Tuning techniques for kinetic MEMS energy harvesters

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***Abstract*— The paper considers piezoelectric vibrating micro electromechanical system (MEMS) harvesters with variable effective spring stiffness. The structure of the harvesters is consists of proof mass suspended by an elastic beam. A viscous damper reflects the energy dissipation of the system. Piezoelectric layers, disposed on the maximum stress zone of the beam surface, are used as generators of electric energy. The stiffness of the beam is varied by capacitors, which are positioned at the maximum deflection zone of the beam. Different types of feedback circuits are used in order to investigate their influence on the resonant frequency of the system. Furthermore, two types of capacitors with transverse and lateral action are applied. According to the type of these capacitors and to the feedback circuits twelve different systems are considered. The dynamic models of those systems have been investigated in order to compare their tuning capability. The results, presented in the paper, could be useful for the designers of tunable kinetic MEMS harvesters.**

Keywords-kinetic energy harvester; electrostatic stiffness; frequency tuninig; resonance.

I. Introduction

The most frequently used techniques for MEMS energy harvesting which use electrical means of interaction with the mechanical part are based on the influence of capacitive, magnetic, piezoelectric and thermal forces [1]. The method to tune the resonant frequency by soften the spring stiffness is based on application of a “negative” spring in parallel to the mechanical spring. Therefore, the effective spring constant of such device, decreases and it can be adjusted in the desired range.

Scheibner et al. [2, 3] reported an oscillating system consisting of an array of eight comb resonators each with a different base resonant frequency. Each resonator comb is tuned by electrostatically softening the structure by applying a tuning voltage to the fixed electrodes. The device was designed so that the resonator array had overlapping tuning ranges which allowed continuous performance in the frequency range of the device from 1 to 10 kHz.

Adams et al. [4] realized a tuning range from 7.7 to 146% of the central frequency of 25 kHz of a resonator with a single comb structure. The tuning of two of their devices requires driving voltage between 0 and 50 V.

Lee et al. [5] presented a frequency-tunable comb resonator with curved comb fingers. Fingers of the tuning comb were designed (curved) to generate a constant electrostatic stiffness or linear electrostatic force that is independent of the displacement of the resonator under a control voltage.

Yao and MacDonald [6] compared frequency tuning by applying either axial force or transverse force on the resonator electrostatically. Frequency tuning by applying transverse force was tested experimentally. It was found that the resonant frequency may increase or decrease with the applied tuning voltage depending on the position of the tuning electrode with respect to the excitation electrode and the resonating rod.

The aim of this paper is to investigate the influence of different feedback circuits on the natural frequency of piezoelectric energy harvester. The feedback input is the piezoelectric voltage and the output is electrostatic force which can be applied by either comb or parallel plate capacitors.

II. General description of the compared enargy harvesters

The common kinetic energy harvester model considered in the paper is described in Fig. 1. A proof mass m is suspended by an elastic beam with stiffness k. The dissipations are presented by a damper with viscous coefficient b. Piezoelectric elements with generalized voltage coefficient d_y are used as voltage sources of the harvester [7]. The natural frequency of the system can by varied by different passive feedback circuits which supply the tunable part of the system.

The body of the harvester oscillates with respect to the ground and its absolute reference coordinate is denoted as X. The relative motion of the mass with respect to the body is denoted by y. The sum of both coordinates

$$Z = X + y \tag{1}$$

is the absolute motion of the mass with respect to the ground.

