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*Lubrication – non-Newtonian properties*

# INVESTIGATION OF THE EFFECT OF NON-ISOTHERMAL FLOW OF NON-NEWTONIAN FLUID IN A THIN LAYER AND THERMAL STATE OF THE TURBOCHARGER RADIAL BEARINGS ON THE ROTOR DYNAMICS

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## ABSTRACT

Most of the theoretical and experimental works on the study of friction units are based on the classical hydrodynamic theory of lubrication. In the present study, a system of equations for hydrodynamic problems that take into account the processes of heat exchange between a lubricant and a solid is given. To model non-Newtonian properties of modern lubricants, a rheological model of the lubricant was used. A series of comparative calculations for evaluating the performance of hydrodynamic units, taking into account their thermal loading, is performed on the example of calculating the dynamics of a flexible asymmetric rotor. The results of the calculations showed that the temperature difference between the rotor bearings was 15-18 degrees. The results of theoretical studies have shown good agreement with the results of experimental studies.

*Keywords:* non-Newtonian fluid, thermal state, bearing, rotor.

## AIMS AND BACKGROUND

The use of modern high-grade lubricants is the key factor for reducing friction losses in tribo-units and improving the power efficiency of an engine as a whole. The majority of hydrodynamic friction units of engines are heavy-loaded; i.e., they are loaded by forces, whose magnitude and direction vary in time.

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A problem of calculating these tribo-units is based on the classical hydrodynamic lubrication theory, which describes the behaviour of a Newtonian fluid in a thin lubricating film that separates the tribo-unit surfaces. However, state-of-the-art lubricant production methods, as well as the permanent improvement and development of new antiwear and other additives, lead to the impossibility of describing the behaviour of lubricating fluids based on the foundations of the lubrication theory. One of the assumptions of this theory as a basis for calculating heavy-loaded tribo-units is that the behaviour of a lubricant obeys the Newton-Stokes law, which implies the following linear dependence of tangential stresses on the shear rate[1]:

$$\tau = \mu \cdot \dot{\gamma}, \tag{1}$$

where $\tau$ is the shear stress; $\mu$ is the Newtonian viscosity; and $\dot{\gamma}$ is the shear rate.

In this dependence, the dynamic (Newtonian) viscosity $\mu$ serves as the coefficient of proportionality; this viscosity depends on the temperature and on the pressure. In this case, the fluid is called Newtonian or perfectly viscous (purely viscous).

The majority of up-to-date high-grade lubricating oils, whose behaviour does not obey condition (1), are non-Newtonian fluids. In the general case, by non-Newtonian behaviour is meant any anomalies observed during the flow of fluids. In this case, there is a need to develop calculation methods based on new, non-Newtonian models of the rheological behaviour of the lubricant.

There are various models of viscoelastic fluids[1], among which the Maxwell model is the best known. In this model, fluids are called Maxwell fluids (viscoelastic Maxwell fluids), and their rheological behaviour is described by the following equations:

$$\tau_{xy} + \lambda \frac{\partial \tau_{xy}}{\partial t} = \mu^* \frac{\partial V_x}{\partial y}; \quad \tau_{yz} + \lambda \frac{\partial \tau_{yz}}{\partial t} = \mu^* \frac{\partial V_z}{\partial y}. \tag{2}$$

where $\tau_{xy}$ and $\tau_{yz}$ are the stress tensor components; $\lambda$ is the relaxation time, which characterizes the lag of changes in the tangential stresses relative to changes in the shear rates; $\partial V_x / \partial y$ and $\partial V_z / \partial y$ are the gradients of the velocities of a lubricant unit volume; $y$ is the coordinate in the direction of the normal to the bearing surface; $t$ is the time; $\mu^*(\grave{O}, p, I_2)$ is the dynamic viscosity coefficient (non-Newtonian viscosity), which, in the general case, depends on the lubricating film temperature $T(x, y, z, t)$, on the pressure $p(\varphi, z)$ and on the second invariant of the shear velocities

$$I_2 \approx (\partial V_x / \partial y)^2 + (\partial V_z / \partial y)^2, \; \dot{\gamma} = \sqrt{I_2} \,. \tag{3}$$

The rheological model of a viscoelastic fluid was used in works of Paranjpe[2], Zhang et al.[3] Den and Elrod[4] constructed a general theory for pseudoplastic non-Newtonian fluids with constant characteristics, whose viscosity was a function of the second invariant of the deformation rate tensor and determined by the following law:

$$\mu^* = k^*\dot{\gamma}^{n-1} \tag{4}$$

where $k^*$ is the measure of the consistency of the fluid and $n$ is the exponent, which characterizes the degree of non-Newtonian behaviour. For pseudo-plastic fluids $n < 1$.

The use of the power law to describe the rheological behaviour of various motor oils requires values of $n$ and $k^*$ to be determined in each particular case, which, unfortunately, involves difficulties. However, Gecim has shown[5] that the use of the power law and its modifications can yield erroneous results, i.e., an underestimated viscosity at high shear rates ($10^6$ $s^{-1}$) or an overestimated viscosity at low shear rates ($10^3$ $s^{-1}$). At low shear rates, the value of the viscosity corresponds to the value of the first Newtonian viscosity $\mu_1$; with an increase in the shear rate, the viscosity tends to the value of the second Newtonian viscosity $\mu_2$. The dependence of the viscosity on the shear rate proposed by Gecim was successfully used in Refs 2, 6, 7. In these works, information on a method for determining the second Newtonian viscosity is lacking.

