Investigation of Digital Procedure for Mains Frequency Measurement

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Abstract – **An investigation of digital procedure for mains frequency measurement has been introduced in this article. The procedure has been implemented as a function in a MatLab environment. The procedure is based on the principles of frequency demodulation. Appropriate digital filters have been synthesized for extracting mains interference from signals and for measuring its frequency. A tests has been performed with synthesized and natural mains interference.**

Keywords – **Mains interference, Frequency measurement, Digital filters synthesis.**

I. Introduction

The efficiency of mains interference suppression depends on the knowledge with high precision for instantaneous value of mains frequency [3]. Special digital filters are the most commonly used for the mains interference suppression [1, 2]. Although maintaining the mains frequency in the energy system within narrow limits [4], its value is altered enough to make it more difficult to suppress. Measurement with a high precision by classical methods requires a long measurement interval and a high sampling rate [5]. In addition, classic methods are often inappropriate for preliminary done records.

A methodology for mains interference (hum) frequency estimation in electrocardiographic signal is represented in [6]. Late on it is developed for mains frequency measurement.

The methodology is based on transfer coefficient determination of a digital filter which is applied to the signal. The transfer coefficient is approximated with a line in the region of the mains frequency. The angle of the line determines the proportion between the frequency shift and the transfer coefficient changing as shown on fig. 1.

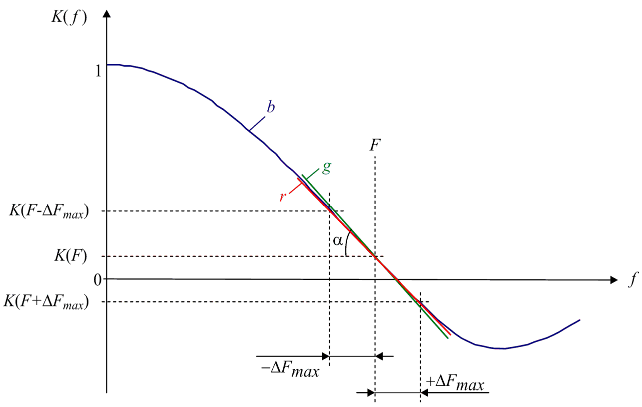


Fig. 1. Linear approximation of the digital filter frequency response in the range of mains frequency deviation.

For the range of mains interference frequency change from $F–\Delta F$max to $F+\Delta F$max around the rated value F the frequency response $K(f)$ of the averaging filter (curve b) is represented as a linear function of the mains frequency with the Cartesian equation $K(f)=K(F)+\tan\alpha.(f-F)$, as shown by curve g.

The tangent of the angle α between the approximated line and the x-axis is represented with the first derivative of the filter transfer coefficient for the rated value F of the mains frequency $\tan\alpha = K^I(f)\big|_{f=F}$. The new value of the coefficient $K(F_{new})$ for the changed mains frequency value F_{new} is represented by eq. (1):

$$K_{Fnew} \equiv K(F_{new}) \approx K(F) + K^I(f)\big|_{f=F}.(F_{new}-F), \quad (1)$$

where the new value of the mains frequency F_{new} can be calculated by eq. (2):

$$F_{new} \approx F + \frac{K(F_{new})-K(F)}{K^I(f)\big|_{f=F}} , \quad (2)$$

If we represent the current value of the mains interference with B_i and the current filtered value with Y_i the current value of $K(F_{new})$ is given by the eq. (3):

$$K(F_{new})_i = \frac{Y_i}{B_i} , \quad (3)$$

The error of the frequency calculation is determined by the accuracy of determination of the angle of the approximation and the difference between the line and frequency response.