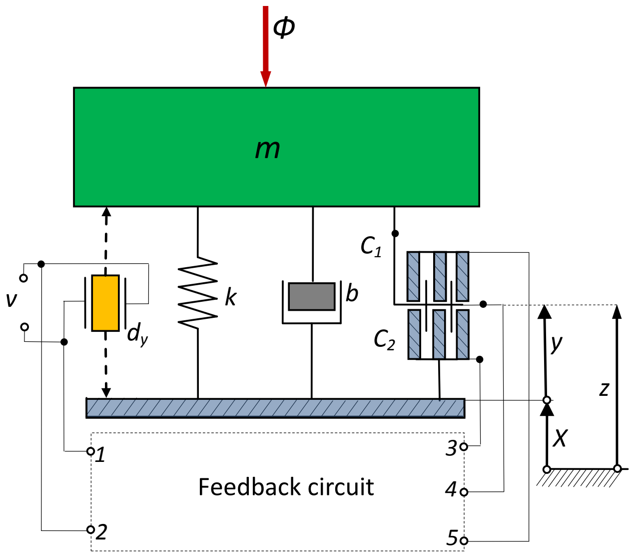

Figure **1.** Common structure of the tunable MEMS energy harvesters

The operational principle of the described kinetic energy harvester is based on the generation of electricity by the piezoelectric elements when the body is subjected to ambient vibrations. In order to improve the efficiency of the harvester, the electrostatic force of a capacitive circuit is used to tune the system's natural frequency according to the ambient vibrations. The main consideration for the appropriate tuning is matching the harvester's natural frequency with the frequency of the ambient vibrating sources.

The applied feedback circuits are shown in Fig. 2. Six passive feedback circuit types are used. The fist one shown in Fig. 2 a) provides parallel connection of the variable capacitors. By the scheme given in Fig. 2 b) a serial connection can be achieved. Next two circuits in Fig. 2 c) and d) are representation of rectifiers by which parallel and serial feedback can by realized respectively. The two last circuits shown in fig. 2 e) and f) are used to ensure unidirectional supply for one of the capacitors. Depending on the direction of the diodes two cases it is possible: synchronous or positive feedback and out of phase feedback.

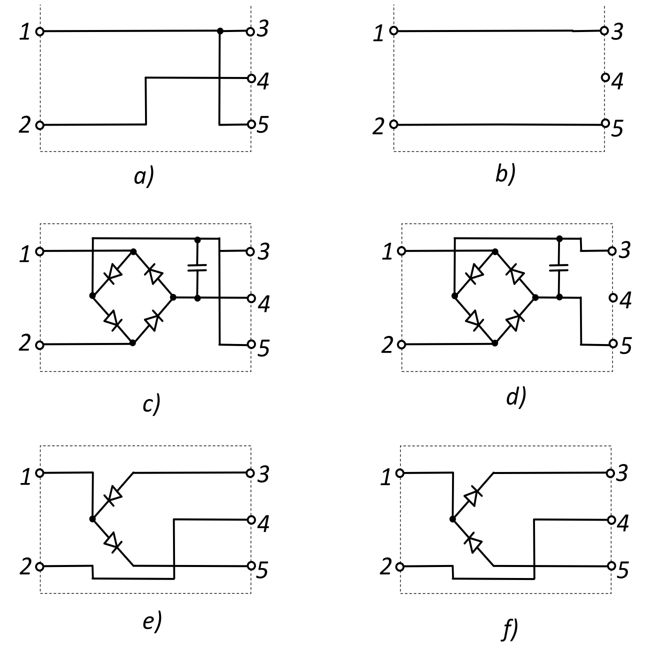

Figure 2. Feedback circuits types: a) direct parallel; b) direct serial; c) constant parallel; d) constant serial; e) unidirectional synchronous; f) out of phase.

The investigated harvester's structures are described in Fig. 3. Both structures are based on double clamped elastic beams. In both structures a) and b) the top piezoelectric layers 1 and the bottom piezoelectric layers 1' are disposed near the fixed ends of the beam.

The structure shown in Fig. 3 a) consists of two arrays of comb capacitors. The first array is disposed at the top surface and the second one is underneath the bottom beam surface. These comb arrays form $2n$ capacitors with longitudinal action, where n is the number of the teeth of the body. The total capacitance of the top array is denoted by C_1 and the for the bottom capacitance the notation C_2 is used.

The structure shown in Fig. 3b has the same piezoelectric layers as these of the previous beam, but both type capacitor are with parallel plates. Here the capacitance of the top capacitor is C_1 and the bottom capacitor has capacitance C_2.

