Load Disturbance Rejection in Hydro Turbine Control Based on Wiener Filtering

Teofana Puleva[1] and Elena Haralanova[2]

Abstract – **In this paper the task of load disturbance rejection in a hydro generator speed and power control is considered. The load change is described as a stochastic process. It is shown that the problem of random load disturbance rejection can be formulated as an optimal controller design in sense of minimum mean-square error (Wiener filter design). Controller structure based on dynamic compensation as well as on direct design with rational transfer function by taking into account the plant non-minimal phase characteristics is obtained.**

Keywords – **Load rejection, Water turbine speed control, Wiener filter design.**

I. INTRODUCTION

The problem of random load disturbance rejection in a hydro generator operation in power limited energy system can be based on the stochastic approach to the control system. The task of load disturbance rejection is formulated as an optimal controller design problem in sense of minimum mean-square error (Wiener filter design) [1-2]. In [3] a robust technique based on weighted multichannel Wiener filter for speech processing is considered. It takes into account the speech distortion due to signal model errors explicitly in its design criterion. The investigations in this paper can be considered as a continuation of previous research works related to multiple model adaptive controller design based on Wiener filtering [4]. The efforts are focused on the determination of the simplified single controller structure by more precise modelling of the stochastic load variation. This approach can be applied to other control design problems describing the local network operation of renewable energy sources.

II. CONTROLLER DESIGN BASED ON WIENER FILTER

The dynamic behavior of a system in which a hydro turbine unit is connected to a power limited energy system is investigated. The main function of the hydro turbine governor in this operation mode is to compensate the unpredictable load variation in order to keep the frequency on its nominal value. The load rate change can be described as a stochastic process that takes random values in particular time intervals. The time intervals between successive load rate changes can be modeled as a stochastic process [1] with an exponential distribution described by the PDF function

$$p_{\dot{v}(x)} = \mu e^{-\mu|\tau|}. \tag{1}$$

The reciprocal value of μ has a sense of a mean value of time intervals between successive load rate changes. The covariance and the spectral density function of this stochastic process are

$$K_{\dot{v}}(\tau) = \sigma^2 e^{-\mu|\tau|}, \quad S_{\dot{v}}(\omega) = \frac{2\sigma_v^2 \mu}{\omega^2 + \mu^2}, \tag{2}$$

where σ_v^2 is the load standard deviation. The load spectral density function can be presented by the following expression

$$S_v(\omega) = \frac{2\sigma_v^2 \mu}{\omega^2(\omega^2 + \mu^2)}.$$

The turbine-generator system dynamic behavior in presence of load disturbance can be presented by the equation

$$y(s) = W_{uy}(s)u(s) + W_{vy}(s)v(s). \tag{3}$$

The transfer functions with respect to the control signal u and to the load disturbance v in (3) are:

$$W_{uy}(s) = \frac{k(-T_w s + 1)}{(T_m s + 1)(0.5T_w s + 1)}, \quad W_{vy}(s) = \frac{k_v}{T_m s + 1}, \tag{4}$$

where T_w is the water column time constant; T_m - mechanical time constant; k and k_v - proportional coefficients with respect to the control signal and to the disturbance respectively.

In Fig. 1 the block diagram of the turbine speed control system in presence of load disturbance and measurement noise is shown. The system output is compared to the desired reference transformation $W_0(s)$.

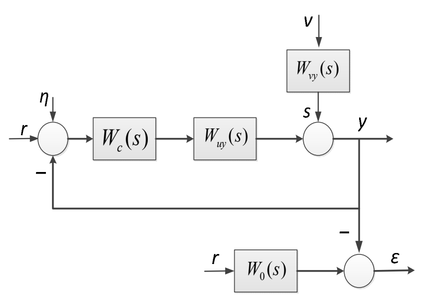


Fig. 1. Load disturbance in speed control of the hydro generator local network operation

Teofana Puleva is with Technical University of Sofia, Faculty of Automatics, Kl. Ohridski St. 8, 1000 Sofia, Bulgaria, E-mail: tpuleva@tu-sofia.bg

Elena Haralanova is with Technical University of Sofia, Faculty of Automatics, Kl. Ohridski St. 8, 1000 Sofia, Bulgaria, E-mail: tpuleva@tu-sofia.bg

The spectral density function of the random signal *s* is

$$S_s(\omega) = \left|W_{vy}(j\omega)\right|^2 S_v(\omega) = \frac{2T_l k_v^2 \sigma_v^2}{\omega^2(1+T_m^2\omega^2)(1+T_l^2\omega^2)}, \quad (5)$$

where $T_l = \mu^{-1}$.