II. Investigation of the "Two-Point" Filter

A numerous of experiments [7, 8] have shown that the best result is achieved by applying the so called "two-point" filter having a differential equation Y and a frequency response $K(f)$ by the equation

$$Y_i = \frac{B_{i-q}+B_{i+q}}{2}, \quad K(f) = \cos\frac{2q\pi f}{Q} , \quad (4)$$

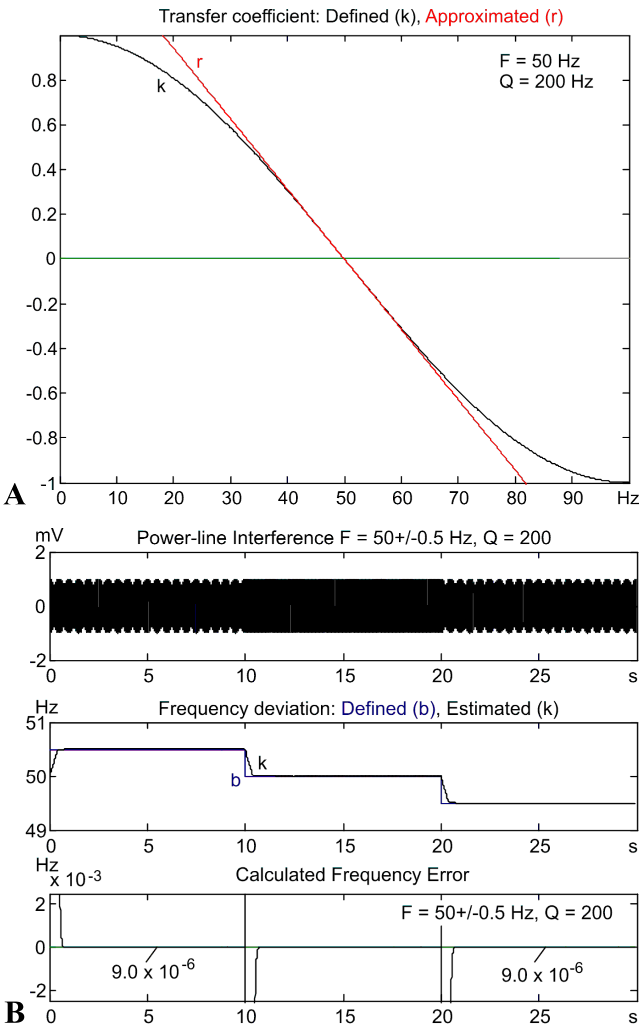
In the examined filter B_i is the ongoing input sample, Y_i is the filtered sample and Q is the sampling rate. The parameter q is the number of samples in the quarter of the mains period. When q is not an integer, a rounded to a lesser or equal integer value is used. In MatLab environment, a function q = floor (Q/F/4) is used.

The illustrations on fig. 2 demonstrates the results from experiments done with a “two-point” filter at sampling rates $Q = 200$, 250, 300 and 350 Hz, rated value of the mains interference $F = 50$ Hz and a deviation of ±0,5 Hz.

A signal with epoch length of 30 s containing a synthesized mains interference with a constant amplitude of 1 mV is used for the experiments. The examined epoch is divided in three parts where the frequency of the interference in the first part is 50.5 Hz, in the second 50 Hz and in the third 49.5 Hz.

For every experiment two illustrations are shown. The illustrations **A** contain the frequency response of the examined filter (curve k), the linear approximation line in the range of the frequency deviation (curve r) and the zero-line with marker on the rated value of the mains interference (curve g).

The illustrations **B** demonstrate the mains frequency calculation procedure. The first subplot is showing the synthesized mains interference, the second is showing the set deviation of the mains interference (curve b) and the calculated frequency (curve r). The third subplot contains the zoomed error that is a difference between the set and the calculated frequency. For the cases of $Q = 250$, 300 and 350 Hz, just zoomed errors are shown.



