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FOUR-PHOTON PARAMETRIC MIXING IN CW AND PULSE REGIMES IN SINGLE MODE OPTICAL FIBERS

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ABSTRACT

The process of four photon parametric mixing can be used to convert the input laser sources, working in CW and pulse regimes, into light at several different frequencies. An effective parametric energy conversion can be observed when phase matching conditions between the waves are satisfied. The basic theoretical investigations are focused on efficienty of the four-photon mixing and parametric gain with applications such as all-optical signal sampling, time-demultiplexing, pulse generation and wavelength conversion. The parametric amplifiers have capacity to provide high gain and low noise at arbitrary wavelengths with proper fiber design and pump wavelength allocation. The problem with the generation of new frequencies on distances less than one coherent length in the process of parametric four-photon mixing was solved in approximation of fixed electric field of the pump wave. The idea of our research is to solve the more general problem in which it is taken into account the mutual action of the first and second order of dispersion and all real $\chi^{(3)}$ nonlinear processes on the parametric four-photon mixing. In CW regime the solutions of the problem, presented above, was solved in the form of Jacobi elliptic functions. In pulse regime we found optimal conditions, where the process of energy exchange is still effective. In this regime a quasi-periodic conversion is observed and group velocity difference between the pump and signal wave is compensated by nonlinear mechanisms.

Keywords: Self-phase modulation, cross-phase modulation, parametric four-photon mixing, analytical solutions, Jacobi functions, numerical calculations

1. INTRODUCTION

Parametric processes are well-known phenomena in materials providing $\chi^{(3)}$ nonlinearity. The process of four-photon parametric mixing is usually used to convert input light pulse into light at several different frequencies. Parametric interaction and energy conversion are strongest, when there are phase matching conditions between the waves [1]. The basic studies are related to the problems of effective amplification of the signal waves with application in optical amplifiers. In regime of amplification usually short cut equations are used and solved [1-7]. The more general problem of periodic energy exchange at long distances between pump, signal and idler waves in pulse regime requires numerical solving of the nonlinear propagation equations, including first and second order of dispersion, self and cross-phase modulation and their influence on the process of parametric four-photon mixing. In CW regime the amplitude equations do not contain terms with group delay and dispersion. In this regime, as it shown in [8-9] the problem can be solved analytically with solutions in form of elliptical Jacoby functions, leading to periodic energy exchange between pump and signal waves. The effective parametric amplifiers in CW regime work with spectral delay between the pump and signal waves of the order of $\Delta\lambda < 40$ nm [7].

We need to ask the question: is it possible an effective energy exchange in pulse regime? This is the main idea of present paper: to investigate the parametric interaction between spectrally close picoseconds pulses $\Delta\lambda = \lambda_p - \lambda_s <$ 40nm, where λ_p and λ_s are respectively the carrier wavelength of the pump and signal waves, with time duration of the

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pulses 1-10 ps ($\Delta\lambda_{\text{pulse}} \approx 3 - 0.3$ nm). The numerical experiment is provided near to zero dispersion region of single mode optical fiber. In this spectral region and for these small spectral shifts between the pulses it is possible to observe a compensation of the group velocity delay by nonlinear mechanisms [10]. As it can be seen from the numerical simulations, this leads to self-confinement of pump and signal waves and to an effective quasi-periodic exchange of energy between them.

2. BASIC EQUATIONS IN CW REGIME

The system of equations that describes the generation of signal and idler waves from a pump wave in the frames of Four Photon Parametric Mechanism (FPPM) at few coherent lengths including in addition the effects of Self-Phase Modulation (SPM) and Cross Phase Modulation (CPM) is well known [7]:

$$\begin{aligned} i\frac{\partial A_p}{\partial z} &= \gamma_p\left(2A_sA_iA_p^*e^{-i\Delta \mathrm{kz}} + \left|A_p\right|^2A_p + 2|A_s|^2A_p + 2|A_i|^2A_p\right), \\ i\frac{\partial A_s}{\partial z} &= \gamma_s\left(A_p^2A_i^*e^{i\Delta \mathrm{kz}} + |A_s|^2A_s + 2\left|A_p\right|^2A_s + 2|A_i|^2A_s\right), \\ i\frac{\partial A_i}{\partial z} &= \gamma_i\left(A_p^2A_s^*e^{i\Delta \mathrm{kz}} + |A_i|^2A_i + 2\left|A_p\right|^2A_i + 2|A_s|^2A_i\right), \end{aligned} \tag{1}$$

