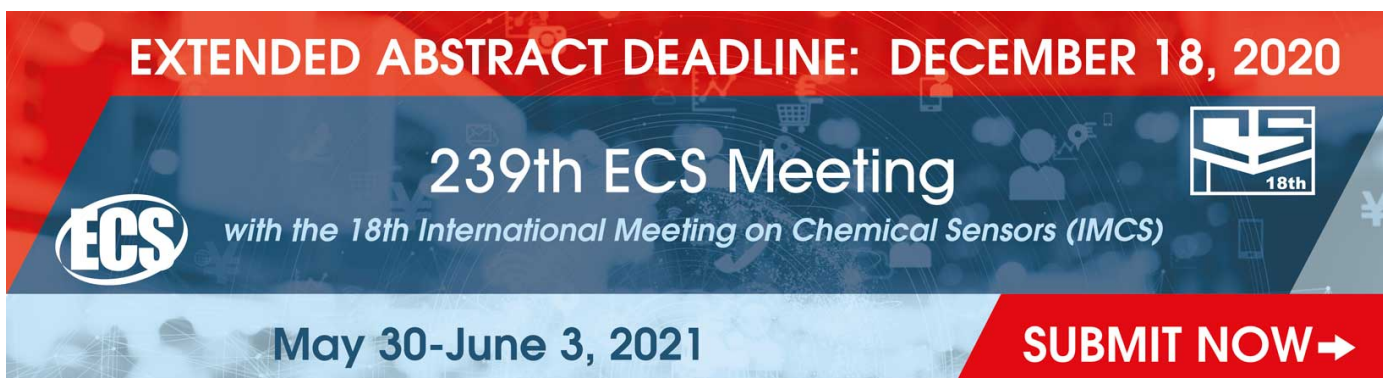**PAPER • OPEN ACCESS**

Determining the values of the Coefficient of Restitution in the meanwhile of a crash between two vehicles

To cite this article: Milena Savova-Mratsenkova and Georgi Djonev 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **618** 012058

View the article online for updates and enhancements.

Determining the values of the Coefficient of Restitution in the meanwhile of a crash between two vehicles

Milena Savova-Mratsenkova, Georgi Djonev

Department of Engines, Automotive Machinery and Transport, Technical University of Sofia, Bulgaria

 savova@tu-sofia.bg

Abstract. The current scientific research aims to present two of the methods, which are used worldwide by the automobile expert practice when investigating the velocity of motor vehicles. These methods are based on theorems from mechanics. Basic quantitative characteristic of energy loss during a two-body impact is the introduced in the impact theory – coefficient of restitution (COR). In reference to automobiles, the values of the coefficient depend on the type of the elements that have crashed as well as on the relative velocity in the meanwhile of the crash. The article presents an approach for determining the values of the above mentioned coefficient when investigating frontal or side collisions of vehicles in a movable barrier, based on multiple regression analysis. The conducted research uses real automobile expertise data from the case law.

1. Introduction

The intensive development of road transport in the past years not only helps to satisfy the constantly increasing need of the society for the use of motor vehicles, but also causes a number of negative consequences as a result of the occurrence of car accidents with either injured or killed casualties. In addition to this, the problems, which are connected with traffic safety and the ways of their solving are subjects of thorough studies and analysis. A fundamental part of them is the process of restoring the whole mechanism of the traffic accident. That process is based on research, analysis and drawing conclusions on the grounds of which the causes and the events that led to the occurrence of the problems in the first place are defined. Furthermore, these actions support the objective description of the facts and the mechanism of occurrence of accidents. Moreover, they are key elements in determining the guiltiness of the participants and in a larger scale they help the process of taking appropriate preventive measures for improving the traffic safety.

A huge part of the conducted up to this moment worldwide researches in the area are based on statistical processing of data about real road traffic accidents [7, 8, 9, 10, 14, 15, 16, 17]. It is unequivocal, however, that the collisions with the most severe consequences of all are the ones that occur between two vehicles. For the study and analysis of vehicle-to-vehicle impacts, knowledge in the field of mechanics and physico-mathematical modeling is required [1, 2, 3, 4, 5, 8, 10, 12]. The main difficulty in the expert study of the above mentioned collisions is determining the redistribution of the participating vehicles' kinetic energy before the impact into energy of deformation during the impact and residual kinetic energy of the vehicles after the impact [2, 3, 8, 9, 10, 11].