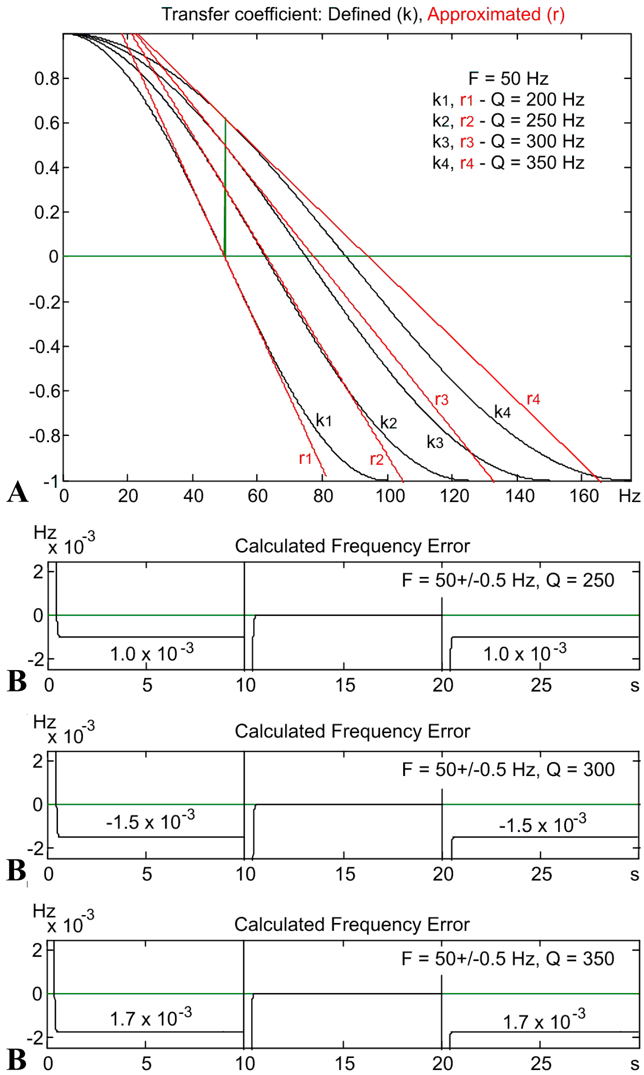


Fig. 2. Experiment with a “two-point” digital filter at $q = 1$.

From the experiment one can observe that the error is non-symmetrically “shifted” in the ends of the frequency range due to the deviation of the approximation line in one direction from the real transfer function of the averaging filter. The lowest error is achieved when the averaging filter is working in the inflection point of its transfer function – the relative error is in the range of ±0,001%.

III. COMBINED “TWO-POINT” FILTER SYNTHESIS

The investigation deals to synthesize a filter with inflection point of the frequency response for the measured frequency. For this purpose, a methodology from [9] is used. Two “two-point” filters $Y1$ and $Y2$ are multiplied by complementary to unity coefficients $(1 - k_y)$ and k_y and are summed.

$$Y_i = \left(1 - k_y\right).Y1_i + k_y.Y2_i, \quad \left| \begin{array}{l} Y1_i = \dfrac{B_{i-q} + B_{i+q}}{2} \\ Y2_i = \dfrac{B_{i-q-1} + B_{i+q+1}}{2} \end{array} \right. , \quad (5)$$

The frequency response of the resultant filter is also expressed by the sum of the frequency responses of the two filters multiplied by complementary to unity coefficients $(1 - k_y)$ and k_y.

$$K(f)=\left(1-k_y\right).K1(f)+k_y.K2(f),$$

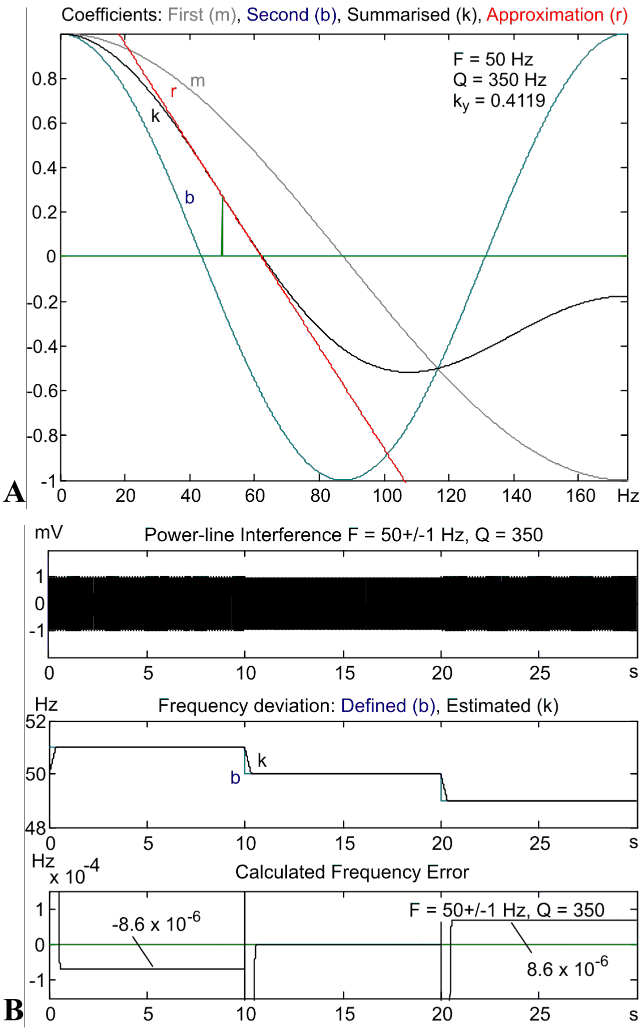
$$\left|\begin{array}{l} K1(f)=\cos\dfrac{2\pi qf}{Q} \\ K2(f)=\cos\dfrac{2\pi(q+1)f}{Q} \end{array}\right. , \qquad (6)$$