where $A_s(z)$, $A_i(z)$ and $A_p(z)$ are complex amplitude functions of signal, idler and pump waves, γ_s, γ_i, γ_p, $2k_p=k_s+k_i+\Delta k$, $2\omega_p=\omega_s+\omega_i$ are respectively the nonlinear coefficients of the medium, the wavevector mismatch and the frequency conversion of the three waves. Usually this system is solved in approximation of fixed strong pump intensities [1, 7]. Thus, conditions for maximal amplification of signal waves were obtained. The real propagation in CW regime and parametric interaction at long distance in the optical fiber can be obtained by solving analytically the system of amplitude equations (1).

We are looking for solution of the basic system of equations (1) by applying well-known mathematical method, described in [9]:

$$\begin{aligned} A_i &= a_ie^{i\phi_i}, \\ A_p &= a_pe^{i\phi_p}, \\ A_s &= a_se^{i\phi_s}. \end{aligned} \tag{2}$$

Thus, the system of equations (1) takes the form:

$$\begin{aligned} i\frac{\partial a_p}{\partial z} - a_p\frac{\partial \phi_p}{\partial z} &= \gamma_p\left[2a_sa_ia_pe^{-i\Psi} + a_p^3 + 2(a_s^2 + a_i^2)a_p\right], \\ i\frac{\partial a_s}{\partial z} - a_s\frac{\partial \phi_s}{\partial z} &= \gamma_s\left[a_p^2a_ie^{-i\Psi} + a_s^3 + 2\left(a_p^2 + a_i^2\right)a_s\right], \\ i\frac{\partial a_i}{\partial z} - a_i\frac{\partial \phi_i}{\partial z} &= \gamma_i\left[a_p^2a_se^{-i\Psi} + a_i^3 + 2\left(a_p^2 + a_s^2\right)a_i\right], \end{aligned} \tag{3}$$

where $a_p(z)=|A_p(z)|$, $a_s(z)=|A_s(z)|$ and $a_i(z)=|A_i(z)|$ are real functions describing the waves amplitudes, while $\phi_p(z)$, $\phi_s(z)$ and $\phi_i(z)$ are phase functions of the waves, and $\psi = 2\phi_p - \phi_s - \phi_i + \Delta kz$ is the *generalized phase*.

In [9] we have solved that system (1) by equalizing the real and imaginary parts on both sides of the equalities (3). By number of transformations and substitutions its analytical solutions can be presented in the form of Jacobi elliptic sine functions [9]:

$$\begin{aligned} a_p^2 &= \frac{1}{2\gamma_s\gamma_i}\left[C_p - 2k\mathrm{sn}(\eta;k)\right], \\ a_s^2 &= \frac{1}{2\gamma_p\gamma_i}\left[C_s + k\mathrm{sn}(\eta;k)\right], \\ a_i^2 &= \frac{1}{2\gamma_s\gamma_p}\left[C_i + k\mathrm{sn}(\eta;k)\right], \end{aligned} \tag{4}$$

where the constants C_p, C_s, and C_i are related to the conservative lows by the following relations:

$$C_p = \frac{C+\mathrm{U\,r}}{1-U^2},$$
$$C_s = C + C_1 - \frac{1}{2}\frac{C+\mathrm{U\,r}}{1-U^2}, \quad (5)$$
$$C_i = C - C_1 - \frac{1}{2}\frac{C+\mathrm{U\,r}}{1-U^2},$$

$$\gamma_s\gamma_i a_p^2 + \gamma_s\gamma_p a_i^2 + \gamma_p\gamma_i a_s^2 = C = \text{const}, \quad (6)$$

$$\gamma_p\gamma_i a_s^2 - \gamma_s\gamma_p a_i^2 = C_1 = \text{const}, \quad (7)$$

$$r = \frac{\Delta k\sqrt{\gamma_s\gamma_i}}{2} + V, \quad (8)$$

$$U = \frac{m_0}{8\gamma_p\sqrt{\gamma_s\gamma_i}}, \quad (9)$$

$$V = \frac{\mathrm{Cm}_1 + C_1 m_2}{4\gamma_p\sqrt{\gamma_s\gamma_i}}, \quad (10)$$