The mechanical interaction between the motor vehicles in the meantime of a collision is extremely complex and due to that it is investigated either in laboratory conditions or by using computer simulation modeling [7, 8, 13].

It is to be noted that the automobile is an intricate, mechanical, nonhomogeneous system of elements, each of which has different sturdiness and elasticity. Therefore, the complex structure of the motor vehicles nowadays is what hinders the physico-mathematical modeling of road traffic accidents [1, 4, 5, 8].

As a main characteristic of the energy loss during a vehicle-to-vehicle collision in most of the conducted researches in the particular scientific field is considered the introduced in the Theory of impact – coefficient of restitution, which mainly depends on the relative velocities of the vehicles before the impact, as well as on the type of the elements that have collided [9, 11, 12].

When analysing traffic safety and investigating the causes and severity of the traffic accidents a regression analysis may be used, with the help of which the complex interconnections between the studied parametres can be described [6, 13, 14, 15, 16, 16].

2. Exhibition

When conducting an expert analysis of a traffic accident by using the provided pre-trial data, which indicates the location of the impact and the location of the vehicles after the accident, the mechanism of the investigated incident is restored by determining the angles of motion of the vehicles and the values after the impact. During the calculation of the speed values at the moment of the impact different methods can be used. Among those methods that are most frequently used are:

2.1. „Momentum 360"

The velocities of the vehicles $\vec{V}_i$ (before the impact) and $\vec{U}_i$ (after the impact) are connected with a relation, based on the Law of storing the amount of traffic $\vec{Q}$, which is expressed by the following vector equation (1):

$$\vec{Q} = m_1 . \vec{V}_1 + m_2 . \vec{V}_2 = m_1 . \vec{U}_1 + m_2 . \vec{U}_2 \quad (1)$$

where m_1 and m_2 are the vehicles' masses; $\vec{V}_1$ and $\vec{V}_2$ - the velocities of vehicles' mass centres before the impact; $\vec{U}_1$ and $\vec{U}_2$ - the velocities of vehicles' mass centres after the impact.
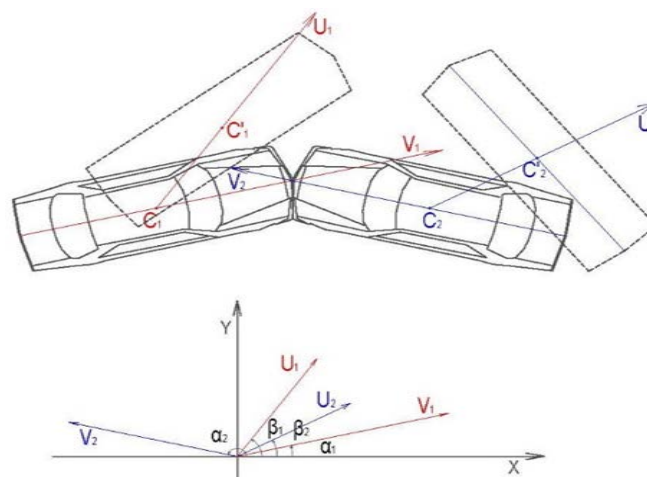

Figure 1: Angles between the vectors of the velocities before (V) and after the impact (U) and the X axis

If we design vector equation (1) on the coordinate axes of an arbitrarily chosen coordinate system, an algebraic system, composed of two equations, (2) is obtained. After we solve the algebraic system we get the values of the vehicles' velocities before the impact:

$$m_1 V_1 cos\alpha_1 + m_2 V_2 cos\alpha_2 = m_1 U_1 cos\beta_1 + m_2 U_2 cos\beta_2$$

$$m_1 V_1 sin\alpha_1 + m_2 V_2 sin\alpha_2 = m_1 U_1 sin\beta_1 + m_2 U_2 sin\beta_2 \quad (2)$$

where α_1 and α_2 are the angles of the vehicles' direction of movement before the impact, while β_1 and β_2 – the angles of the vehicles' direction of movement after the impact.