Based on experimental studies on various motor oils[8], a rheological model of the viscosity of oil was proposed, in which the viscosity was a power function of the temperature, pressure, and shear rate simultaneously. A method for determining the second Newtonian viscosity using up-to-date tribological equipment was also proposed.

Along with this, when studying the efficiency of tribo-units, thermal processes that occur in heavy-loaded fluid-lubricated bearings are of great importance. They are usually considered based on a solution of the generalized equation of energy (heat transfer) for a thin film of a viscous incompressible fluid, which separates two arbitrarily moving surfaces. This equation allows for both convective heat transfer by the lubricant and heat transfer due to thermal conductivity. In this case, the temperature distribution $T(x, y, z, t)$ in the lubricating film is described by the following equation[9–11]:

$$\rho c_0 \frac{\partial T}{\partial t} + \rho c_0 \left( V_x \frac{\partial T}{\partial x} + V_y \frac{\partial T}{\partial y} + V_z \frac{\partial T}{\partial z} \right) - \lambda_0 \frac{\partial^2 T}{\partial y^2} = D \tag{5}$$

where $\rho$ is the density of the lubricant; $c_0$ and $\lambda_0$ are the specific heat and the thermal conductivity of the lubricant, which are usually assumed to be constant; $V_x, V_y$ and $V_z$ are the components of the velocity vector of the unit lubricant volume located between the joint surfaces; and $D$ is the dissipative function determined by the following approximate expression:

$$D = \mu I_2. \tag{6}$$

The three following approaches to the integration of Eq. (5) can be used, depending on applied assumptions on the temperature distribution in a thin lubricating film: thermo-hydrodynamic (nonisothermal), adiabatic, and isothermal distribution.

In the adiabatic approach, it is assumed that no temperature changes across the lubricating film occur and that the journal and the bearing are perfect heat insulators. A calculation temperature $T = T(x, t)$ averaged over the bearing width is introduced; the substitution of this temperature in Eq. (5) yields the differential equation for the temperature distribution along the coordinate $x$. Since, in this approach, heat removal to the journal and the bearing is not allowed for, calculated temperatures are highly overestimated, which reduces the validity of the results.

In the isothermal approach, it is assumed that the calculation (equivalent) current temperature $T_{eq} = T_{eq}(t)$ is the same on all points of the lubricating films. This temperature is a fairly inertial parameter and determined, when solving the equation of heat balance, which reflects the equality of cycle-averaged values of the heat dissipated in the lubricating film and the heat removed by the lubricant flowing out through the bearing ends.

In the thermo-hydrodynamic approach, the temperature is assumed to change in all directions[10,12], including across the lubricating film. In this case, the boundary conditions are best fit for real thermal processes. This approach yields information on the local characteristics of the temperature field over the lubricating film, i.e., maximum and instantaneous average values of the temperature, as well as zones of increased thermal load.

Many researchers attempt to allow for the most realistic properties of lubricants, as well as a number of design, process, operational, and other parameters, which affect the operation of tribo-units. The significance and topicality of the thermo-hydrodynamic lubrication problem for heavy-loaded sliding bearings with allowance for the rheological behaviour of a lubricating fluid increase.

Additionally, researchers should take into account the mutual influence of bearings on each other. For example, in paper[13] authors presented an algorithm for solving the problem of rotor dynamics. The authors take into account the flexibility of the rotor.

However, experimental studies[14,15] had shown that the bearings of the turbine and compressor operate under different thermal conditions. The temperature difference for the turbine and compressor bearings can be 15–30°C. Studies on thermal aspects of bearings considered at different bearing temperatures are very limited.

Consequently, the evaluation of the thermal state of each bearing and accounting their thermal state in the calculation of the rotor dynamics is an urgent task. At the same time the position of the rotor relative to the bearing must be determined in each time point. The flexibility of the rotor must be taken into account. The lubrication regime in the thin layer must be taken into account too.

## METHOD OF SOLUTION. PROBLEM FORMULATION

The scope of problems in the theory of hydrodynamic tribo-units is characterized by the set of methods for solving the following interrelated problems.

(1) Solving equations of motion to determine the trajectory of the journal center in the bearing.

(2) Determining pressures in the lubricating film, which separates the friction surfaces having imperfect geometry with an arbitrary law of their motion with allowance for properties of the lubricant.

(3) Assessing the temperature state of the 'shaft - lubricating film - bearing' system with allowance for properties of the structural materials.

(4) Determining and optimizing geometrical and hydro-mechanical characteristics of the bearing.

The complex solution of these problems is a key stage in improving the reliability of tribo-units and developing friction units that meet current requirements. However, this solution involves substantial difficulties, since it requires precise and highly efficient calculation methods and algorithms to be developed.

The result of the modeling of heavy-loaded fluid lubricated bearings is usually assessed by the calculation of hydro-mechanical characteristics, determined during a bearing loading cycle. The minimal permissible lubricating film thickness and the maximum of permissible hydrodynamic pressure are commonly used as criteria for the efficiency of tribo-units[16–18].

A solution of the above-listed problems in isothermal formulation with allowance for determining the lubricating film thickness is presented in Refs 7, 8, 17. However, in many cases of calculation, there is a need to find a temperature distribution across the lubricating film, which is only possible in the thermo-hydrodynamic approach. This approach allows one to consider both the non-Newtonian behaviour of lubricating oil and the geometry of the friction surfaces, which restrict a thin lubricating film.

