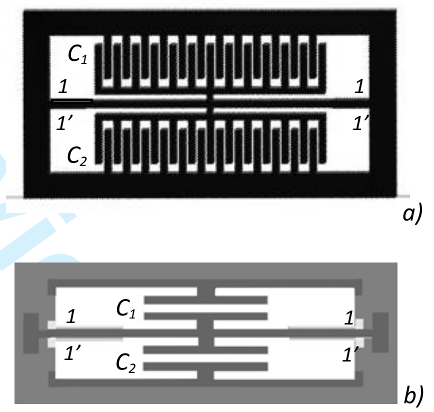

Figure 3. Basic MEMS structures with doubly clamped beam: a) piezoelectric layers and comb capacitors; b) piezoelectric layers and parallel plate capacitor

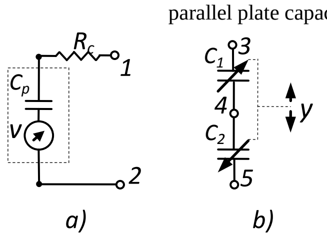

Figure 4. Equivalent schemes of: a) piezoelectric element; b) capacitive group

and 5 which are connected to the correspondingly numbered terminals of the feedback circuit block. Taking into account the six feedback circuits presented in Fig.2 and the two structures described in Fig.3 it follows that there are 12 available options for creating tunable harvesters. A special classifying notation is introduced, shown in Table I.

TABLE I. NOTATIONS OF TNE AVALABLE HARVESTER COMBINATION

Circuit \ Structure	Comb capacitors	Parallel plate capacitors
Parallel	H_{11}	H_{12}
Serial	H_{21}	H_{22}
Rectilinear parallel	H_{31}	H_{32}
Rectilinear serial	H_{41}	H_{42}
In-phase	H_{51}	H_{52}
Inverted phase	H_{61}	H_{62}

III. INVESTGATION OF TUNABLE PARAMETERS OF THE MODELS

A common model with lumped parameters can describe the motion of the considered above harvesters. It is assumed that the mass of the system is lumped at the center of the beam. The motion of this mass can be described by the differential equation

$$m\ddot{y} + b\dot{y} + ky = F_C - m\ddot{X}\ , \qquad (2)$$

where $\ddot{y} = \frac{d^2y}{dt^2}$, $\ddot{y} = \frac{dy}{dt}$, $\ddot{X} = \frac{dX}{dt}$ and F_C is the total capacitive force. Assuming that the ambient vibrations are given by

$$X = a_x \sin \Omega t\ , \qquad (3)$$

the differential equation is rewritten in the view

$$m\ddot{y} + b\dot{y} + ky = F_C + m\Omega^2 \sin \Omega t\ , \qquad (4)$$

If the capacitive force can be presented as a polynomial of y then the coefficient before the first derivative of y gives the so called effective or electrostatic stiffness of the system. Depending on the sign of this electrostatic stiffness the suspension stiffness can be increased if it is negative or decreased in case of a positive value.

Independently of the feedback circuit a common approach here is used in order to determine the total capacitive force. The main steps of this approach are:

1. Determination of the capacitive co-energy of the system as function of mass displacement y [8].

2. Expressing the piezoelectric voltage as a function of the beam deflection.

3. The derivative of the co-energy function with respect to displacement y yields the total capacitive force.

Let us consider the **harvester H_{11}** which structure corresponds to the accepted conventions in Table I. The electrostatic co-energy of the capacitors is

$$E_{C_1} = \frac{n\varepsilon\left(l_c + y\right)w_c v^2}{g_c} \qquad E_{C_2} = \frac{n\varepsilon\left(l_c - y\right)w_c v^2}{g_c}, \qquad (5)$$

where ε is the permittivity, l_c is the initial overlapping length, w_c is the width and g_c is the gap between the capacitor teeth.