The values of the angles of vehicles' direction of movement before and after the impact shall be determined in accordance with the direction of the X axis of the selected coordinate system, which is directed along the axis of the traffic lane.

2.2. *Theory of impact* – when this method is applied a coefficient of restitution k [1, 3] is used. The coefficient of restitution is a quantitative characteristic of the loss of energy in the meantime of an impact between two vehicles. When an expert analysis of a two-vehicle traffic accident is conducted, the coefficient of restitution is presented as a ratio of the relative velocity's projection between the contact points after the impact on the axis of the directrix of the impact impulse.

When the coordinate system is rotated at an angle α_S (the angle between the vector of the impact impulse and the X axis) as a normal axis (n) is assumed the axis of the impact impulse, while the tangential axis (τ) is its perpendicular. Therefore, the formulas for determining the coefficient of restitution k and the coefficient of momentary friction λ, which are defined by the projections of the velocities on the axes of the new coordinate system, are (3):

$$k = \pm \frac{U_{n2} - U_{n1}}{V_{n1} - V_{n2}} \; ; \qquad 1 - \lambda = \frac{U_{\tau2} - U_{\tau1}}{V_{\tau1} - V_{\tau2}} \approx 1. \quad (3)$$

In a case of a *perfectly elastic collision k =1*, the whole kinetic energy of the bodies before the impact is further transformed into kinetic energy after the impact, hence there is no loss of kinetic energy.

However, in a case of a *perfectly inelastic collision k=0* the whole kinetic energy before the impact is further transformed into energy of deformation, thermal and acoustic energy.

For every distinct material object, the coefficient of restitution alters in the range: $0 < k < 1$.

When taking into account the axis n of the coordinate system, the scalar equation is as follows (4):

$$[(-\cos(\alpha_1 - \alpha_s) - sin(\alpha_1 - \alpha_s))V_1 + (\cos(\alpha_2 - \alpha_s) + sin(\alpha_2 - \alpha_s))V_2](\pm k) =$$

$$= [(-\cos(\beta_2 - \alpha_s) - \sin((\beta_2 - \alpha_s))U_2 + (\cos(\beta_1 - \alpha_s) + \sin(\beta_1 - \alpha_s))U_1]. \quad (4)$$

The value of the coefficient of restitution (k) can be estimated from the last equation, if the values of all other elements are known and if the restrictive conditions are observed (5):

$$S = m_1 . \Delta V_1 = m_2 . \Delta V_2 \, , \; N.s; \quad 1 - \lambda \approx 1. \quad (5)$$

The change of the velocity of the mass centre in each of the vehicles for the time duration of the crash is represented by the vector $\Delta \vec{V}_i$, which has the following form (6):

$$\Delta \vec{V}_i = \vec{U}_i - \vec{V}_i \quad (6)$$

The coefficient of restitution depends basically on the type of the impact, the construction of the colliding elements from the vehicles and the relative velocities of the vehicles themselves before the impact.

Modern automobiles are complex mechanical systems with various shape and different internal structure, which is defined by numerous details with various sturdiness, as well as plastic and elastic qualities, dictated by passive safety requirements. For this reason, the reliable data about the values of today vehicles' coefficient of restitution are very limited and are mainly from studies, conducted by separate car manufacturers.

The authors of the current scientific paper have set themselves the goal of developing an approach for determining the values of the coefficient of restitution in the meantime of frontal or side collisions between vehicles. This new approach is based on multiple regression analysis.

The present research uses data from 14 real judicial automotive expert reports, describing actual traffic accidents with either frontal or side collisions between the vehicles. In table 1 and 2 are shown the values of the automobile parametres, which have been used for the development of the currently presented by the authors regression model.

Table 1.