Along with this approach to determine the temperature distribution in the lubricant layer, the use of the finite volume method (FVM) is of great interest. The application of this method makes it possible to obtain more accurately the temperature values in each lubricating layer. In this case, the properties of construction and lubricants, as well as the boundary layer on the surfaces of friction mates are taken into account.

The main provisions of the FVM are conveniently stated by considering the 'standard' balance equation of a certain value φ in the control volume Ω, which is bounded by a surface with an external normal $\vec{n}$:

$$\int_{\Omega} \frac{\partial \rho \phi}{\partial t} d\Omega + \sum_{k} \int_{S_k} \vec{n} \cdot \vec{q} ds = \int_{\Omega} Q d\Omega, \quad \vec{q} = \rho \vec{V} \phi - \alpha \nabla \phi, \tag{7}$$

where $\vec{q}$ is the flux density vector of $\vec{\varphi}$, which includes the convective and diffusion components, $Q$ is the distribution density of bulk sources; $\vec{V}$ is the velocity vector; ρ is the density of the medium; α is the diffusion coefficient. The φ could include, for example, the internal energy of fluid, the additive concentration; the kinetic energy of turbulence, etc.

In the limit, with volume we compress to a point, based on the Ostrogradsky-Gauss formula, this equation can be rewritten in differential form: $\partial\rho\phi / \partial t + \nabla \vec{q} = Q$.

Calculated areas are divided into small control volumes, for each of which the balance ratio (7) is recorded. One nodal point is located in each center of control volume. When solving three-dimensional problems with a complex geometry of the regions, in most cases, the cells of the computational grid are used as a control volume.

There are two options for solving problems with the help of the FVM:

1. The boundaries of the control volume coincide with the boundaries of the element.

2. The faces of the control volumes pass through the centers of the faces of the elements, into which the region is divided. The required variables are stored in the vertices of the elements. A control volume is built around each vertex.

FVM has several advantages:

• the main quantities are stored throughout the region, for example, system energy, mass, heat flux, and so on. This condition is satisfied even for a coarse computational grid;

• the calculation speed is high. Many estimated values can be calculated by splitting a region into elements, and there is no need to calculate them on each time step;

• the method is easily used for problems with complex geometry and curvilinear boundaries. The ease is due to using different geometric types of elements - triangles, polygons.

## BASIC EQUATIONS

The methodology of the dynamics calculation of the flexible non-symmetric rotor on the multi-layer sliding bearings is based on methods of integrating the motion equations of movable elements of the bearings and rotor. The motion equations include forces that are associated with the presence in the system 'rotor – bearings' of lubricating layers having substantially nonlinear characteristics.

The nonlinear reactions of the lubricating layer are determined by integrating the hydrodynamic pressures diagram. These diagrams are calculated on the each time step by numerical integration of the differential equation of Reynolds.

The hydrodynamic pressure field in a thin lubricating film is usually determined using the universal Elrod equation for the degree of filling of the clearance with a lubricating fluid[19,20] or the generalized Reynolds equation[10,17]; for a non-Newtonian fluid, we write the generalized Reynolds equation as follows:

$$\frac{\partial}{\partial\varphi}\left[\bar{h}^{n+2}\left(\bar{\phi}_2-\frac{\bar{\phi}_1^2}{\bar{\phi}_0}\right)\bar{\rho}\frac{\partial\bar{p}}{\partial\varphi}\right]+\frac{\partial}{\partial\bar{z}}\left[\bar{h}^{n+2}\left(\bar{\phi}_2-\frac{\bar{\phi}_1^2}{\bar{\phi}_0}\right)\bar{\rho}\frac{\partial\bar{p}}{\partial\bar{z}}\right]=\frac{\partial}{\partial\varphi}\left[\bar{\omega}_{21}\bar{\rho}\bar{h}\left(1-\frac{\bar{\phi}_1}{\bar{\phi}_0}\right)\right]+\frac{\partial}{\partial\bar{t}}\left(\bar{\rho}\bar{h}\right) \quad (8)$$

where φ and $z$ are the angular and axial coordinates; $\bar{\rho}=\rho/\rho_0$ is the dimensionless density; $\rho_0$ is the density of the Newtonian fluid; $\bar{p}=(p-p_a)\psi^2/\mu_0\omega_0$, $\psi=h_0/r_2$, $\bar{z}=z/r_2$, $-a\le\bar{z}\le a$, $\bar{t}=\omega_0 t$ are the dimensionless hydrodynamic pressures, relative radial clearance, coordinate along the bearing width, and time; $h_0$ is the radial installation clearance; $a=B/D$ is the relative bearing width; $\mu_0$ is the characteristic viscosity of the lubricant; $pa$ is the atmospheric pressure; $B$, $D=2r_2$, $r_2$ respectively, are the width, diameter, and radius of the journal; $\bar{\omega}_{21}=(\omega_2-\omega_1)/\omega_0$ is the dimensionless angular velocity of the journal; $\bar{h}$ is the dimensionless lubricating film thickness. $\bar{h}$ and its derivatives $\partial\bar{h}/\partial\bar{t}$ are determined as follows:

$$\bar{h}=1-\chi\cos(\varphi-\delta),\quad \partial\bar{h}/\partial\bar{t}=-\dot{\chi}\cos(\varphi-\delta)-\chi\dot{\delta}\cos(\varphi-\delta), \quad (9)$$

where χ is the relative eccentricity and δ is the phase angle of the line of centers); $\bar{\phi}_k=\int_0^{\bar{h}}\bar{y}^k/\bar{\mu}^*\,d\bar{y}$, $k=0;1;2$, $\bar{\mu}^*$ is the dimensionless non-Newtonian viscosity of the lubricant, which depends on the shear rate, on the temperature, and on the pressure; $\bar{y}$ is the dimensionless coordinate across the lubricating film.