The equivalent elements of the electrical part are described in Fig. 4. The piezoelectric element can be presented as a voltage source, capacitor C_p and series resistor R [8]. The capacitors for both structures shown in Fig. 3 have the same

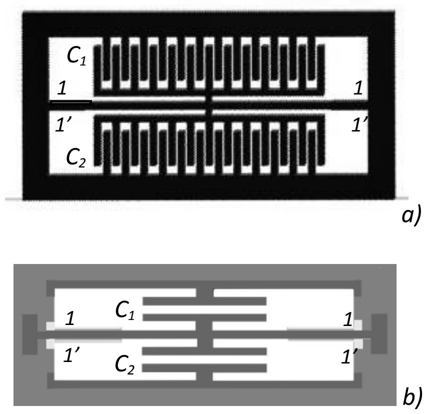

Figure 3. Basic MEMS structures with doubly clamped beam: a) piezoelectric layers and comb capacitors; b) piezoelectric layers and parallel plate capacitor

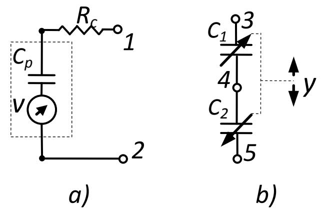

Figure 4. Equivalent schemes of: a) piezoelectric element; b) capacitive group

schematic representation. In Fig. 4 b) the variable capacitors C_1 and C_2 form a net with common point 4 and free ends 3 and 5 which are connected to the same digits of the feedback circuit block. Taking into account the six feedback circuit schemes presented in Fig. 2 and the two structures described in Fig. 3 it is seen that there are 12 available options for creating a tunable harvesters.

III. INVESTGATING THE TUNABLE PARAMETERS OF THE MODELS

A common model with lumped parameters can describe the motion of the considered above harvesters. It is assumed that the mass of the system is lumped at the center of the beam. The motion of this mass can be described by the differential equation

$$m\ddot{y} + b\dot{y} + ky = F_C - m\ddot{X} \ , \tag{2}$$

where $\ddot{y} = \frac{d^2 y}{dt^2}$, $\dot{y} = \frac{dy}{dt}$, $\ddot{X} = \frac{dX}{dt}$ and F_C is the total capacitive force. Assuming that the ambient vibrations are given by the Law

$$X = a_x \sin \Omega t \ , \tag{3}$$

the differential equation is rewritten in the view

$$m\ddot{y} + b\dot{y} + ky = F_C + m\Omega^2 \sin \Omega t \ , \tag{4}$$

If the capacitive force can be presented as a polynomial of *y* then the coefficient before the first degree of *y* gives the so called effective or electrostatic stiffness of the system. Depending on the sign of this electrostatic stiffness the suspension stiffness can be increased if it is negative or decreased in case of a positive value.

Independently of the feedback circuit a common approach here is used in order to determine the total capacitive force. The main steps of this approach are:

1. Determination of the capacitive co-energy of the system as function of mass displacement *y* [8].

2. Expressing the piezoelectric voltage as function of the beam deflection.

3. The derivative of the co-energy function with respect to displacement *y* yields the total capacitive force.

Let us consider the first **case numbered 11** which means that we have a harvester with a feedback circuit shown in Fig 1 a) and structure described in Fig. 2 a). The co-energies of the capacitors are

$$E_{C_1} = \frac{n\varepsilon \;\; l_c + y \;\; w_c v^2}{g_c} \qquad E_{C_2} = \frac{n\varepsilon \;\; l_c - y \;\; w_c v^2}{g_c} \ , \tag{5}$$

where ε is the permittivity, l_c is the initial overlapping length, w_c is the width and g_c is the gap between the tooth capacitor.