№	U_1 m/s	U_2 m/s	m_1 kg	m_2 kg	$m_1.U_1$ kg.m/s	$m_2.U_2$ kg.m/s	β_1^0	β_2^0	α_1^0	α_2^0
1	9,3	9,91	1250	1420	11625,0	14072,2	339	186	0	188
2	13,88	13,14	2010	1530	27898,8	20104,2	353	11	355	40
3	6,79	4,36	1350	1650	9166,5	7194,0	345	2	0	213
4	1,72	3,64	1150	1275	1978,0	4641,0	250	235	0	200
5	10,65	24,93	1360	1600	14484,0	39888,0	287	151	4	176
6	9,01	8,79	2000	1250	18020,0	10987,5	214	355	210	350
7	17,96	9,82	1160	1300	20833,6	12766,0	1	153	10	180
8	7,86	6,23	1600	1070	12576,0	6666,1	348	10	355	98
9	6,89	7,95	1320	1840	9094,8	14628,0	356	309	15	270
10	14,69	16,31	1470	1400	21594,3	22834,0	20	2,5	84	7
11	8,58	12,53	1440	1500	12355,2	18795,0	27	344	1	0
12	5,18	3,88	1380	1460	7148,4	5664,8	336	347	0	228
13	9,19	14,15	970	1315	8914,3	18607,2	11	0,6	3	31
14	13,66	8,7	2500	1270	34150,0	11049,0	343	80	0	179

Table 2.

№	U_1 m/s	U_2 m/s	$m_1U_1+m_2U_2$ kg.m/s	$(\beta_1-\beta_2)^0$	α_S^0	k
1	9,3	9,91	25697,2	153	9,06	0,311
2	13,88	13,14	4800,3	18	359.6	0,047
3	6,79	4,36	16360,5	17	13.29	0,107
4	1,72	3,64	6619,0	15	7,18	0,072
5	10,65	24,93	54372,0	136	38,42	0,143
6	9,01	8,79	29007,5	141	295,0	0,661
7	17,96	9,82	33599,6	152	22,34	0,482
8	7,86	6,23	19242,1	22	9,76	0,093
9	6,89	7,95	23722,8	47	31,08	0,240
10	14,69	16,31	44428,3	17,5	12,54	0,052
11	8,58	12,53	31150,2	43	17,25	0,102
12	5,18	3,88	12813,2	11	17,39	0,038
13	9,19	14,15	27521,5	10,4	7.00	0,184
14	13,66	8,7	45199,0	97	13,45	0,136

In table 2 [β1-β2] is equal to the absolute value of the difference in the angles β1 and β2.

In the conducted research are determined the values of the coefficient of restitution in case of a frontal or side collision between two vehicles, i.e. in case of an impact between two objects in motion. The above mentioned values of the coefficient of restitution are not the same with the ones in a case of an impact of a vehicle with a fixed barrier (with a distinct material object which is not in motion) [1].

On the basis of the acquired statistics data, with the help of correlation and regressive analysis, is studied the dependence of the coefficient of restitution on some of the other factors, which are shown in the tables.

The correlation analysis allows to be defined the dependencies between random variables. The change in the value of one of these random variables leads to a change in the mathematical expectation of another. The next stage in the analysis of the collected statistics data includes creating a regression equation, which aims to describe the connection between the values of the dependent variable k (the coefficient of restitution) and the chosen five independent variables [$(m_1.U_1 + m_2.U_2)$; $(\beta_1-\beta_2)$; α_S; U_1; U_2] (7):

$$k = f[(m_1.U_1 + m_2.U_2), (\beta_1 - \beta_2), \alpha_S, U_1, U_2, \varepsilon]. \quad (7)$$

In the equation ε represents the random residual components of the model.

The process of creating a regression equation is connected with the necessity of solving a series of interrelated tasks: establishing the shape of the model, determining the coefficients of the regression equation, checking the model for relevance and adequacy.

In order for the set tasks to be solved a software product for statistics processing - „Statgraphics Centurion" has been used. A polynomial model of multiple linear regression has been obtained. The said polynomial model represents the correlation between the dependent parameter *k* and the five independent variables.

In table 3 are shown the values of the coefficients in the derived model. These values are determined by the Least-squares method, which minimizes the sum of the squared residuals. So as to exist a statistically significant correlation between the factors, it is essential that P-Value < 0,05 at 95,0% or higher confidence level.