Assuming that the piezoelectric voltage can be substituted by the relation

$$v = d_y y\ , \qquad (6)$$

where d_y is called generalized piezoelectric voltage constant depending on the beam elastic parameters and piezoelectric transducer properties [7], the total co-energy of the capacitors can be presented as

$$E_C = \frac{2n\varepsilon l_c w_c d_y^2 y^2}{g_c}. \qquad (7)$$

From (7) the total electrostatic force

$$F_C = \frac{\partial E_C}{\partial y} = \frac{4n\varepsilon l_c w_c d_y^2 y}{g_c} \qquad (8)$$

is obtained. Then the electrostatic stiffness is

$$k_{es} = \frac{4n\varepsilon l_c w_c d_y^2}{g_c}. \qquad (9)$$

For the **harvester H_{21}**, which combines serial feedback circuit and comb capacitors the electrostatic co-energies are

$$E_{C_1} = \frac{n\varepsilon\left(l_c + y\right)w_c}{g_c}u_1^2 \qquad E_{C_2} = \frac{n\varepsilon\left(l_c - y\right)w_c}{g_c}u_1^2 \qquad (10)$$

In the above formulae the voltages of the capacitors u_1 and u_2 are derived from the system

$$\begin{vmatrix} u_1 + u_2 = v \\ C_1 u_1 - C_2 u_2 = 0 \end{vmatrix} \qquad (11)$$

which has the solution

$$u_1 = \frac{C_2}{C_1 + C_2}v \quad u_2 = \frac{C_1}{C_1 + C_2}v \qquad (12)$$

Taking into account that the capacitances of the capacitors are

$$C_1 = \frac{2n\varepsilon\left(l_c + y\right)w_c}{g_c} \qquad C_2 = \frac{2n\varepsilon\left(l_c - y\right)w_c}{g_c} \qquad (13)$$

and after substituting (13) and (12) in (10) the electrostatic co-energies are found

$$E_{C_1} = \frac{n\varepsilon w_c\left(l_c + y\right)\left(l_c - y\right)^2}{4g_c l_c^2}v^2 \qquad E_{C_2} = \frac{n\varepsilon w_c\left(l_c - y\right)\left(l_c + y\right)^2}{4g_c l_c^2}v^2 \quad (14)$$

Through substituting of (6) in (14) the total electrostatic co-energy is

$$E_C = \frac{n\varepsilon w_c d_y^2}{2g_c l_c}\left(l_c^2 y^2 - y^4\right) \quad (15)$$

For the electrostatic force of both capacitors it is found that

$$F_C = \frac{n\varepsilon w_c d_y^2}{2g_c l_c}\left(2l_c^2 y - 3y^3\right) \quad (16)$$

The coefficient before y is the electrostatic stiffness

$$k_{es} = \frac{n\varepsilon w_c d_y^2 l_c}{g_c} \quad (17)$$

For the **harvester *H₃₁*** which combines a rectifier with a parallel feedback circuit it can be assumed that the rectifier input voltage is

$$u_{in} = u_m \sin\Omega t = y_{\max} d_y^2 \sin\Omega t\,, \quad (18)$$

where u_m is the amplitude of the piezoelectric voltage and $y_{\max}$ is the maximum deflection of the beam. For the rectifier output voltage can be written

$$u_{out} \approx y_{\max} d_y^2 = const \quad (19)$$

The electrostatic co-energies in this case are

$$E_{C_1} = \frac{n\varepsilon(l_c + y) w_c d_y^2 y_{\max}^2}{g_c}\,, \quad E_{C_2} = \frac{n\varepsilon(l_c - y) w_c d_y^2 y_{\max}^2}{g_c} \quad (20)$$

The total co-energy of the capacitors becomes

$$E_C = \frac{2n\varepsilon l_c w_c d_y^2 y_{\max}^2}{g_c} \quad (21)$$

After differentiation the total electrostatic force and electrostatic stiffness are obtained

$$F_C = \frac{\partial E_C}{dy} = 0, \quad k_{es} = 0 \quad (22)$$

The above result shows that there is no electrostatic force (both capacitive forces are in equilibrium) and this system has no tunable properties.