Assuming that the piezoelectric voltage can be substituted by the relation

$$v = d_y y \ , \tag{6}$$

where d_y is called generalized piezoelectric voltage constant which depends on beam elastic parameters and piezoelectric transducer properties [7], the total co-energy of the capacitors can be presented in the view

$$E_C = \frac{2n\varepsilon l_c w_c d_y^2 y^2}{g_c} \ . \tag{7}$$

From (7) the total electrostatic force

$$F_C = \frac{\partial E_C}{\partial y} = \frac{4n\varepsilon l_c w_c d_y^2 y}{g_c} \tag{8}$$

can be obtained. Then the electrostatic stiffness is

$$k_{es} = \frac{4n\varepsilon l_c w_c d_y^2}{g_c} \ . \tag{9}$$

For the **case 21**, which combines serial feedback circuit and comb capacitors the electrostatic co-energies are

$$E_{C_1} = \frac{n\varepsilon \;\; l_c + y \;\; w_c}{g_c} u_1^2 \quad E_{C_2} = \frac{n\varepsilon \;\; l_c - y \;\; w_c}{g_c} u_1^2 \tag{10}$$

In the above formulae the voltages of the capacitors u_1 and u_2 are derived from the system

$$\left| \begin{array}{l} u_1 + u_2 = v \\ C_1 u_1 - C_2 u_2 = 0, \end{array} \right. \tag{11}$$

which has the solution:

$$u_1 = \frac{C_2}{C_1 + C_2} v \quad u_2 = \frac{C_1}{C_1 + C_2} v \ . \tag{12}$$

Taking into account that the capacitances of the capacitors are:

$$C_1 = \frac{2n\varepsilon \;\; l_c + y \;\; w_c}{g_c} \qquad C_2 = \frac{2n\varepsilon \;\; l_c - y \;\; w_c}{g_c} \tag{13}$$

and after substituting (13) and (12) in (10) the electrostatic co-energies

$$E_{C_1} = \frac{n\varepsilon w_c \;\; l_c + y \;\; l_c - y \;\; ^2}{4 g_c l_c^2} v^2 \qquad E_{C_2} = \frac{n\varepsilon w_c \;\; l_c - y \;\; l_c + y \;\; ^2}{4 g_c l_c^2} v^2 \tag{14}$$

are found. Through substituting of (6) in (14) the total electrostatic co-energy

$$E_C = \frac{n\varepsilon w_c d_y^2}{2 g_c l_c} \;\; l_c^2 y^2 - y^4 \tag{15}$$

is expressed. For the electrostatic force of both capacitors it is found that

$$F_C = \frac{n\varepsilon w_c d_y^2}{2 g_c l_c} \;\; 2 l_c^2 y - 3 y^3 \;\; . \tag{16}$$

The coefficient before y gives the electrostatic stiffness

$$k_{es} = \frac{n\varepsilon w_c d_y^2 l_c}{g_c}. \qquad (17)$$

For the **case 31** which combines rectifier with parallel feedback circuit it can be assumed that the rectifier input voltage is:

$$u_{in} = u_m \sin\Omega t = y_{\max} d_y^2 \sin\Omega t\,,. \qquad (18)$$

where u_m is the amplitude of the piezoelectric voltage and $y_{\max}$ is the maximum deflection of the beam. For the rectifier output voltage can be written:

$$u_{out} \approx y_{\max} d_y^2 = const\,. \qquad (19)$$

The electrostatic co-energies in this case are:

$$E_{C_1} = \frac{n\varepsilon (l_c + y) w_c d_y^2 y_{\max}^2}{g_c}, \quad E_{C_2} = \frac{n\varepsilon (l_c - y) w_c d_y^2 y_{\max}^2}{g_c}. \qquad (20)$$

The total co-energy of the capacitors becomes:

$$E_C = \frac{2n\varepsilon l_c w_c d_y^2 y_{\max}^2}{g_c}. \qquad (21)$$

After differentiating for the total electrostatic force and electrostatic stiffness is obtained, respectively:

$$F_C = \frac{\partial E_C}{dy} = 0, \quad k_{es} = 0. \qquad (22)$$

The above result shows that there is no electrostatic force (the two capacitive forces are in equilibrium) and this system has no tunable properties.