Table 3.

Parameter	Estimate	Error	Statistic	P-Value
$m_1.U_1+m_2.U_2$	-0,0000142227	0,0000053958	-2,63588	0,0271
$\beta_1 - \beta_2$	0,00228072	0,00046788	4,87465	0,0009
α_S	0,000865039	0,00024815	3,48598	0,0069
U_1	0,0286416	0,00990357	2,89205	0,0178
U_2	0,0132796	0,00910193	1,45899	0,1786

R-squared = 92,4718 percent
R-squared (adjusted for d.f.) = 89,126 percent
Standard Error of Est. = 0,0893627
Mean absolute error = 0,0573742
Durbin-Watson statistic = 2,15845
Lag 1 residual autocorrelation = -0,132544

The multiple regression equation, representing the obtained model is as follows (8):

$$k = -0,0000142227(m_1.U_1 + m_2.U_2) + 0,00228072(\beta_1 - \beta_2) +$$

$$+0,000865039.\alpha_S + 0,0286416.U_1 + 0,0132796.U_2 \quad (8)$$

The R-Squared statistic indicates that the model as fitted explains 92,4718% of the variability in k. The standard error of the estimate shows the standard deviation of the residuals to be 0,0893627. The mean absolute error (MAE) of 0,0573742 is the average value of the residuals.

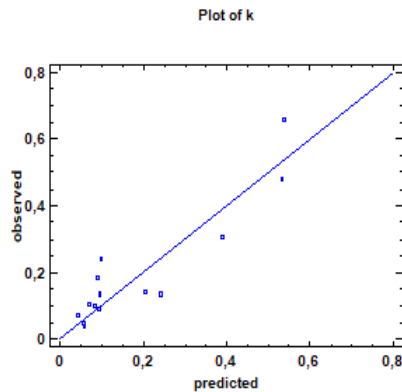On figure 1 and figure 2 are shown the matches between the observed and the predicted values of k.
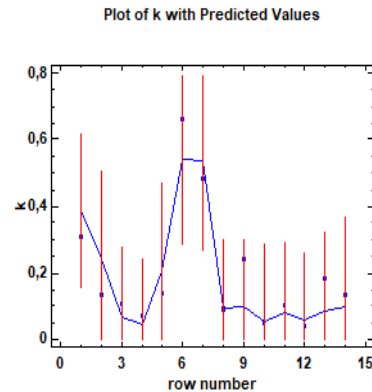

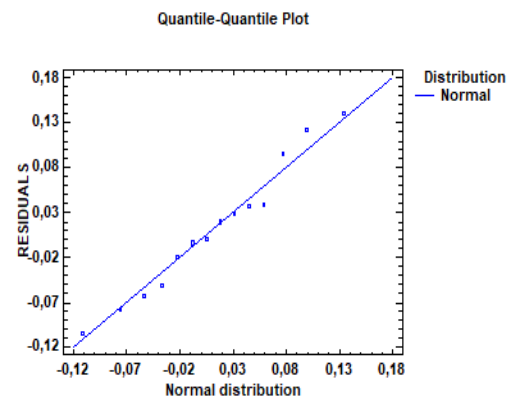**Figure 2:** Plot of k **Figure 3:** Plot of k with predicted values

Table 4 contains information about the predicted values of k, which have been generated by the fitted model.

Table 4.

№	Observed Value	Fitted Value	Residual	Studentized Residual	Stnd. Error CL for Forecast	Lower 95,0% CL for Forecast
1	0,311	0,38927	-0,078274	-1,07146	0,102698	0,15696
2	0,136	0,24143	-0,10543	-3,44107	0,117906	-0,02535
3	0,107	0,069955	0,037045	0,408831	0,0923063	-0,13886
4	0,072	0,043884	0,028117	0,300998	0,0901522	-0,160065
5	0,143	0,206193	-0,06319	-1,49528	0,118068	-0,060896
6	0,661	0,538994	0,122006	3,01269	0,1134	0,282464
7	0,482	0,532929	-0,05093	-1,00577	0,115776	0,271025
8	0,093	0,0928	0,000200	0,002203	0,0929675	-0,117508
9	0,24	0,099592	0,140408	1,79688	0,0913971	-0,107163
10	0,052	0,056208	-0,004208	-0,0552	0,103945	-0,178934
11	0,102	0,082093	0,0199069	0,222952	0,0940257	-0,130608
12	0,038	0,057782	-0,0197817	-0,21324	0,090986	-0,148043
13	0,184	0,089468	0,0945317	1,39705	0,104444	-0,146802
14	0,136	0,096792	0,0392078	1,06639	0,120826	-0,176536