To determinate the coefficient k_y, the second derivative of the frequency response of the summed filter $K^{II}(f)\Big|_{f=F}=\left(1-k_y\right).K1^{II}(f)\Big|_{f=F}+k_y.K2^{II}(f)\Big|_{f=F}$ should have value 0 for $f = F$ from where:

$$k_y=\frac{K1^{II}(F)}{K1^{II}(F)-K2^{II}(F)},$$

$$\left|\begin{array}{l} K1^{II}(F)=-\left(\dfrac{2\pi q}{Q}\right)^2\sin\dfrac{2\pi Fq}{Q} \\ K2^{II}(f)=-\left(\dfrac{2\pi(q+1)}{Q}\right)^2\sin\dfrac{2\pi F(q+1)}{Q} \end{array}\right. . \qquad (7)$$

This way, the summed filter has inflection point for the value of the mains frequency F.



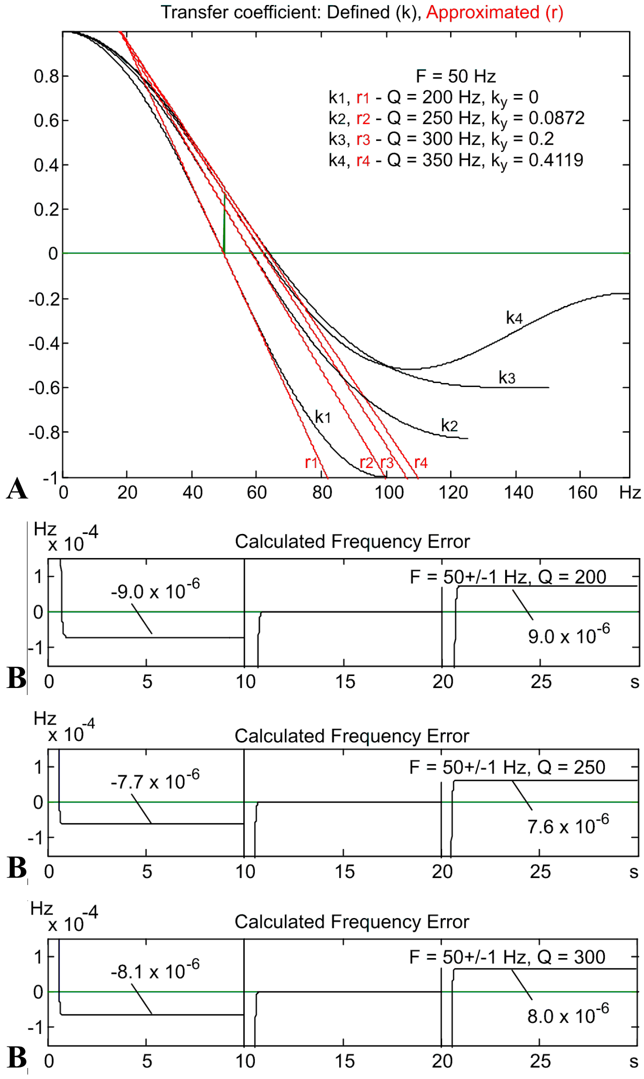


Fig. 3. Experiment with summed "two-point" filter at $q = 1$.