Combining feedback circuit with rectifier and the capacitors in series (**case 41**) by the help of the total co-energy of the system, it can be written:

$$E_C = \frac{n\varepsilon w_c d_y^2}{2 g_c l_c}\left(l_c^2 - y^2\right) y_{\max}^2 \qquad (23)$$

Using the same approach as in the previous systems for the capacitive force and capacitive stiffness respectively we get:

$$F_C = -\frac{n\varepsilon w_c d_y^2 y_{\max}^2}{g_c l_c} y\,, \qquad (24)$$

$$k_{es} = -\frac{n\varepsilon w_c d_y^2 y_{\max}^2}{g_c l_c}. \qquad (25)$$

The combination of unidirectional circuit and comb capacitors (**case 51**) leads to the total co-energy presented by the expression:

$$E_C = \begin{cases} \dfrac{n\varepsilon (l_c + y) w_c d_y^2 y^2}{g_c} & y > 0 \\ \dfrac{n\varepsilon (l_c - y) w_c d_y^2 y^2}{g_c} & y < 0. \end{cases} \qquad (26)$$

After differentiating the total capacitive force

$$F_C = \begin{cases} \dfrac{n\varepsilon w_c d_y^2}{g_c}\left(2 l_c y + 3y^2\right) & y > 0 \\ \dfrac{n\varepsilon w_c d_y^2}{g_c}\left(2 l_c y - 3y^2\right) & y < 0 \end{cases} \qquad (27)$$

is found. The electrostatic stiffness for the considered case is

$$k_{es} = \frac{2n\varepsilon w_c l_c d_y^2}{g_c}. \qquad (28)$$

The last possible **case 61** with comb capacitors involves out of phase feedback circuit. The capacitive co-energy and corresponding force in this case have the opposite to the previous case sign. Therefore the co-energy, total capacitive force and the electrostatic stiffness respectively are obtained from:

$$E_C = \begin{cases} -\dfrac{n\varepsilon (l_c + y) w_c d_y^2 y^2}{g_c} & y > 0 \\ -\dfrac{n\varepsilon (l_c - y) w_c d_y^2 y^2}{g_c} & y < 0 \end{cases} \qquad (29)$$

$$F_C = \begin{cases} -\dfrac{n\varepsilon w_c d_y^2}{g_c}\left(2 l_c y + 3y^2\right) & y > 0 \\ -\dfrac{n\varepsilon w_c d_y^2}{g_c}\left(2 l_c y - 3y^2\right) & y < 0 \end{cases} \qquad (30)$$

$$k_{es} = -\frac{2n\varepsilon w_c l_c d_y^2}{g_c}. \qquad (31)$$

Determination of electrostatic stiffness of the harvester with parallel plate capacitors and parallel feedback circuit (**case 12**) requires the electrostatic energy of the capacitors

$$E_{C_1} = \frac{\varepsilon A_p}{2\left(g_0 + y\right)} v^2 \qquad E_{C_1} = \frac{\varepsilon A_p}{2\left(g_0 + y\right)} v^2 \qquad (32)$$

to be expressed. Here $A_p = l_p w_p$ is the area of the plate, l_p is the length, w_p is the width of the plate, and g_0 is the initial gap between plates. The total electrostatic co-energy is

$$E_{C_1} = \frac{\varepsilon A_p}{2\left(g_0 + y\right)} v^2 \qquad E_{C_2} = \frac{\varepsilon A_p}{2\left(g_0 - y\right)} v^2 \qquad (33)$$

Taking into account (6) the total co-energy is found in the form

$$E_C = \frac{\varepsilon A_p d_y^2 g_0}{g_0^2 - y^2} y^2 . \qquad (34)$$

After differentiating (34) the electrostatic force

$$F_C = \frac{2\varepsilon A_p d_y^2 g_0^3}{\left(g_0^2 - y^2\right)^2} y . \qquad (35)$$

is obtained. Using Taylor series for (35) the electrostatic force is presented by the approximate expression

$$F_C \approx \frac{2\varepsilon A_p d_y^2}{g_0} y + \frac{4\varepsilon A_p d_y^2}{g_0^3} y^3 + O\left(y^5\right) . \qquad (36)$$