Combining feedback circuit with a rectifier and capacitors in series (**harvester H_{41}**) with the help of the total co-energy of the system, it can be written:

$$E_C = \frac{n\varepsilon w_c d_y^2}{2g_c l_c}\left(l_c^2 - y^2\right) y_{\max}^2 \quad (23)$$

Using the same approach as for the previous systems, the capacitive force and capacitive stiffness are obtained respectively

$$F_C = -\frac{n\varepsilon w_c d_y^2 y_{\max}^2}{g_c l_c} y \quad (24)$$

$$k_{es} = -\frac{n\varepsilon w_c d_y^2 y_{\max}^2}{g_c l_c} \quad (25)$$

The combination of the in-phase circuit and comb capacitors (**harvester H_{51}**) leads to the total co-energy expressed by

$$E_C = \begin{cases} \dfrac{n\varepsilon(l_c + y) w_c d_y^2 y^2}{g_c} & y > 0 \\ \dfrac{n\varepsilon(l_c - y) w_c d_y^2 y^2}{g_c} & y < 0 \end{cases} \quad (26)$$

After differentiation the total capacitive force

$$F_C = \begin{cases} \dfrac{n\varepsilon w_c d_y^2}{g_c}\left(2l_c y + 3y^2\right) & y > 0 \\ \dfrac{n\varepsilon w_c d_y^2}{g_c}\left(2l_c y - 3y^2\right) & y < 0 \end{cases} \quad (27)$$

is found. The electrostatic stiffness for the considered case is

$$k_{es} = \frac{2n\varepsilon w_c l_c d_y^2}{g_c} \quad (28)$$

The last possible type **harvester H_{61}** with comb capacitors involves an inverted-phase feedback circuit. The capacitive co-energy and corresponding force in this case have the opposite sign with respect to the previous case. Therefore the co-energy, the total capacitive force and the electrostatic stiffness respectively are

$$E_C = \begin{cases} -\dfrac{n\varepsilon(l_c + y) w_c d_y^2 y^2}{g_c} & y > 0 \\ -\dfrac{n\varepsilon(l_c - y) w_c d_y^2 y^2}{g_c} & y < 0 \end{cases} \quad (29)$$

$$F_C = \begin{cases} -\dfrac{n\varepsilon w_c d_y^2}{g_c}\left(2l_c y + 3y^2\right) & y > 0 \\ -\dfrac{n\varepsilon w_c d_y^2}{g_c}\left(2l_c y - 3y^2\right) & y < 0 \end{cases} \quad (30)$$

$$k_{es} = -\frac{2n\varepsilon w_c l_c d_y^2}{g_c} \quad (31)$$

Determination of electrostatic stiffness of the harvester with parallel plate capacitors and parallel feedback circuit (**harvester H_{12}**) requires the expression for the electrostatic energy of the capacitors

$$E_{C_1} = \frac{\varepsilon A_p}{2(g_0 + y)} v^2 \qquad E_{C_1} = \frac{\varepsilon A_p}{2(g_0 + y)} v^2 \quad (32)$$

Here $A_p = l_p w_p$ is the area of the plate, l_p is the length, w_p is the width of the plate, and g_0 is the initial gap between plates. The total electrostatic co-energy is

$$E_{C_1} = \frac{\varepsilon A_p}{2(g_0 + y)} v^2 \qquad E_{C_2} = \frac{\varepsilon A_p}{2(g_0 - y)} v^2 \qquad (33)$$

Taking into account (6), the total co-energy is found

$$E_C = \frac{\varepsilon A_p d_y^2 g_0}{g_0^2 - y^2} y^2 \qquad (34)$$

After differentiation of (34) the electrostatic force is

$$F_C = \frac{2\varepsilon A_p d_y^2 g_0^3}{\left(g_0^2 - y^2\right)^2} y \qquad (35)$$

Using the Taylor series for (35) the electrostatic force is presented by the approximated expression

$$F_C \approx \frac{2\varepsilon A_p d_y^2}{g_0} y + \frac{4\varepsilon A_p d_y^2}{g_0^3} y^3 + O\left(y^5\right) \qquad (36)$$