Equation (8) was integrated using the multigrid method under the Swift–Stieber boundary conditions with account for the presence of lubrication sources (holes and grooves) on the friction surfaces[21]:

$$\bar{p}(\varphi,\bar{z}=\pm a)=0;\ \bar{p}(\varphi,\bar{z})=\bar{p}(\varphi+2\pi,\bar{z});\ \bar{p}(\varphi,\bar{z})\ge 0$$

$$\text{on } (\varphi,\bar{z})\in\Omega_S\ \ \bar{p}(\varphi,\bar{z})=\bar{p}_S,\ \ S=1,2...S^*,$$

where $\Omega_S$ is the lubrication source area, in which the pressure is constant and equal to the feed pressure $\bar{p}_S$, and $S^*$ is the number of lubrication sources.

The dependence of the viscosity of the lubricant on the shear rate and on the pressure was represented as follows[8]:

$$\bar{\mu}^*=\begin{cases}\bar{\mu}_1\cdot\exp(\beta\cdot\bar{p}), & I_2<10^4, s^{-1};\\ \bar{\mu}\cdot I_2^{(n-1)/2}\cdot\exp(\beta\cdot\bar{p}), & 10^4<I_2<10^6, s^{-1};\\ \bar{\mu}_2\cdot\exp(\beta\cdot\bar{p}), & I_2>10^6, s^{-1}.\end{cases} \quad (10)$$

where $n$ is the parameter that characterizes the degree of non-Newtonian behaviour and β is the pressure coefficient of the viscosity of the lubricant, which depends on the temperature.

In accordance with model (10), over portion 1 ($I_2 < 10^4\, s^{-1}$), oil behaves as a Newtonian fluid with the viscosity $\mu_1(T_{eq}, p)$. Over portion 2 ($10^4 s^{-1} < I_2 < 10^6 s^{-2}$), the viscosity decreases following the power law. Over portion 3 ($I_2 > 10^6 s^{-1}$), oil is considered as a Newtonian fluid with the viscosity $\mu_1(T_{eq}, p)$.

The volume flow rates of the lubricant through the sections with the unit length along the coordinates $x$ and $z$ are written in the following form:

$$\overline{q}_{xy} = \int_0^{\overline{y}} \overline{V}_x d\overline{y}\,;\; \overline{q}_{zy} = \int_0^{\overline{y}} \overline{V}_z d\overline{y}\,.$$

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Equation (5) was written in the following dimensionless form[10,12]:

$$\frac{\partial \overline{T}}{\partial \overline{t}} = -\overline{V}_x \frac{\partial \overline{T}}{\partial \varphi} - \overline{D}\frac{\partial \overline{T}}{\partial \overline{y}} - \overline{V}_z\left(\frac{1}{a}\right)\frac{\partial \overline{T}}{\partial \overline{z}} + \frac{1}{Pe}\cdot\frac{1}{\overline{h}^2}\cdot\frac{\partial^2 \overline{T}}{\partial \overline{y}^2} + \frac{1}{k_T}\cdot\frac{1}{\overline{h}^{n+1}}\overline{D}_e. \tag{11}$$

where $\overline{T} = T/T_0$ is the dimensionless temperature in a point of the lubricating film; $T_0$ is the characteristic temperature;

$$\overline{D} = -\frac{\overline{y}}{\overline{h}}\frac{\partial \overline{h}}{\partial \overline{t}} - \overline{V}_x \frac{\overline{y}}{\overline{h}}\frac{\partial \overline{h}}{\partial \varphi} + \frac{1}{\psi \overline{h}}\overline{V}_y\,;\; \overline{D}_e = \overline{\mu}\left|\frac{\partial \overline{V}_x}{\partial \overline{y}}\right|^{n-1}\left[\left(\frac{\partial \overline{V}_x}{\partial \overline{y}}\right)^2 + \left(\frac{\partial \overline{V}_z}{\partial \overline{y}}\right)^2\right];$$

$Pe = \rho_0\overline{\rho}c_0\omega_0\Delta_0^2/\lambda_0$ is the Peclet number; $k_T = \rho_0\overline{\rho}c_0T_0\psi^2/(\omega_0\mu_0)$, $\Delta_0 = 2\cdot h_0$.

To simplify the solution of the problem, we considered the two-dimensional equation of energy. In order to substantiate a possibility of this simplification, heavy-loaded bearings with various ways of lubricant supply from the sources (grooves and holes) located on the $z = 0$ axis were considered. The experimental results have shown that, with these ways of lubricant supply, changes in the temperatures on the surfaces of the bushing and the journal, which contact the lubricating film, along the bearing width are not so significant as their changes along the coordinate φ (Ref. 22).