The next experiment (illustrated in fig. 3) is shows the results from the tests with summed "two-point" filters with inflection points of the frequency response coinciding with the mains frequency. The experiment is done the same way as experiment on fig. 2. The mains frequency deviation is increased twice (from 49 to 51 *Hz*), and the scale of the error is increased 10 times. The contents of the subplots are the same as in fig. 2. In illustrations **A**, frequency responses of the first (curve *m*) and the second (curve *b*) summed filters are added.

From the experiment done one can conclude that the frequency calculation error is insignificant in case of working in the inflection point of the filter.

IV. Synthesis of Mains Frequency Extraction Filter

Normally in the input signal, other components than the mains frequency are present. One is the zero line shifting. For this purpose a filter has been constructed which has 0 for frequency $f = 0$ and 1 for $f = F$.

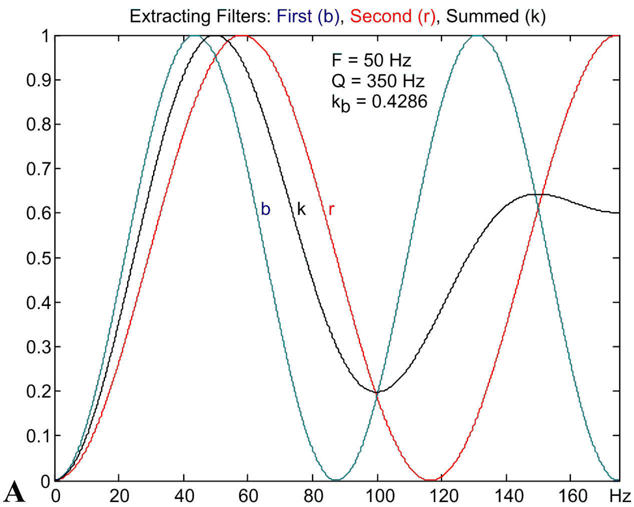
For the purpose of extracting the mains frequency and reject the zero line shifting, the above-mentioned methodology for summing two filters is applied. Now both filters are so-called "three-point" filters, having zero transfer coefficient at $f = 0$:

$$B_i=(1-k_b).B1_i+k_b.B2_i,$$
$$\begin{cases} B1_i=\dfrac{-0,5X_{i-m}+X_i-0,5X_{i+m}}{2} \\ B2_i=\dfrac{-0,5X_{i-m-1}+X_i-0,5X_{i+m+1}}{2} \end{cases} \quad . \qquad (8)$$

In the examined filter X_i is the ongoing input sample, B_i is the extracted sample of the mains frequency. The parameter m is the number of samples in the semi-period of the mains. When m is not an integer, a rounded to a lesser or equal integer value is used. In MatLab environment, a function m = floor (Q/F/2) is used.

To determinate the coefficient k_b, the first derivative of the frequency response of the summed filter should have value 0 for $f = F$ i.e.:

$$k_b=\left[1-\frac{(m+1)\cdot\cos\dfrac{\pi F(m+1)}{Q}\cdot\sin\dfrac{\pi F(m+1)}{Q}}{m\cdot\cos\dfrac{\pi Fm}{Q}\cdot\sin\dfrac{\pi Fm}{Q}}\right]^{-1} \quad . \qquad (9)$$



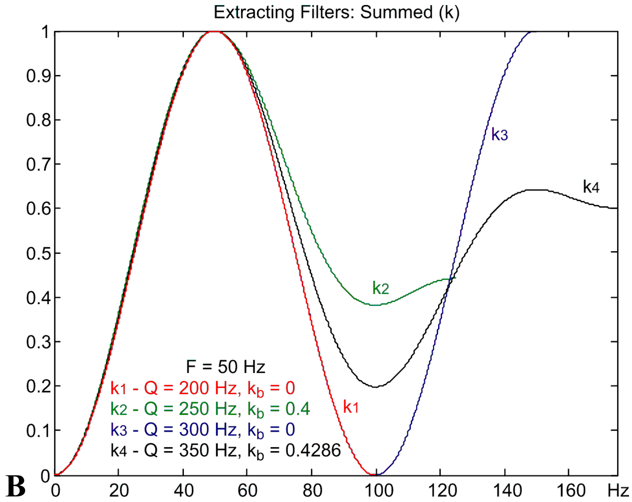


Fig. 4. Extraction filters frequency responses.