The electrostatic stiffness in this case is

$$k_{es} = \frac{2\varepsilon A_p d_y^2}{g_0} \qquad (37)$$

For the combination of parallel plate capacitors and serial feedback (**harvester *H*$_{22}$**) the solution of (11) leads to the following expressions for the voltages

$$u_1 = \frac{v(g_0 + y)}{2g_0} \qquad u_2 = \frac{v(g_0 - y)}{2g_0} \qquad (38)$$

Then for the capacitive co-energies of both capacitors follows

$$E_{C_1} = \frac{\varepsilon A_p (g_0 + y)}{8g_0^2} v^2 \qquad E_{C_2} = \frac{\varepsilon A_p (g_0 - y)}{8g_0^2} v^2 \qquad (39)$$

Taking into account (6) the sum of both co-energies gives

$$E_C = \frac{\varepsilon A_p d_y^2}{4g_0} y^2 \qquad (40)$$

The derivative of (40) with respect to *y* gives the electrostatic force

$$F_C = \frac{\varepsilon A_p d_y^2}{2g_0} y \qquad (41)$$

From this expression the electrostatic stiffness is found as

$$k_{es} = \frac{\varepsilon A_p d_y^2}{2g_0} \qquad (42)$$

The next **harvester *H*$_{32}$** combines a parallel plate capacitor structure and feedback consisting of a rectifier and a parallel capacitor circuit. The electrostatic co-energies are described by (33) but in those expressions it will be necessary to substitute the voltage with the obtained in (19). Following the same approach as described by formulae (33) to (35) the total electrostatic co-energy results in

$$E_C = \frac{\varepsilon A_p d_y^2 g_0 y_{\max}^2}{g_0^2 - y^2} \qquad (43)$$

The electrostatic force for this case is

$$F_C = \frac{2\varepsilon A_p d_y^2 g_0 y_{\max}^2}{\left(g_0^2 - y^2\right)^2} y \qquad (44)$$

The application of the Taylor series expansion for (44) gives

$$F_C \approx \frac{2\varepsilon A_p d_y^2 y_{\max}^2}{g_0^3} y + \frac{4\varepsilon A_p d_y^2 y_{\max}^2}{g_0^5} y^3 + O\left(y^5\right) \qquad (45)$$

From this polynomial the electrostatic stiffness is approximated as:

$$k_{es} = \frac{2\varepsilon A_p d_y^2 y_{\max}^2}{g_0^3} \qquad (46)$$

The harvester *H*$_{42}$ employs parallel plate capacitors. It involves a rectifier and a serial circuit feedback. Through substituting (19) in (39), i.e. assuming that $v = u_{out}$, the total electrostatic energy is obtained as

$$E_C = \frac{\varepsilon A_p d_y^2 y_{\max}^2}{4g_0} \qquad (47)$$

This result shows that the capacitive forces of both capacitors are in equilibrium and the electrostatic stiffness is zero as it was observed in (22). Note that instead of case described by (22) here the capacitors are connected in series.

The electrostatic energy for in-phase (synchronous) feedback circuit applied to the parallel plate capacitor structure (**harvester *H*$_{52}$**) is described by the combined expressions

$$E_C = \begin{cases} \dfrac{\varepsilon A_p d_y^2 y^2}{2(g_0 + y)} & y > 0 \\ \dfrac{\varepsilon A_p d_y^2 y^2}{2(g_0 - y)} & y < 0 \end{cases} \qquad (48)$$

The derivative of (48) with respect to y gives the electrostatic force:

$$F_C = \begin{cases} \dfrac{\varepsilon A_p d_y^2 y(2g_0 + y)}{2(g_0 + y)} & y > 0 \\ \dfrac{\varepsilon A_p d_y^2 y(2g_0 - y)}{2(g_0 - y)} & y < 0 \end{cases} \qquad (49)$$

The linear part of this force can be found by the Taylor series expansion

$$F_C \approx \begin{cases} \dfrac{\varepsilon A_p d_y^2}{g_0} y - \dfrac{3\varepsilon A_p d_y^2}{2g_0^2} y^2 + \dfrac{2\varepsilon A_p d_y^2}{g_0^3} y^3 - O\left(y^4\right) & y > 0 \\ \dfrac{\varepsilon A_p d_y^2}{g_0} y + \dfrac{3\varepsilon A_p d_y^2}{2g_0^2} y^2 + \dfrac{2\varepsilon A_p d_y^2}{g_0^3} y^3 + O\left(y^4\right) & y < 0 \end{cases} \quad (50)$$