After block diagram algebra in Fig. 1 the topology of the diagram shown in Fig. 2 can be obtained. The controller will be designed for small perturbations around the equilibrium point r=0.

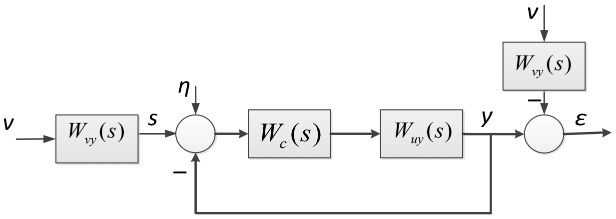


Fig. 2. Modified block diagram

The error spectral density function from the general Wiener problem is

$$S_\varepsilon = \left|1-W\right|^2 S_s + \left|W\right|^2 S_\eta, \quad (6)$$

where W is the closed loop transfer function. Therefore the problem of random load disturbance rejection in a hydro generator operation in power limited energy system can be formulated as an optimal controller design problem in sense of minimum mean-square error (*Wiener filter design*). The linear least-squares filter performs optimal separation of the measured signal and additive noise, based on their spectral characteristics. The optimal control system transfer function in sense of minimum mean-square error is [2, 5-7].

$$W_{opt}(s) = \frac{1}{\varphi}\left[\frac{W_0 S_s}{\overline{\varphi}}\right]_+, \ \varphi\overline{\varphi} = S_s + S_\eta, \ S_{s\eta} = 0, \quad (7)$$

where operations of factorization and separation have to be carried out in order to obtain a causal solution. In case $W_0(s) = 1$ and in presence of measurement noise modeled as a white noise, the separation in Eq. (7) may be avoided. Hence the Wiener filter design can be carried out by the formula

$$W_{opt}(s) = \frac{1}{\varphi}\left(\varphi - \sqrt{S_\eta}\right), \ S_\eta = N^2. \quad (8)$$

The factorization of Eq. (8) leads to the following expression:

$$\varphi = \frac{b_0 s^3 + b_1 s^2 + b_2 s + b_3}{s(T_l s+1)(T_m s+1)}. \quad (9)$$

The coefficients b_i, $i = \overline{0,3}$ are calculated from the following relationships:

$$b_0 = NT_m T_l, \ b_1^2 = N^2\left(T_m^2 + T_l^2\right) + 2NT_m T_l b_2, \quad (10)$$

$$b_2^2 = N^2 + 2\sqrt{D_s}, \ b_3 = k\sigma_v\sqrt{2T_l},$$

where $S_\eta = N^2$, $D_s = 2T_l k_v^2 \sigma_v^2$.

For a given spectral density function of the random signal *s*, the optimal system transfer function is

$$W_{opt}(s) = \frac{c_1 s^2 + c_2 s + c_3}{b_0 s^3 + b_1 s^2 + b_2 s + b_3}, \quad (11)$$

$$c_1 = b_1 - N(T_m + T_l), \ c_2 = b_2 - N, \ c_3 = k\sigma_v\sqrt{2T_l}. \quad (12)$$