The illustrations on fig. 4 demonstrates the methodology of a summed "two-point" building (fig. **A**) end frequency responses of summed filters at sampling rates $Q = 200$, 250, 300 and 350 Hz, with rated value of the mains $F = 50$ Hz (fig. **B**).

Similarly, digital filters are designed to suppress the second and third harmonics of the mains frequency, which are presented in [10].

V. Program Implementation of the Digital Procedure for Mains Frequency Measurement

The procedure for mains frequency measurement is implemented as a function in MatLab environment and is shown on fig. 5. After parameters initialization (line 8-13) two modules are developed.

The first module includes extracting filter coefficient calculating (lines 14-30) and extracts the mains frequency from the input signal (lines 31-34). Extracting filter can be twice applied by doubling lines 32-34.

```
1  function [FPL] = PLI_Measurement_V19(X,Q,F,Res)
2      %%% Procedure for Power-Line Interference Measuring %%%
3      %%% Input parameters:                  %%% Output parameters:
4      %%% X - Input signal;              %%% FPL - Calculated mains frequency;
5      %%% Q - Sampling rate, Hz;
6      %%% F - Mains rated value, Hz;
7      %%% Res - Resolution;
8                          %%% Initialization %%%
9      LX = length(X);
10     B = horzcat(zeros(1,LX));     % Mains buffer definition
11     FPL = horzcat(zeros(1,LX));   % Output buffer definition
12     m = floor(Q/F/2);             % Mains semi-period
13     q = floor(Q/F/4);             % Mains quarter-period
14                         %%% Extracting filter calculation %%%
15     %******************************* I - impulse
16     I=horzcat(zeros(1,m+1),1,zeros(1,m+1));
17     %******************************* K1 - First "three-point" filter
18     K1=horzcat(0,0.5,zeros(1,m-1),1,zeros(1,m-1),0.5,0)/2;
19     K1F=cos((pi*F*m)/Q).^2;
20     K1Fi=-2*(pi*m)/Q*cos((pi*F*m)/Q)*sin((pi*F*m)/Q);
21     %******************************* K2 - Second "three-point" filter
22     K2=horzcat(0.5,zeros(1,m),1,zeros(1,m),0.5)/2;
23     K2F=cos((pi*F*(m+1))/Q).^2;
24     K2Fi=-2*(pi*(m+1))/Q*cos((pi*F*(m+1))/Q)*sin((pi*F*(m+1))/Q);
25     %******************************* K - Summed "three-point" filter synthesis
```

```
26      kb=K1Fi/(K1Fi-K2Fi);
27      KF=(1-kb)*K1F+kb*K2F;
28      K=((1-kb)*K1+kb*K2);
29      %******************************** Ks - Normalized K-filter
30      Ks=(I-K)/(1-KF);
31      %******************************** Mains interference extracting
32      for i=1+(2*m+1): 1: LX-2*m-1;
33         B(i)=0; for j=1:1:2*m+3; B(i)=B(i)+Ks(j)*X(i+j-m-2); end
34      end
35                      %%% Measuring filter calculation %%%
36      %******************************** First "two-point" derivate
37      K1Fi=-(2*pi*q/Q)*sin(2*pi*F*q/Q);
38      K1Fii=-(2*pi*q/Q)^2*cos(2*pi*F*q/Q)*1000;
39      %******************************** Second "two-point" derivate
40      K2Fi=-(2*pi*(q+1)/Q)*sin(2*pi*F*(q+1)/Q);
41      K2Fii=-(2*pi*(q+1)/Q)^2*cos(2*pi*F*(q+1)/Q)*1000;
42      %******************************** Summed "two-point" filter synthesis
43      ky = K1Fii/(K1Fii-K2Fii);
44      KFi=(1-ky)*K1Fi+ky*K2Fi;
45                     %%% Transfer coefficients calculation %%%
46      KF01 = cos(2*pi*q*F/Q);      % Initial value of First filter coefficient
47      KF02 = cos(2*pi*(q+1)*F/Q); % Initial value of Second filter coefficient
48      KF0=(1-ky)*KF01+ky*KF02;     % Initial value of Summed filter coefficient
49      KFmax=KF0+(-F*0.02)*KFi;     % Summed filter coefficient maximal value
50      KFmin=KF0+(+F*0.02)*KFi;     % Summed filter coefficient minimal value
51      KFspd = (KFmax-KFmin)/(Q/(1*q)); % Filter coefficient maximal speed
52      KF = KF0;
53                          %%% Algorithm %%%
54      for i=1+q+1: 1: LX-q-2;
55          Y1=(B(i-q)+B(i+q))/2;
56          Y2=(B(i-q-1)+B(i+q+1))/2;
57          Y=(1-ky)*Y1+ky*Y2;
58          if abs(B(i))>Res; % Division zero protection
59              KFnew=Y/B(i); else KFnew=KF;
60          end
61          if KFnew-KF>KFspd; KFnew=KF+KFspd; end  % KF speed protection (rising)
62          if KFnew-KF<-KFspd; KFnew=KF-KFspd; end % KF speed protection (falling)
63          if KFnew>KFmax; KFnew=KFmax; end;       % KF maximal value protection
64          if KFnew<KFmin; KFnew=KFmin; end;       % KF minimal value protection
65          KF=KF/2+KFnew/2;                        % KF filtering
66          FPL(i)=F+(KFnew-KF0)/KFi;               % Mains frequency calculating
67      end
68  end
```