The electrostatic stiffness of the system in this case is

$$k_{es} = \frac{\varepsilon A_p d_y^2}{g_0} \quad (51)$$

The last 12th case **(harvester H_{62})** contains a combination of an Inverted phase circuit and parallel plate capacitors. Since the direction of the forces in this case is opposite to the forces in the previous case, for simplicity it can be stated that the elastic stiffness will have the opposite sign to that in (51), i.e.

$$k_{es} = -\frac{\varepsilon A_p d_y^2}{g_0} \quad (52)$$

The negative value of this electrostatic stiffness shows that this system can increase its effective stiffness which is equivalent to hardening of the elastic beam.

IV. Conclusions

The results for the electrostatic stiffness are presented in Table II. The theoretical investigations show that out of the 12 considered cases, only 10 combinations can be used for tuning of the system natural frequency.

TABLE II. COMPARISON OF ELLECTROSTATIC STIFFNESS

Circuit \ Structure	Comb capacitors	Parallel plate capacitors
Parallel	$k_{es} = \dfrac{4n\varepsilon l_c w_c d_y^2}{g_c}$	$k_{es} = \dfrac{2\varepsilon A_p d_y^2}{g_0}$
Serial	$k_{es} = \dfrac{n\varepsilon w_c d_y^2 l_c}{g_c}$	$k_{es} = \dfrac{\varepsilon A_p d_y^2}{2g_0}$
Rectilinear parallel	0	$k_{es} = \dfrac{2\varepsilon A_p d_y^2 y_{\max}^2}{g_0^3}$
Rectilinear serial	$k_{es} = -\dfrac{n\varepsilon w_c d_y^2 y_{\max}^2}{g_c l_c}$	0
Unidirectional	$k_{es} = \dfrac{2n\varepsilon w_c l_c d_y^2}{g_c}$	$k_{es} = \dfrac{\varepsilon A_p d_y^2}{g_0}$
Out of phase	$k_{es} = -\dfrac{2n\varepsilon w_c l_c d_y^2}{g_c}$	$k_{es} = -\dfrac{\varepsilon A_p d_y^2}{g_0}$

For the systems which combine a rectifier with a parallel feedback circuit and comb capacitor ($\boldsymbol{H_{31}}$) as well as for the case of parallel plate capacitors with a rectifier and a serial feedback (circuit $\boldsymbol{H_{42}}$), it was proved that the total electrostatic force will be zero and the electrostatic stiffness will be zero as well.

It was shown that the maximum influence of the tuning capacitance of the systems can be achieved by parallel feedback circuits for harvesters $\boldsymbol{H_{11}}$ and $\boldsymbol{H_{12}}$. The series feedback circuits provide 4 times weaker tuning influence for harvesters $\boldsymbol{H_{21}}$ and $\boldsymbol{H_{22}}$. The in-phase circuit gives medium values for the electrostatic stiffness.

There are three options for hardening the elastic beam. Two of them are when the feedback is out of phase ($\boldsymbol{H_{61}}$, $\boldsymbol{H_{62}}$) and the third one is for the comb capacitors and rectifier with series feedback $\boldsymbol{H_{41}}$.

The co-energies of all systems as presented in this analysis have only one independent variable, which is the deflection *y* of the beam. For most typical capacitive actuators without feedback the voltage does not depend on the mechanical coordinate. In contrast to those cases, here the driving voltage is a linear function of the mass displacement. This feature changes the form of the electrostatic energy and hence, alters all following results for forces and electrostatic stiffness.

More detailed research on the capacitive energy harvesters has already been started and will involve mostly the assessment of the energy consumption and the efficiency of the feedback circuits. The feedback is consuming from the harvested energy. Therefore, it is necessary to minimize the total energy loses provoked by the listed above (possible) tuning circuits. This study will be used as the basis for the planned practical implementations of the tunable energy harvesters, so that harvesters with proper characteristics could be produced by the industry.

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