$$\eta = 2z\sqrt{\frac{1-U^2}{\gamma_s\gamma_i}}, \quad (11)$$

$$m_0 = 8\gamma_s\gamma_p + 8\gamma_i\gamma_p - 4\gamma_s\gamma_i - 4\gamma_p^2 - \gamma_s^2 - \gamma_i^2,$$
$$m_1 = \gamma_s^2 + \gamma_i^2 + 4\gamma_s\gamma_i - 4(\gamma_s+\gamma_i)\gamma_p, \quad (12)$$
$$m_2 = \gamma_s^2 - \gamma_i^2 - 4\gamma_s\gamma_i - 4(\gamma_s+\gamma_i)\gamma_p,$$

$$k^2 = \frac{1}{2}\left(\frac{C+\mathrm{U\,r}}{1-U^2}\right) - 1 - \frac{2B_0 r}{(C+\mathrm{U\,r})}, \quad (13)$$

where $0 < k < 1$, constants m_0, m_1 and m_2 count for the influence of the processes of SPM and XPM. The parameters C and C_1 are constants connected with the conservation lows; B_0 is integration constant; r, U, V and η are additional constants simplifying the expressions. These expressions (4)-(13) characterize the periodic energy exchange between the intensity of the pump a_p^2 from one hand and the intensities of the signal a_s^2 and idler a_i^2 waves from other hand during their propagation in optical fiber. It is important to mention that this periodic energy exchange between the three waves is under the influence of processes of FPPM, SPM and CPM. They are defined by the values of the constants Δk, C, and C_1, which depend on the initial conditions and the initial generalized phase $\Psi(0)$.

3. BASIC EQUATIONS IN PULSE REGIME

The pulses at different frequencies as pump, signal and idler waves propagate in single mode fibers with different group velocities and dispersion constants. The normalized envelope equations governing their propagation are well known [10-11] and in a coordinate system, moving with the group velocity of the pump wave $v_p$can be written as:

$$i\frac{\partial A_p}{\partial z} + \frac{1}{2}\frac{\partial^2 A_p}{\partial t^2} = \gamma_p\left(2A_sA_iA_p^*e^{-i\Delta \mathrm{kz}} + \left|A_p\right|^2 A_p + 2|A_s|^2A_p + 2|A_i|^2A_p\right),$$
$$i\left(\frac{\partial A_s}{\partial z} + \frac{1}{\Delta v_s}\frac{\partial A_s}{\partial t}\right) + \frac{D_s}{2}\frac{\partial^2 A_s}{\partial t^2} = \gamma_s\left(A_p^2A_i^*e^{i\Delta \mathrm{kz}} + |A_s|^2A_s + 2\left|A_p\right|^2A_s + 2|A_i|^2A_s\right), \quad (14)$$
$$i\left(\frac{\partial A_i}{\partial z} - \frac{1}{\Delta v_i}\frac{\partial A_i}{\partial t}\right) + \frac{D_i}{2}\frac{\partial^2 A_i}{\partial t^2} = \gamma_i\left(A_p^2A_s^*e^{i\Delta \mathrm{kz}} + |A_i|^2A_i + 2\left|A_p\right|^2A_i + 2|A_s|^2A_i\right).$$

We normalize the equations to the material and wave characteristic of the pump wave and in that way we obtained the following constants:

$$z = z/z_{\mathrm{disp}};\ z_{\mathrm{disp}} = t_{0p}^2/k_p^{"};\quad \gamma_p = k_p n_2\left|A_0^p\right|^2 t_{0p}^2/k_p^{"};\quad A_p = A_0^pA_p;\ A_s = A_0^pA_s;\ A_i = A_0^pA_i, \quad (15)$$

$$\frac{1}{\Delta v_s} = \frac{v_p - v_s}{v_p^2};\quad \frac{1}{\Delta v_i} = \frac{v_p - v_i}{v_p^2};\quad D_s = \frac{k_s^{"}}{k_p^{"}};\quad D_i = \frac{k_i^{"}}{k_p^{"}}, \quad (16)$$

$$\gamma_s = \frac{k_s}{k_p}\gamma_p;\quad \gamma_i = \frac{k_i}{k_p}\gamma_p, \quad (17)$$