Fig. 5. MatLab program code of the function for mains frequency measurement.

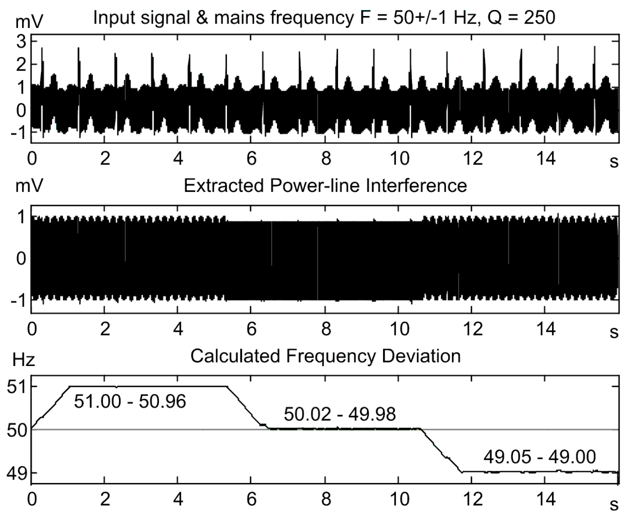
The second module consists of measuring filter building (lines 35-44), its transfer coefficients initialization (lines 45-52) and applying the algorithm for mains frequency measurement (lines 53-67).

VI. Experiments

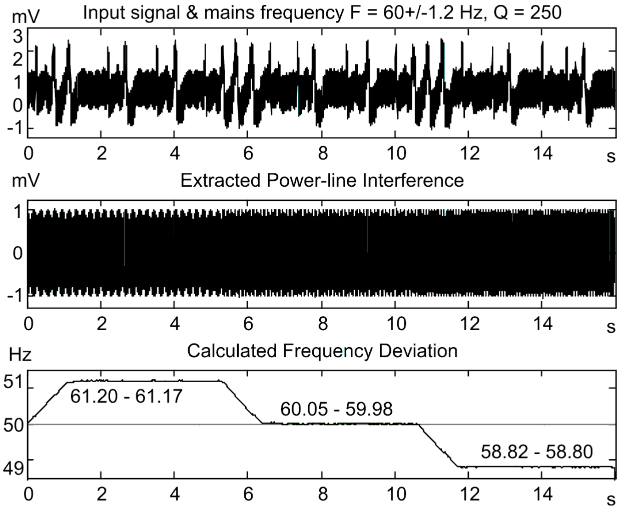
On fig. 6 are shown results from experiments with two different ECG signals with added synthesized 1 *mV* mains interference. The tested epoch is 16 s length. In the experiments No 1 and 3 a step change in the frequency of the mains interference was applied, in the first third of the studied epoch the frequency of the mains interference is F+0.02F, the second is F and the third F-0.02F.