It is assumed that the thermal state of the lubricating film is fairly completely characterized by the temperature distribution $T(\varphi, y, t) = T(\varphi, z_j, y, t)$ in a section $z_j$. With allowance for the above-made assumption on the constancy of the temperature along the $z$ axis, Eq. (11) for $T$ is written as follows:

$$\frac{\partial \overline{T}}{\partial \overline{t}} = -\overline{V}_x \frac{\partial \overline{T}}{\partial \varphi} - \overline{D}\frac{\partial \overline{T}}{\partial \overline{y}} + \frac{1}{Pe}\cdot\frac{1}{\overline{h}^2}\cdot\frac{\partial^2 \overline{T}}{\partial \overline{y}^2} + \frac{1}{k_T}\cdot\frac{1}{\overline{h}^{n+1}}\overline{D}_e \tag{12}$$

On each step of calculating the trajectory of the journal center, when the Reynolds (Elrod) equation and the equation of energy are integrated, we calculated the heat, which flows from the lubricating film per unit of surface of the bushing $Q_1$ ($y$ = 0) and the journal $Q_2$ ($y = h$), i.e., the unit of heat flows in the section $z_j$:

$$Q_1\,(x, z_j, t) = -\lambda_0 \left( \frac{\partial T}{\partial y} \right)_{y=0} ; \quad Q_2(x, z_j, t) = -\lambda_0 \left( \frac{\partial T}{\partial y} \right)_{y=h} . \tag{13}$$

Upon completion of the loading cycle, the unit of heat flows from the lubricating film to the bushing and journal averaged over the cycle duration $t_c$ were calculated with allowance for (13):

$$Q_1^*(x, z_j) = \frac{1}{t_c} \int_0^{t_c} Q_1\,(x, z_j, t)\, dt\, ; \; Q_2^*(x, z_j) = \frac{1}{t_c} \int_0^{t_c} Q_2(x, z_j, t)\, dt\, .$$

The temperature distribution $T_1$ (φ, $R$, $t$) in the bushing, where $R$ is the radial coordinate, is determined by solving the equation for the transient heat flow; in cylindrical coordinates and the dimensionless variables, this equation is written as follows:

$$\frac{\partial \bar{T}_1}{\partial \bar{t}} = \bar{\alpha}_1 \left( \frac{\partial^2 \bar{T}_1}{\partial \bar{R}^2} + \frac{1}{\bar{R}} \frac{\partial \bar{T}_1}{\partial \bar{R}} + \frac{1}{\bar{R}^2} \frac{\partial^2 \bar{T}_1}{\partial \varphi^2} \right), \tag{14}$$

where $\bar{R} = R/r_2$; $\bar{T}_1 = T_1/T_0$; $\bar{\alpha}_1 = \lambda_1/(c_1\rho_1 r_2\omega_0)$ is the dimensionless coefficient of heat transfer from the bushing surface to the environment; and $\rho_1$, $c_1$, $\lambda_1$ are the density, specific heat, and specific thermal conductivity of the bushing material.

To calculate the temperatures in the bushing, we introduce the calculation system of coordinates $Ox_1y_1z_1$ and the dimensionless variables $\bar{y}_1 = (R - r_1)(r_3 - r_1) = (\bar{R} - 1)$, $\bar{r}_3 = r_3/r_1$, where $r_1$ and $r_3$ are the calculation radii of the inner and outer surfaces of the bushing.

With allowance for the transition of the dimensionless variables, Eq. (14) is written in the following form:

$$\frac{\partial \bar{T}_1}{\partial \bar{t}} = \bar{\alpha}_1 \left( \frac{1}{(\bar{r}_3 - 1)^2} \frac{\partial^2 \bar{T}_1}{\partial \bar{y}_1^2} + \frac{1}{[1 + (\bar{r}_3 - 1)\bar{y}_1](\bar{r}_3 - 1)} \frac{\partial \bar{T}_1}{\partial \bar{y}_1} + \frac{1}{[1 + (\bar{r}_3 - 1)\bar{y}_1]^2} \frac{\partial^2 \bar{T}_1}{\partial \varphi^2} \right). \tag{15}$$

The scheme proposed in Ref. 2 seems promising for calculating the journal temperature; in this scheme, the journal is considered as a thermal element with a uniform temperature field. In this case, the calculation equation for the dimensionless journal temperature $\bar{T}_2$ is written as follows:

$$\frac{d\bar{T}_2}{d\bar{t}} = \sum \bar{Q}_2^* - \bar{\alpha}_2(\bar{T}_2 - \bar{T}_c), \tag{16}$$

where $\bar{T}_2 = T_2/T_0$; $\bar{T}_C = T_C/T_0$; $\bar{Q}_2^* = Q_2^*/(m_2c_2\omega_0)$; $\bar{\alpha}_2 = \alpha_2 S_2/(m_2c_2\omega_0)$, $m_2$, $c_2$ are the mass and specific heat of the journal; $\alpha_2$ is the average coefficient of heat transfer from the journal to the environment; $S_2$ is the area of the heat exchange surface of the journal; $T_C$ is the ambient temperature; and $\sum Q_2^*$ is the total loading-cycle-averaged heat flow from the lubricating film to the journal.

Because of substantial thermal inertia, the journal and bushing respond only slightly to any thermal changes in the lubricating film during the loading cycle $t_C$, which is equivalent, e.g., for internal combustion engines, to two revolutions of a crankshaft.