The electrostatic stiffness in this case is

$$k_{es} = \frac{2\varepsilon A_p d_y^2}{g_0} . \qquad (37)$$

The combination of parallel plate capacitors and serial feedback (**case 22**) the solution of (11) leads to the following expressions for the voltages:

$$u_1 = \frac{v\left(g_0 + y\right)}{2g_0} \qquad u_2 = \frac{v\left(g_0 - y\right)}{2g_0} . \qquad (38)$$

Then for the capacitive co-energies of both capacitors follows:

$$E_{C_1} = \frac{\varepsilon A_p \left(g_0 + y\right)}{8g_0^2} v^2 \qquad E_{C_2} = \frac{\varepsilon A_p \left(g_0 - y\right)}{8g_0^2} v^2 . \qquad (39)$$

Taking into account (6) the sum of both co-energies gives:

$$E_C = \frac{\varepsilon A_p d_y^2}{4g_0} y^2 . \qquad (40)$$

The derivative of (40) with respect to *y* gives the electrostatic force:

$$F_C = \frac{\varepsilon A_p d_y^2}{2g_0} y . \qquad (41)$$

From this expression the electrostatic stiffness

$$k_{es} = \frac{\varepsilon A_p d_y^2}{2g_0} . \qquad (42)$$

is found.

Next **case numbered 32** combines parallel plate capacitor structure and feedback consisting of rectifier and parallel capacitor circuit. The electrostatic co-energies are described by (33) but in those expressions it will be necessary to substitute the voltage with the obtained in (19). Following the same approach as this described by formulae (33) to (35) for the total electrostatic co-energy results:

$$E_C = \frac{\varepsilon A_p d_y^2 g_0 y_{\max}^2}{g_0^2 - y^2} . \qquad (43)$$

The electrostatic force in this case is:

$$F_C = \frac{2\varepsilon A_p d_y^2 g_0 y_{\max}^2}{\left(g_0^2 - y^2\right)^2} y \qquad (44)$$

The application of Taylor series expansion for (44) gives

$$F_C \approx \frac{2\varepsilon A_p d_y^2 y_{\max}^2}{g_0^3} y + \frac{4\varepsilon A_p d_y^2 y_{\max}^2}{g_0^5} y^3 + O\left(y^5\right) \qquad (45)$$

From this polynomial the electrostatic stiffness

$$k_{es} = \frac{2\varepsilon A_p d_y^2 y_{\max}^2}{g_0^3} . \qquad (46)$$

is determined.

C**ase 42** utilizes parallel plate capacitors. It involves a rectifier and serial circuit feedback. Through substituting (19) in (39), i.e. assuming that $v = u_{out}$, for the total electrostatic energy is obtained:

$$E_C = \frac{\varepsilon A_p d_y^2 y_{\max}^2}{4g_0} \qquad (47)$$

This result shows that the capacitive forces of both capacitors are in equilibrium and the electrostatic stiffness is zero as it was observed in (22). Note that instead of case described by (22) here the capacitors are connected in series.

The electrostatic energy for unidirectional synchronous feedback circuit applied to the parallel plate capacitor structure (**case 52**) is described by the formulae:

$$E_C = \begin{cases} \dfrac{\varepsilon A_p d_y^2 y^2}{2\left(g_0 + y\right)} & y > 0 \\ \dfrac{\varepsilon A_p d_y^2 y^2}{2\left(g_0 - y\right)} & y < 0. \end{cases} \qquad (48)$$

The derivative of (48) with respect to y gives the electrostatic force:

$$F_C = \begin{cases} \dfrac{\varepsilon A_p d_y^2 y\left(2g_0 + y\right)}{2\left(g_0 + y\right)} & y > 0 \\ \dfrac{\varepsilon A_p d_y^2 y\left(2g_0 - y\right)}{2\left(g_0 - y\right)} & y < 0. \end{cases} \qquad (49)$$