In experiments No 2 and 4, a smooth change of mains interference by linear law within the range of F-0.02F to F+0.02F is applied. In all experiments the sampling rate is 250 Hz. At tests 1 and 2 the rated value of the mains frequency is $F = 50$ Hz, and for tests 3 and 4 it is $F = 60$ Hz.

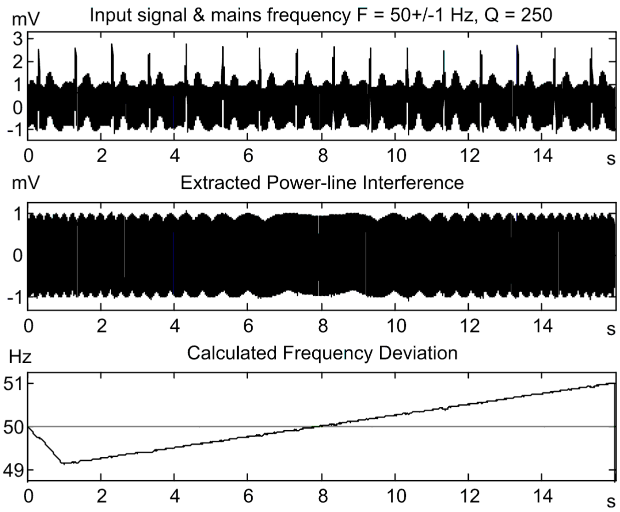
The first subplot shows the tested signal, the second subplot – the extracted mains interference. On the third subplot is shown the calculated frequency of the main interference. One can observes that the absolute error does not exceed 0.05 Hz (relative error 0.1 %).



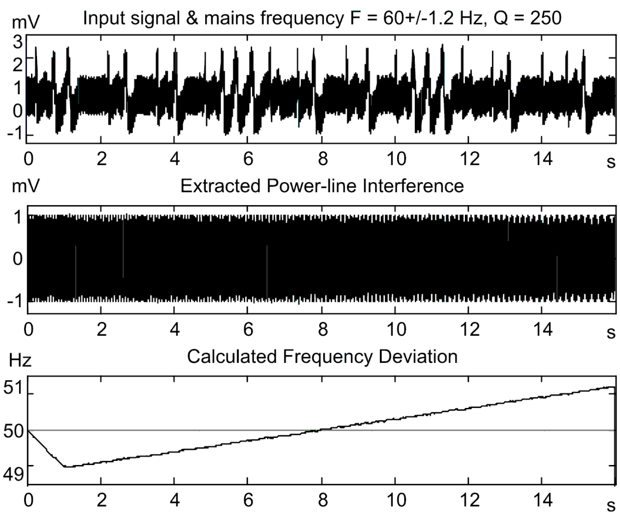
Test No 1: Signal ECGprobe, $Q = 250$ Hz, $F = 50\pm1$ Hz.



Test No 2: Signal AHA_7009, Q = 250 Hz, F = 60±1,2 Hz.



Test No 3: Signal ECGprobe, Q = 250 Hz, F = 50±1 Hz.



Test No 4: Signal AHA_7009, Q = 250 Hz, F = 60±1,2 Hz.

Fig. 6. Tests for mains interference frequency estimation.

VII. CONCLUSION

In the present work, a procedure for measuring the frequency of mains interference has been investigate. The procedure uses a frequency detection method based on the relationship between the measured frequency and the transfer coefficient of a digital averaging filter. For this purpose, a suitable "two point" filter has been synthesized. The inflection point in its frequency response coincides with the rated value of the mains frequency.

A suitable "three point" filter for extracting the mains interference from preliminary recorded signals has been synthesized. The filter completely suppresses the constant component in the input signal and has a unit transmission coefficient for the mains frequency.

The procedure for mains frequency measurement has been implemented as a function in MATLAB environment. Experiments with a stepwise and smooth changing in mains frequency indicate that the relative measurement error does not exceed 0.1%. The method can be easy applied at relatively low sampling rates.

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