where z_{disp}, t_{0p}, n_2, A_0^p are respectively dispersion length, time duration of the pump wave, nonlinear refractive index for the pump wave, initial amplitude of the pump wave; v_i and v_s are correspondingly group velocity of the signal and idler waves; $k_p^{"}$, $k_s^{"}$ and $k_i^{"}$ are group velocity dispersions of the pump, signal and idler waves. The normalized group velocities differences of the signal and idler waves with respect to the pump wave and the normalized dispersions are presented as follow Δv_s, Δv_i, D_s, D_i.
In our numerical calculation we used the pump wave, which propagates in fused silica single mode fiber. The main wavelength of the pump was chosen to be in negative dispersion region of the fiber at $\lambda_p = 1.36$μm and with time duration $t_{0p} = 1$ ps ($\Delta\lambda_p \simeq 3$ nm). By two color scheme it is possible to be generated small signal and idler waves at spectral distances $\Delta\lambda = \lambda_p - \lambda_s = 40$ nm which is typical for parametric amplifiers. Thus, the signal wave is at $\lambda_s \simeq$ 1.4nm, while the idler wave is at $\lambda_i \simeq 1.32$ nm and they are well spectrally separated. The dispersion characteristics of the pump, signal and idler waves are presented in the following Table 1.

Table 1.

Carrying wavelenght [μm]	Group velocity [m/s]	Dispersion of the group velocity [s^2/m]
$\lambda_p = 1.36$	$v_p = 2.05078 \times 10^8$	$k_p^{"} = -0.7855\ \times 10^{-28}$
$\lambda_s \simeq 1.4$	$v_s = 2.05095 \times 10^8$	$k_s^{"} = -1.17 \times 10^{-28}$
$\lambda_i \simeq 1.32$	$v_i = 2.05105 \times 10^8$	$k_i^{"} = -0.416 \times 10^{-28}$

The dimensionless group velocities, dispersion parameters and the nonlinear coefficients can be obtained from equations (15)-(17) and are presented in Table 2.

Table 2.

Carrying wavelenght [μm]	Normalized Group velocity	Dispersion parameter	Nonlinear coefficient
$\lambda_p = 1.36$	$1/v_p = 0$	1	1
$\lambda_s \simeq 1.4$	$1/\Delta v_s = 0.0027$	$D_s = -1.5$	$\gamma_s = 1.0294$
$\lambda_i \simeq 1.32$	$1/\Delta v_i = 0.0017$	$D_i = -0.4$	$\gamma_i = 0.97$

The numerical experiment is provided by solving the nonlinear system (14) with split-step Fourier method and parameters from Table 2. The initial condition in the experiment is strong pump wave with intensity near to the critical for soliton regime:

$$A_p^{\text{init}} = \text{sec}h(t). \qquad (18)$$

The signal and idler waves are with initial phase $\phi = \pi/2$, $\Delta k=0.2$ and very small initial amplitudes:

$$A_s^{\text{init}} = A_i^{\text{init}} = 0.01\text{sec}h(t)e^{i\pi/4}. \qquad (19)$$

The time evolution of the three waves is presented on Fig. 1. As it can be seen significant energy exchange between the pump and signal waves is observed. Other important result is that there is no walk-off effect. It is expected for spectrally close laser pulses and as it is predicted in [10], it is due to the nonlinear self-confinement by SPM and FPPM effects.