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Under the periodically applied loads, the solution of the entire problem was continued until the moment, when the calculation coordinates of the journal center $\overline{U}(t) = (\overline{x}_2, \overline{y}_2, \overline{z}_2)$; the pressures, and the temperature fields in two next loading cycles (periods) coincide, i.e., the following conditions that substitute the initial conditions are satisfied:

$$\overline{U}(t) = \overline{U}(\bar{t} + \bar{t}_c); \quad \overline{p}(\varphi, \overline{z}, \bar{t}) = \overline{p}(\varphi, \overline{z}, \bar{t} + \bar{t}_c);$$
$$\overline{T}(\varphi, \overline{y}, \bar{t}) = \overline{T}(\varphi, \overline{y}, \bar{t} + \bar{t}_c); \; \overline{T}_1(\varphi, \overline{R}, \bar{t}) = \overline{T}_1(\varphi, \overline{R}, \bar{t} + \bar{t}_c); \; \overline{T}_2(\bar{t}) = \overline{T}_2(\bar{t} + \bar{t}_c). \tag{17}$$

The boundary conditions, under which Eqs. (12), (15), and (16) of the heat subproblem were integrated, are formulated as follows. For the temperatures of the lubricating film and the bushing, the conditions of periodicity in the circumferential direction are true:

$$\overline{T}(\varphi, \overline{y}, \bar{t}) = \overline{T}(\varphi + 2\pi, \overline{y}, \bar{t}); \quad \overline{T}1(\varphi, \overline{R}, \bar{t}) = \overline{T}1(\varphi + 2\pi, \overline{R}, \bar{t})$$

On the outer bushing surface, the following free convection hypothesis is assumed to be true:

$$\left.\frac{\partial \overline{T}_1}{\partial \overline{R}}\right|_{\overline{R}=\overline{r}_3} = \frac{\alpha_1}{\lambda_1}\left(\left.\overline{T}_1\right|_{\overline{R}=\overline{r}_3} - \overline{T}_c\right).$$

On the surface, which is common for the lubricating film and the bushing, the following conditions of continuity of the heat flow (conjugation conditions) are as

$$\left.\frac{\partial \overline{T}_1}{\partial \overline{y}_1}\right|_{\overline{y}_1=0} = -\left(\overline{r}_3 - 1\right)\frac{\lambda_0}{\lambda_1 \overline{h}\psi}\left.\frac{\partial \overline{T}}{\partial \overline{y}}\right|_{\overline{y}=0}.$$

On the surfaces of the lubricating films that are common for the bushing and for journal surfaces, the following temperature equality conditions are assigned:

$$\overline{T}(\varphi, \overline{y} = 0, \bar{t}) = \overline{T}_1(\varphi, \overline{y}_1 = 0, \bar{t} - \bar{t}_c); \; \overline{T}(\varphi, \overline{y} = 1, \bar{t}) = \overline{T}_2(\bar{t} - \bar{t}_c).$$

The general scheme for solving the problem is represented in Refs 23, 24.

## RESULTS AND DISCUSSION

During the first stage of calculation for each bearing lubricant layer, the calculation of temperature fields was performed. Mesh creation in ANSYS meshing was performed for the fluent solver. At the same time, the sweep method was used to create the mesh of the model, which allows building a computational mesh based on prismatic elements using the operation of pulling elements of one layer along a certain axis. This method can be used for a class of geometric models, obtained as a body of rotation / pulling. In this case, the choice of the source surface and the target surface was made manually. This is because the source surface and the target surface have common nodes or edges.

The prismatic layers were built automatically using the sweep bias type options, which determine the direction of thickening, and the sweep bias option, which determines the degree of thickening of the elements. Also, the Inflation tool was used to create a computational grid with thickening layers of prismatic cells near the surfaces of a geometric model. Smooth transition method was used in this tool. Smooth transition uses the size of the grid non-prismatic element in the model to calculate the thickness of the last prismatic layer and the total thickness of all the prismatic layers.

The main variable parameters of this method are as follows:

• Transition ratio – shows how many times the thickness of elements in the last prismatic layer is less than the characteristic cell sizes of the next, non-prismatic layer. In our case, the value is $5\times10^{-2}$.

• Maximum layers – sets the maximal number of prismatic layers created. The number of elements in the boundary layer was assumed to be 20.

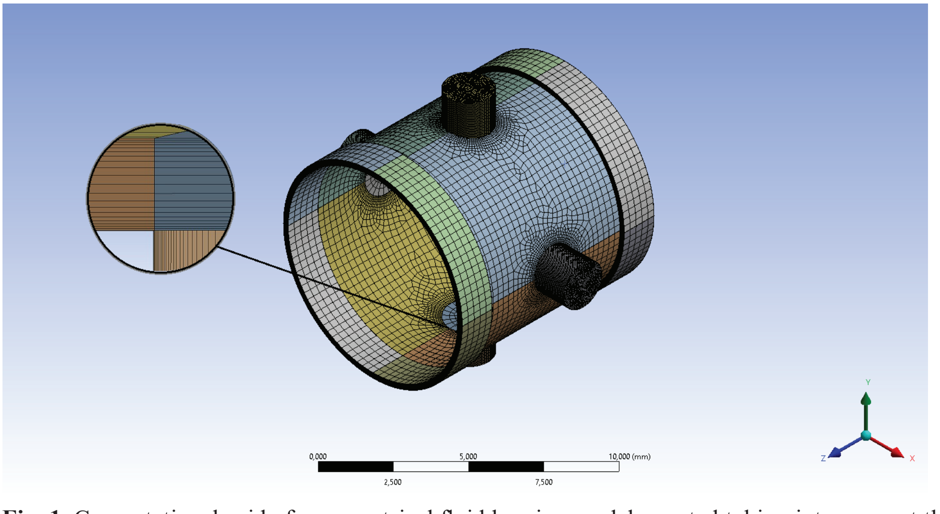



**Fig. 1**. Computational grid of a geometrical fluid bearing model, created taking into account the internal prismatic layers