A linear part of this force can be found by Taylor series expansion which gives:

$$F_C \approx \begin{cases} \dfrac{\varepsilon A_p d_y^2}{g_0} y - \dfrac{3\varepsilon A_p d_y^2}{2g_0^2} y^2 + \dfrac{2\varepsilon A_p d_y^2}{g_0^3} y^3 - O\left(y^4\right) & y > 0 \\ \dfrac{\varepsilon A_p d_y^2}{g_0} y + \dfrac{3\varepsilon A_p d_y^2}{2g_0^2} y^2 + \dfrac{2\varepsilon A_p d_y^2}{g_0^3} y^3 + O\left(y^4\right) . & y < 0. \end{cases} \qquad (50)$$

The electrostatic stiffness of the system in this case can be assumed as:

$$k_{es} = \frac{\varepsilon A_p d_y^2}{g_0} \quad (51)$$

The last 12th **case 62** contains a combination of out of phase circuit and parallel plate capacitors. Since the direction of the forces in this case is opposite to the forces in the previous case, for simplicity can be stated that the elastic stiffness will have the opposite sign to that in (51), i.e.:

$$k_{es} = -\frac{\varepsilon A_p d_y^2}{g_0} \quad (52)$$

The negative value of this electrostatic stiffness shows that this system will increase its effective stiffness which is equivalent to hardening of the elastic beam.

IV. CONCLUSIONS

The expressions of the electrostatic stiffness are presented in Table 1.

The theoretical investigations show that out of the 12th considered cases, only 10 combinations can be used for tuning of the system natural frequency.

For the systems which combine rectifier with parallel feedback circuit and comb capacitor as well as the case having parallel plate capacitors with a rectifier and serial feedback circuit, it was proved that the total electrostatic force will be zero and the electrostatic stiffness will be zero as well.

TABLE I. COMPARISON OF ELLECTROSTATIC STIFFNESSES

Circuit \ Structure	Comb capacitors	Parallel plate capacitors
Paralel	$k_{es} = \frac{4n\varepsilon l_c w_c d_y^2}{g_c}$	$k_{es} = \frac{2\varepsilon A_p d_y^2}{g_0}$
Serial	$k_{es} = \frac{n\varepsilon w_c d_y^2 l_c}{g_c}$	$k_{es} = \frac{\varepsilon A_p d_y^2}{2g_0}$
Rectilinear parallel	0	$k_{es} = \frac{2\varepsilon A_p d_y^2 y_{max}^2}{g_0^3}$
Rectilinear serial	$k_{es} = -\frac{n\varepsilon w_c d_y^2 y_{max}^2}{g_c l_c}$	0
Unidirectional	$k_{es} = \frac{2n\varepsilon w_c l_c d_y^2}{g_c}$	$k_{es} = \frac{\varepsilon A_p d_y^2}{g_0}$
Out of phase	$k_{es} = -\frac{2n\varepsilon w_c l_c d_y^2}{g_c}$	$k_{es} = -\frac{\varepsilon A_p d_y^2}{g_0}$

It was shown that the maximum influence of the tuning capacity of the systems can be achieved by parallel feedback circuits. The series feedback circuits provide 4 times weaker tuning influence. The unidirectional circuit gives medium values of the electrostatic stiffness.

There are three options for hardening of the elastic beam. Two of them are when the feedback is out of phase and the third one is for the comb capacitors and rectifier with series feedback.

The obtained in this analysis co-energies of all systems have only one independent variable, which is the deflection *y* of the beam. In the most typical capacitive actuators without feedback the voltage does not depend on mechanical coordinate. In contrast of these cases here the driving voltage is linear function of the mass displacement. This feature changes the form of the electrostatic energy and all the following results as forces and electrostatic stiffness.

The further investigations of the capacitive energy harvesters involve the assessment of energy consumption of the feedback circuits. Therefore it, is necessary to minimize the total energy loses of the tunable circuits, so that a harvester with proper characteristics could be built.

This investigations are the basis for the further practical implementation of the tunable energy harvesters.

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