If the optimal system transfer function is already determined $W = W_{opt}$, the controller transfer function can be obtained:

$$W_c(s) = \frac{W}{W_{pl}(1-W)}, \quad (13)$$

where W_{pl} is the plant transfer function ($W_{pl} = W_{uy}$). The turbine speed and power controller design takes into account special features of the plant related to its non-minimal phase characteristic. The Eq. (13) can not be applied to controller design when the plant exhibits non-minimal phase behaviour. In this case another controller structure and design algorithm can be developed. One approach for solving this problem is to apply direct design method based on rational transfer functions description by taking into account plant non-minimal phase characteristics [8-9] when the desired closed loop trasfer function is already obtained. The control signal u is formed by the following polynomial expression:

$$Ru = Sr - Ty, \quad (14)$$

where *R, S* и *T* are polinomials of s satisfying the corresponding degree requirements, r and y are the reference and the output signal respectively. The polinomials *R*, *S* and *T* can be determined by solving the Diophantine equation

$$(AR + BT)y(s) = BSr(s), \quad (15)$$

where the polynomials *B* and *A* are the plant transfer function numerator and denominator, and the ratio $y(s)/r(s)$ is the desired transfer function W_{opt}. In presence of non-minimal phase characteristics, the numerator is presented as a product of two components $B(s) = B^+(s)B^-(s)$, where the non-minimal phase term is $B^-(s)$. The optimal transfer function in case of functional constraints is [1-2, 7]

$$W_{opt}(s) = \frac{E}{\overline{E}\varphi}\left[\frac{W_0 \overline{E} S_s}{E\overline{\varphi}}\right]_+, \quad (16)$$

$$E = -T_w s + 1, \ \overline{E} = T_w s + 1, \quad (17)$$

where the non-minimal fraction *E* is analytical function in the left half-plane and $\overline{E}$ is analytical in the right half-plane.

The factorization of the input signal spectral density function is $\varphi\overline{\varphi} = S_s + S_\eta$, where φ is given by Eq. (9).

The separation determines the causal term which can be written as:

$$\left[\frac{W_0 \bar{E} S_s}{E\bar{\varphi}}\right]_+ = \frac{m_1 s^2 + m_2 s + m_3}{s(T_l s+1)(T_m s+1)}, \quad (18)$$

$m_1 = d_0 T_m T_l + d_1 T_l + d_2 T_m$, $m_2 = d_0(T_m + T_l) + d_1 + d_2$, $m_3 = d_0$, where the coefficients d_i, $i = \overline{0,3}$ depend on the plant and the load parameters

$$d_0 = \lim_{j\omega=0} \frac{j\omega \bar{E} S_s}{E\bar{\varphi}}, \; d_1 = \lim_{j\omega=-\frac{1}{T_m}} \frac{\left(j\omega+\frac{1}{T_m}\right)\bar{E}S_s}{E\bar{\varphi}},$$

$$d_2 = \lim_{j\omega=-\frac{1}{T_l}} \frac{\left(j\omega+\frac{1}{T_l}\right)\bar{E}S_s}{E\bar{\varphi}}.$$

The optimal transfer function with functional constraints is

$$W_{opt}(s) = \frac{(m_1 s^2 + m_2 s + m_3)(-T_w s+1)}{(b_0 s^3 + b_1 s^2 + b_2 s + b_3)(T_w s+1)}. \quad (19)$$

If the transfer function of the optimal system is already determined $W = W_{opt}$, the controller transfer function can be obtained in accordance with Eq. (13) and can be described by the following general expression

$$W_c(s) = \frac{p_0 s^3 + p_1 s^2 + p_2 s + p_3}{s(T_l s+1)(T_m s+1)}. \quad (20)$$

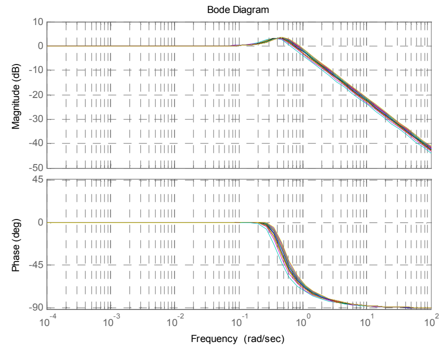
In general the controller coefficients p_i, $i = \overline{0,3}$ depend on the variable load characteristics descibed by load variance σ_v^2 and load time constant T_l.