When assessing the relevance of the regression equation it is necessary for the residuals to be analysed. For the model to be adequate, the residuals must be independent and normally distributed.

The hypothesis that in the studied model the residuals are normally distributed is then further checked.
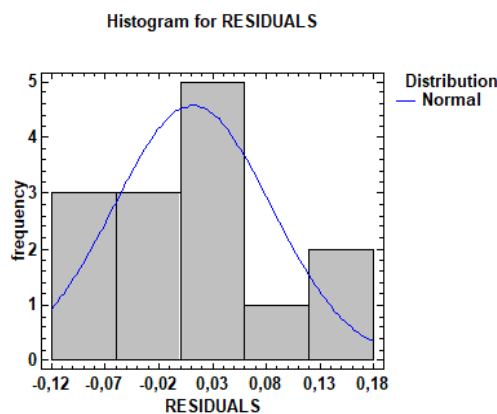
Figure 4: Histogram for residuals   **Figure 5:** Normal distribution of the residuals

The parametres of the estimated normal distribution are mean = 0,0114005 and standard deviation = 0,073407.

In order for the hypothesis to be checked the test of Kolmogorov-Smirnov has been used. The results from the test are shown in table 5:

Table 5. Kolmogorov-Smirnov Test

	Normal
DPLUS	0,138127
DMINUS	0,0855679
DN	0,138127
P-Value	0,952154

The acquired results from the test give no argument for rejection of the residuals' normal distribution.

Therefore, the obtained multiple regression equation is relevant and can be used for further forecasting of the coefficient of restitution's values.

3 Conclusion

The present scientific article presents two of the methods, which are used worldwide by the automobile expert practice when investigating the velocities of motor vehicles during an impact between them with the help of theorems from mechanics. Each road accident is characterized by its own specificity, which furthermore requires the choice of an adequate method when conducting an expert analysis.

Basic quantitative characteristic of the energy loss in the meantime of a two-body collision is the introduced in the Theory of impact – coefficient of restitution. When automobiles are to be concerned, the values of the coefficient of restitution mainly depend on the type of the elements that have crashed, as well as on the relative velocities of the vehicles. The authors of the present article have created a specifically developed for the purpose model for forecasting the values of the coefficient of restitution, based on multiple regression analysis.

For the achieving of the goal that has been set, a software product for statistical processing – 'Statgraphics Centurion' has been used. With the help of that software product a polynomial model of multiple linear regression, describing the correlation between the parameter k and the five selected independent variables - $(m_1.U_1+m_2.U_2)$; $(\beta_1-\beta_2)$; α_S; U_1; U_2 is obtained.

During the conducted research real automobile expertise data has been used, from which the values of the independent variables in the acquired model have been extracted.

In order for the multiple regression equation to be obtained the following interrelated tasks have been solved: the shape of the model has been established, the coefficients of the regression equation have been determined, the model has been checked for significance and adequacy.

Moreover, the estimated through the acquired multiple regression equation value of the coefficient of restitution may be used in further analysis for determining the velocities of the vehicles at the moment of the impact with the help of a research method, based on the Theory of impact.