**Fig. 2**. Estimated 3D model of Fluent

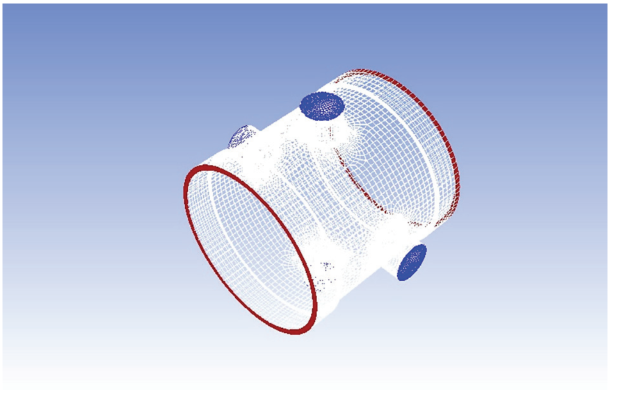


**Table 1**. The boundary conditions

| | Gauge total pressure, MPa | Initial gauge pressure, MPa | Total temperature, C° |
|---|---|---|---|
| Inlet | 0.5 | 0.45 | 90 |
| Outlet | 0.1 | – | 50 |

• Growth rate – determines the change in the thickness of the next prismatic layer with respect to the thickness of the previous one. In our case, the value is 1.2. Four input lubrication sources were provided, when building the model.

An example of a breakdown is represented in Fig. 1.

To calculate the thermal fields in the ANSYS mathematical model, the fluent module was used. In the model, the energy equation was connected, necessary to describe the variable temperature field. The viscosity model was chosen turbulent, with the *k*-epsilon realizable turbulence model. Figure 2 shows the calculated 3D model, in which the inputs are highlighted in blue and the outputs in red. The calculations were carried out under the boundary conditions indicated in Table 1.

An example of calculating the temperature for one of the lubricant layers is shown in Fig. 3.

The obtained temperature values were used in the further calculation of the dynamics of the flexible rotor as boundary conditions.

The rotor was represented as five masses interconnected by flexible massless rods. The scheme of the separation of the rotor into parts is shown in Fig. 4. The system of equations of motion of the rotor elements are integrated by the Runge-Kutta method. Variable step integration over time is automatically selected by using Merson's amendment. The position of the rotor in the bearing space was considered as a superposition of two displacements. The first one of these is the displacement of the undeformed rotor as a rigid whole. The second one is the bending of the rotor (Fig. 5).