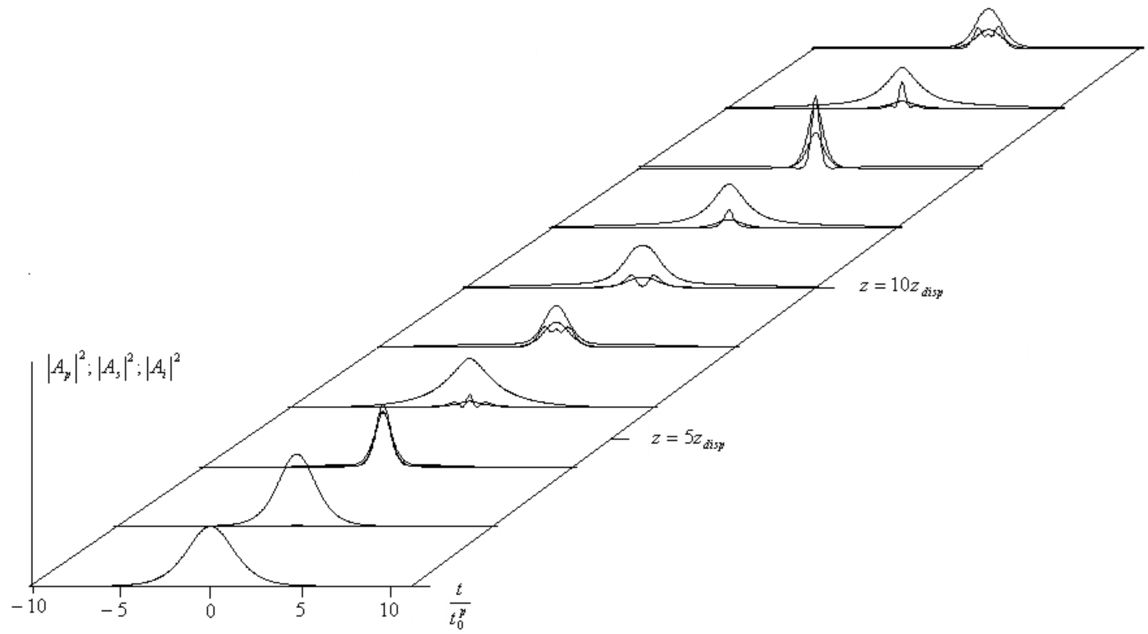

Figure 1. Time evolution of pump, signal and idler waves propagating in the negative dispersion region of single mode optical fiber. The pump wave is clearly seen in the beginning, while at distance of one-two dispersion lengths signal and idler waves are generated. The pulses are spectrally closer and there is no walk-off effect. The self-confinement is due to influence of CPM and FPPM effects on the relative motion of the waves [10]. Significant energy exchange is observed.

The graph of energy transformation of each wave $E_j = \int_{-\infty}^{\infty} |A_i| \, \mathrm{dt}$; $j = p, s, i$ is presented on Fig 2. Comparing with the results, obtained for CW regimes, in pulse regime instead of periodic exchange we observe quasi-periodical energy exchange between the pump and signal waves.

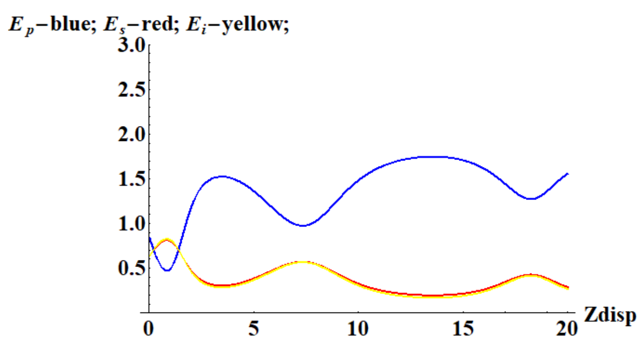

Figure 2. Graph of energy transformation of each wave $E_j = \int_{-\infty}^{\infty} |A_i|^2 \, \mathrm{dt}$; $j = p, s, i$. Instead of periodic exchange as it is possible in CW regime, in pulse regime we observed a quasi-periodical exchange with very long period of oscillations of order of several diffraction lengths.

4. CONCLUSIONS

In the presented work it is investigated the FPPM effect in two different regimes. In CW regime the corresponding nonlinear system of envelope equations (1) is solved analytically. The analytical solutions are described by Jacobi functions. A periodic exchange with period determinated by the parameter k of the elliptical $\mathrm{sn}(\eta; k)$ function is obtained. The mathematical method can be applied for waves with arbitrary initial intensities, generalized phase and wave number mismatch.

In pulse regime of propagation the parametric interaction is investigated in negative dispersion region of single mode optical fiber near to zero dispersion. The time duration of the pulses in the experiment was *1 ps*. The spectral differences between pump, signal and idler waves was chosen to be around $\Delta\lambda = \lambda_p - \lambda_s = 40$nm. Due to small group delay between pump and signal waves, there are no walk-off effect and an effective generation is possible. In pulse regime, instead of periodic, a quasi-periodic exchange of energy can be observed. The result can be used for parametric amplifiers, working in pulse regime in single mode fibers.

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