References
[1]   Karapetkov St., Auto-technical expertise. Sofia, Technical University, 2005.
[2]   Savova-Mratsenkova M., Djonev G., Influence of the angle of the directory on the accuracy of the measurement when determining vehicles' velocities in case of an impact between them. *Conference BulTrans, Sozopol*, 11-13 September 2017.
[3]   Djonev G., Savova-Mratsenkova M., Methods for determining the velocity of vehicles in case of an impact between them. Conference BulTrans, Sozopol, 11-13 September 2017.
[4]   Pisarev, A. and others. Course in Theoretical Mechanics - Part II. Sofia, Technika, 1988.
[5]   Ilarionov V., Examination of an accident, Moscow, "Transport", (1989).
[6]   Kachaunov T., Modelling and Optimization of Transport Processes, VTU - T. Kableshkov, 2005.
[7]   Hoxha G., Shala A., Likaj R., Pedestrian crash model for vehicle speed calculation at road accident. *International Journal of Civil Engineering and Technology*, Volume **8**, Issue 9, September 2017, pp. 1093–1099, ISSN: 0976-6308, https://pdfs.semanticscholar.org/8eac/c99fba42ae9d81a2ce77fcd800240f5a8293.pdf
[8]   Scott W., Bain C., Manoogian S., Cormier J., Simulation Model for Low-Speed Bumper-toBumper Crashes. *SAE International*, 04.12.2010, ISSN 0148-7191, http://www.brconline.com/files/consultants/101/simulation_model_for_low-speed_bumper-to-bumper_crashes.pdf
[9]   Rose N., Fenton S., Beauchamp G*.,* Restitution Modeling for Crush Analysis: Theory and Validation. *SAE International* 2006-01-0908, 2006, http://kineticorp.com/wp-content/uploads/2017/05/2006-01-0908-restitution-crush-analysis-theory-and-validation.pdf
[10]   Rose N., Fenton S., Beauchamp G., Analysis of Vehicle-to-Ground Impacts during a Rollover with an Impulse-Momentum Impact Model. *SAE International* 2008-01-0178, http://kineticorp.com/wp-content/uploads/2016/08/2008-01-0178-Analysis-of-Vehicle-to-Ground-Impacts-During-a-Rollover-with-an-Impulse-Momentum-Impact-Model.pdf
[11]   Batista M., On the Mutual Coefficient of Restitution in Two Car Collinear Collisions. 01.2006, https://arxiv.org/vc/physics/papers/0601/0601168v1.pdf
[12]   Tseveennamjil B., Hudák A., Rievaj V., Determining the speed of vehicles before and after Crash. Number III, Volume **VI**, July 2011, http://pernerscontacts.upce.cz/22_2011/Tseveennamjil.pdf
[13]   Doris E., The Analysis of Injury Mechanisms from Frontal Car Accidents. *University of Iceland Reykjavik, January 2017*, https://skemman.is/bitstream/1946/26755/1/The%20Analysis%20of%20Injury%20Mechanisms%20from%20Frontal%20Car%20Accidents.pdf
[14]   Agyemang B., Abledu G., Semevoh R., Regression Analysis of Road Traffic Accidents and Population Growth in Ghana. *International Journal of Business and Social Research (IJBSR)*, Volume -**3**, No.-10, October, 2013, https://290-757-2-PB.pdf
[15]   Gupta H., Rokade S., Development of crash prediction model using multiple regression analysis. *International Journal of Current Engineering and Scientific Research, 2017*, http://troindia.in/journal/ijcesr/vol4iss6/82-86.pdf
[16]   Awe1 O., Adarabioyo M., Multivariate Regression Techniques for Analyzing AutoCrash Variables in Nigeria*. Journal of Natural Sciences Research*, ISSN 2225-0921 (Online) Vol.**1**, No.1, 2011, https://www.researchgate.net/profile/Olushina_Awe/publication/266165185_Multivariate_Regression_Techniques_for_Analyzing_Auto-Crash_Variables_in_Nigeria/links/55b38e4608aed621de00f818/Multivariate-Regression-Techniques-for-Analyzing-

Auto-Crash-Variables-in-Nigeria.pdf

[17]   Desai M., Patel A., *Road Accidents Study Based On Regression Model: A Case Study of Ahmedabad City*. National Conference on Recent Trends in Engineering & Technology, 13-14 May 2011,
http://bvmengineering.ac.in/misc/docs/published-20papers/civilstruct/Civil/101027.pdf