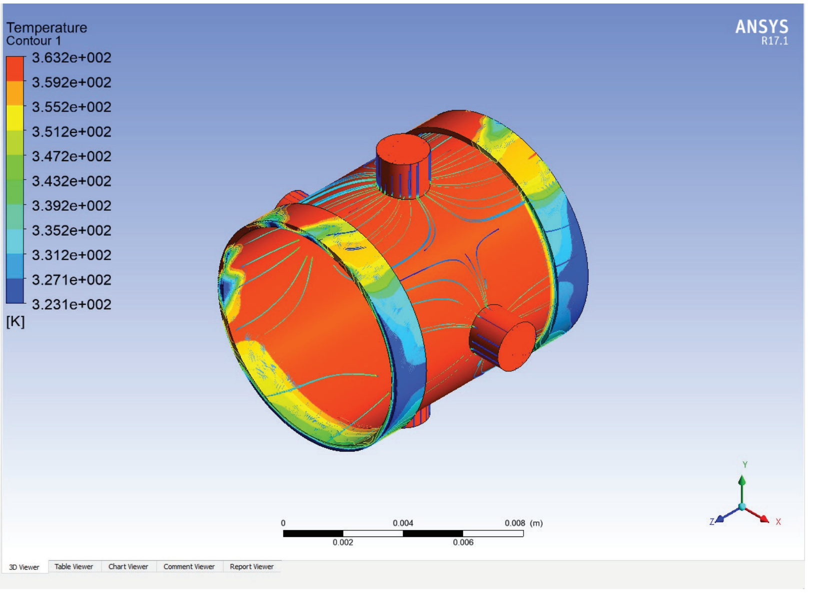



**Fig. 3**. The result of the calculation of thermal fields for a fluid bearing model

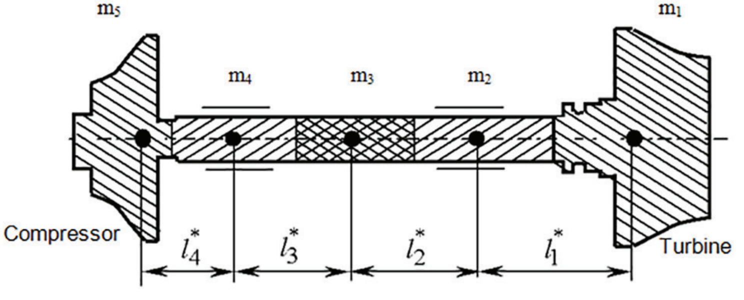



**Fig. 4**. The scheme of the separation of the rotor into parts

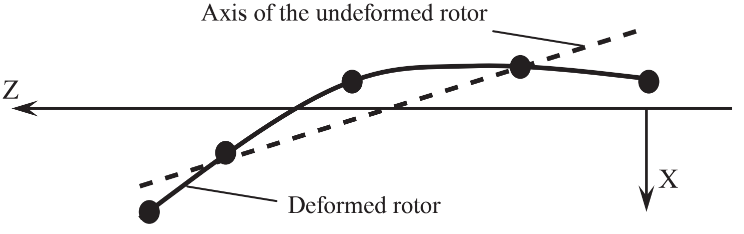



**Fig. 5**. The deformed rotor in plane *OXZ*

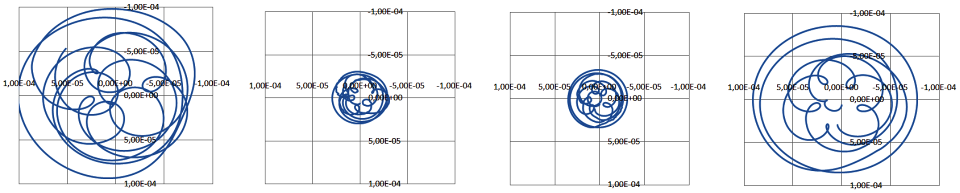
The investigations were carried out using the example of calculating the dynamics of the flexible asymmetrical rotor of the turbocharger TKR-10 (the radius of the journal of each bearing is 5.98×10$^{-3}$ m; the internal radial clearance is 22×10$^{-6}$ m; the width of the inner lubricating layer is 12.5×10$^{-3}$ m; the external radius of the ring is 9.6×10$^{-3}$ m; the external clearance is 35.75×10$^{-6}$ m; the width of the external lubricating layer is 12.5×10$^{-3}$; the weight of the each ring is 0.018 kg; the rotor mass in the turbine bearing $m_2$ is 0.031 kg; the rotor mass in the compressor bearing $m_4$ is 0.045 kg; the rotor mass is 1.276 kg). Rotor speed was varied from 500 to 13000 rad./s.

The calculating results of the temperature in the turbine bearing $T_t$ and the compressor bearing $T_C$ are shown in the Table 2.

The trajectories of the moving parts of the rotor were calculated for each speed value. For example, in Fig. 6 the trajectory, corresponding speeds of 9000 rad./s are shown. At the same time, the position of the rotor at each time was determined. Additionally, the position of the rotor at different times was determined. Elastic rotor lines corresponding to the speed of 9000 rad./s and different moment of time ($t_i, i = 0...n$) are shown in Fig. 7. The change in the vibration amplitude of the moving parts $A_j$ of the rotor as a function of its rotational speed is shown in Table 3.

**Table 2**. The temperature in the bearings

| Rotor speed, rad./s | Temperature in the turbine bearing, °C | Temperature in the compressor bearing, °C |
|---|---|---|
| 500 | 101,5 | 91,2 |
| 1000 | 104,4 | 93,3 |
| 3000 | 121,4 | 101,9 |
| 5000 | 130,4 | 117,9 |
| 7000 | 131,4 | 119,2 |
| 9000 | 145,6 | 121,7 |
| 11000 | 140,1 | 120,9 |
| 13000 | 142,7 | 124,0 |




*a*) *b*) *c*) *d*)

**Fig. 6**. Trajectories of rotor elements ($\omega_1$ = 9000 rad./s): *a*) compressor center of mass; *b*) journal center of mass in compressor bearing; *c*) journal center of mass in turbine bearing; *d*) turbine center of mass

**Table 3**. Maximal amplitude of the moving parts of the rotor

| | Rotor speed, rad./s | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $A_j$, μm | 500 | 1000 | 3000 | 5000 | 7000 | 9000 | 11000 | 13000 |
| $A_1$ | 3.85 | 2.08 | 45.2 | 54.5 | 68.9 | 84.3 | 105 | 129 |
| $A_2$ | 0.32 | 1.76 | 17.4 | 23.5 | 31.4 | 33.1 | 38.1 | 42.1 |
| $A_4$ | 2.34 | 1.68 | 19.3 | 20.1 | 27.9 | 29.8 | 33.2 | 38.9 |
| $A_5$ | 9.3 | 1.94 | 50.1 | 55.8 | 80.1 | 91.4 | 127 | 172 |

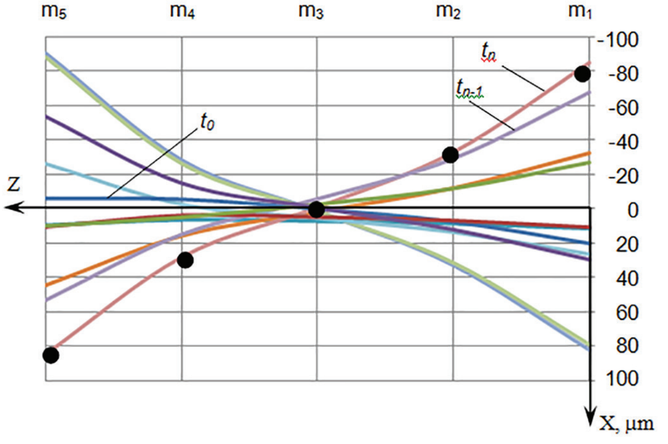



**Fig. 7.** Elastic rotor lines ($\omega_1$ = 9000 rad./s). The position of the rotor axis at different times

The results of the calculation were compared with similar results obtained with an isothermal approach.

## CONCLUSIONS

The difference in temperature values of the turbine bearings and the compressor one ranged from 2 to 12 degrees at different speeds.

With an increase in the speed of the rotor rotation from 3000 to 13000 rad./s, the amplitudes of the oscillations of its elements increased by 2.5 times. The shape of the oscillations varies from conic to cylindrical. The use of the non-isothermal approach to solve equation of energy allows us to clarify the thermal state of each lubricating layer. In addition, it makes possible to estimate the thickness of the lubricating layer in the bearing. An increase in the amplitude by 2-2,5 times leads to a sharp decrease of the lubricant layer thickness. In this case, the lubrication mode in the bearing goes from liquid to mixed mode.

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