III. Experimental Results

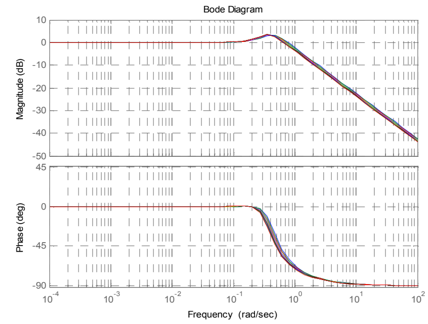
The hydro turbine model is described by the transfer function Eq. (4) with $k = 1.25$, $T_m = 12.8\,s$ and $T_w = 1.18\,s$. The shaping filter parameters are: load time constant $T_l = 30\,s$ and coefficient $k_v = 1.25$. The variance fluctuation interval is $\sigma_v^2 = [0.1, 1.1]$ and the load time constant interval is $T_l = [20, 65]\,s$.

The optimal transfer functions are determined in both cases: without functional constraints defined by Eq. (11) and by taking into account non-minimal phase plant characteristics (Eq. 19). The experiment consists of random varying load conditions. The idea is to present the system performance in the whole operational range. Although, hydro turbine generators typically work long time intervals under almost constant load conditions, such sudden changes in the load conditions can accrue. The frequency responses under variable load characteristics are obtained. In Figs. 3 and 4 the optimal system frequency responses under variable load variance and variable load time constant are shown.

It can be noticed the optimal system frequency response ensures a zero steady-state error in the low frequency range up to 0.1 rad/s that corresponds to the load signal frequency band.

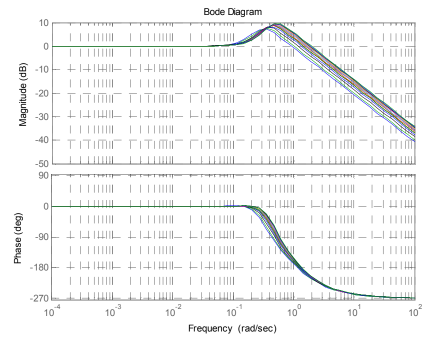


(a)



(b)

Fig. 3. Optimal system frequency response under (a) variable load variance; (b) variable load time constant



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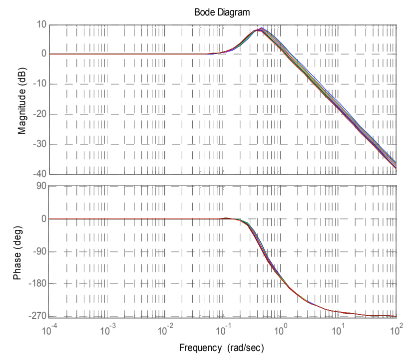
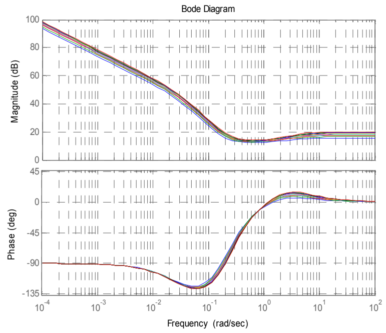


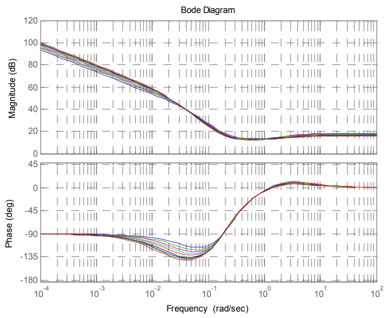
(b)

Fig. 4. Optimal system frequency response with functional constraints under (a) variable load variance; (b) variable load time constant

The load characteristics variation reflects on the natural frequency in a relatively small range less than half of decade.



(a)



(b)

Fig. 5. Controller frequency response under (a) variable load variance (b) variable load time constant

The both control algorithms use the optimal system transfer function (Wiener filter) that requires a factorization of the spectral density function. The direct design with rational transfer function method avoids the separation Eq. (18) that makes easily to design the control algorithm.

In Table I some results from the design procedure based on minimum mean square error method (Wiener filter design) are shown. It can be noticed the mean square error increases significantly in case of functional constraints. The controller structure based on dynamic compensation as well as on direct design with rational transfer function method is determined. The both controllers have the same order and an integral term.

TABLE I
OPTIMAL TRANSFER FUNCTION

Without functional constraints	With functional constraints
$D_v = 2$, $T_l = 30\ s$	
$W_{opt}(s) = \frac{0.72s^2+0.34s+0.07}{s^3+0.84s^2+0.35s+0.07}$	$W_{opt}(s) = \frac{-1.70s^3+0.84s^2+0.42s+0.07}{1.18s^4+1.99s^3+1.24s^2+0.43s+0.07}$
min mean-square error $0.72\,10^{-2}$	*min* mean-square error $3.5\,10^{-2}$

In case without functional constraints the controller structure following the design procedures given by Eqs. (13) and (14) is:

$$W_c(s) = \frac{7.37s^3+15.07s^2+4.74s+0.62}{s^3+3.04s^2+0.1s}$$

$$R(s) = 0.1s^3+1.13s^2+1.39s\ ,\quad S(s) = 1.72s^2+4.5s+0.86$$

$$T(s) = 4.37s^3+10.81s^2+4.57s+0.86$$

The simulation results shown in Fig. 6 show a very good system performance in tracking the load signal and keeping the frequency about its nominal value in Fig. 7.

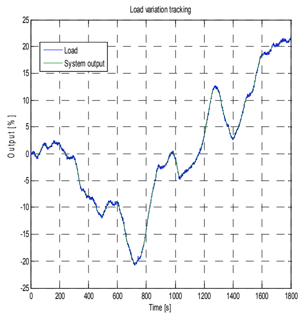


Fig. 6. Load variation tracking

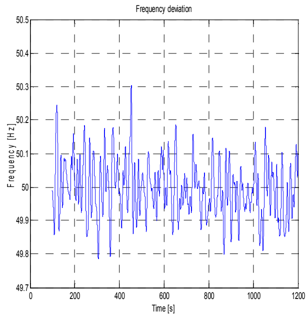


Fig. 7. Frequency deviation

IV. CONCLUSION

The problem of random load disturbance rejection in a hydro generator operation in power limited energy system is formulated as an optimal controller design problem in sense of minimum mean-square error (Wiener filter design). The turbine speed and power controller design in case of functional constraints is investigated. The controller structure based on dynamic compensation as well as on direct design with rational transfer function is determined. Simulations with proposed algorithms are carried out in order to explore the systems performance. This approach can be applied to other control design problems describing the renewable energy sources (RES) operation in electric power system where the both generated power from RES as well as the load have a random nature. In case of stochastic characteristics varying in a wide range an appropriate adaptation procedure applied to the controller parameters can be carried out [10].

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REFERENCES

[1] A. Pervozwanski, *Automatic Control Theory*, Nauka, Moskva, 1996 (*in Russian*).
[2] R. Gray, *An Introduction to Statistical Signal Processing*, Cambridge University Press, 2004.
[3] S. Doclo, A. Spriel, J. Wouters and M. Moonen, “Frequency-Domain Criterion for the Speed Distortion Weighted Multichannel Wiener Filter for Robust Noise Reduction”, Speech Communication, vol. 49, 2007, pp. 636-656.
[4] T. Puleva, “Multiple Model Wiener Filter for Load Disturbance Rejection in Hydro Generator Local Network Operation”, Proceedings of the Technical University of Sofia, vol. 58, Book 1, pp. 33-43, 2008.
[5] R. Brown, P. Hwang, *Introduction to Random Signal and Applied Kalman Filtering*, John Wiley & Sons, Inc., 1992.
[6] P. Diniz, *Adaptive Filtering: Algorithms and Practical Implementation*, Springer, 2008.
[7] M. Kolos, I. Kolos. *Methods of Linear Optimal Filtration*, Lomonosov Moskow State University Press, 2000.
[8] K. Astrom, B. Wittenmark, *Computer Controlled Systems: Theory and Design*, 3th edition, Prentice Hall, 1997.
[9] M. Lund, *Pole Placement Design*, Toolbox for MATLAB.
[10] R. Horowitz, B. Li, and J.W. McCormick, “Wiener-filter-based Minimum Variance Self-tuning Regulation”, Automatica, vol. 34, no. 5, pp. 531-545, 1998.