
Optimization of railway vehicles circulation in passenger transport

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Abstract

One of the basic principles of railway passenger transport is regularity, which is represented by passenger trains timetable. Regularity of railway passenger transport ensures that railway vehicles circulation indicates exact number of vehicles, which are necessary to fulfill the timetable of passenger trains. Railway vehicles circulation is a sequence of passenger trains, what determines which train is ensured by which vehicle. In the paper, selected mathematical methods, mostly from operational research, are applied to optimization of railway vehicle circulation in passenger transport. Theoretical basis of railway vehicles circulation, which is supported by mathematical application, are used in contemporary conditions of railway infrastructure and railway vehicles. Results of the railway vehicles optimization are costs reduction for the transport company and transport offer improvement for traveling public. The paper includes also a case study of the railway vehicles circulation optimization.

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1. Introduction

Railway vehicles involve mobile part of the operating basis on railways. The term railway vehicles means all vehicles, which are used for the movement on the track. Railways accomplish the main service – transportation by these vehicles. One of the basic characteristics of railway vehicles is use of the principle that steel wheel is rolling on...
the rail. Another characteristic sign is that railway vehicles can be either carried or led. Railway vehicles can be joined into train set, what increases the requirements for their construction.

The term railway vehicles operation means mostly assignment of these vehicles to various trains and all technological acts and processes, which are related to it. There are for instance ensuring periodical maintenance and repairs, which are necessary for retention of the railway vehicles in operative-capable conditions. In railway passenger transport, all trains operate according to a pre-established plan – timetable of trains. Majority of railway passenger trains are regular so scheduling the assignment of the railway vehicles to the various trains can be efficient. There is also a possibility to plan all technological acts, which are related to the operation on the railways.

2. Similar contemporary research

There are not so many researches focusing on issues that deal with optimization of railway vehicles. Similar research is done with issues of city bus transport, where main topics are vehicles scheduling and also bus-drivers scheduling. Mathematical approaches of bus-drivers scheduling is a main field of Stanislav Paluch’s (University of Zilina) research where the author describes some mathematical algorithms that are used to optimize bus-drivers scheduling and gives some model examples. Applications of flow-networks are elaborated in his publications. Other author Jaroslav Kleprlik (University of Pardubice) deals with scheduling in general and in his publications, he suggests using of the Hungarian method to solve optimization in transport as the assignment problem. These mathematical approaches that were mentioned above are the subject of the paper and its purpose in railway passenger transport is also part of the paper. Some authors, for example Matuš Dlugos, Peter Züber, Rudolf Dzuruš, Jozef Štaffen, Jozef Bujna (all from University of Zilina) and Robert Geci (University of Presov) deal with similar issues in their diploma thesis.

3. Railway vehicles characteristics

According to possibility of own movement, railway vehicles can be divided into groups of propulsive, driven and special railway vehicles. The group of propulsive railway vehicles consists of locomotives and motor wagons. Significant difference between locomotive and motor wagon is that the locomotives are intended only for generating the traction power while motor wagons generate traction power and also there is a place for passengers and their luggage. Very similar to motor wagons are motor units, which consist of some propulsive and driven vehicles permanently joint into one railway unit. These Railway units are indivisible train sets dedicated to transportation of passengers and their luggage. Their big advantage is that they can drive both directions without shunting of the propulsive vehicle. Railway units usually consist of motor wagons, embedded wagons and control wagons.

Railway vehicles in railway passenger transport could be divided also according to train type considering their maximum velocity or construction therefore railway vehicles are intended to:

- long-haul transportation,
- regional (Suburban) transportation,
- city (urban) transportation,
- shunting.

Propulsive railway vehicles are principally intended to generating the traction power. For this purpose, they are equipped with aggregate for generating the traction power and in some cases they are also equipped with energy storage for propulsion of this aggregate. Propulsive railway vehicles can be divided according to traction type into:

- railway vehicles of independent traction – steam traction and diesel traction,
- railway vehicles of semi-dependent traction - accumulator traction and flywheel traction,
- railway vehicles of dependent traction – electric traction.
In railway passenger transport, driven railway vehicles – wagons are basic transport vehicles, which are dedicated to transportation of passengers and their luggage. Railway passenger wagons can be divided according to their specification into:

- passenger wagons (compartments, open-space, first class, second class, double-floor, combined),
- couchette wagons,
- sleeper wagons,
- saloon wagons,
- restaurant wagons,
- bistro wagons,
- service wagons,
- luggage wagons.

4. Railway vehicles circulation in general

One of the main goals of transport companies is decreasing the operating costs. This can be possible by efficient use of railway vehicles, what depends on railway vehicle circulation – the basic plan for railway vehicles usage. Railway vehicles circulation is necessary for determination of exact number of railway vehicles for accomplishing the train timetable. The term railway vehicles circulation means the sequence of trains in exact time, what designates the assignment of each railway vehicle to selected trains. The circulation is divided into operating days therefore there is a sequence of trains for one railway vehicle per each day and there is also a sequence of these operating days. All necessary technological processes are taken into account, for instance planned repairs and maintenance, refuelling, cleaning, filling water or emptying faecal tanks.

Railway vehicles circulation is a basic support for calculation of railway vehicles kilometrage per specific time. Kilometrage is always calculated as average values, because each railway vehicle have got different amount of kilometres per day. However when the planned railway vehicles circulation is accomplished, after a certain period (number of operating days in the railway vehicles circulation) all railway vehicles have got the same amount of kilometrage. It is necessary to plan the railway vehicles circulation separately for time or date limitations in the train timetable. Very important task is to ensure fluent passage of railway vehicles between operating days even in the case of railway vehicles are passing between various circulations with some time or date limitations. In the railway vehicles circulation, there is also indicated a fact whether the railway vehicle rides individually or more railway vehicles ride together on the selected train. In this case, there must be provided also the order of these vehicles on the selected train and its composition must be abided in starting and terminal station.

Factors which influence the circulation of railway vehicles from operational point of view:

- train route length,
- number of stations and stops,
- throughput of railway line sections,
- depot location,
- platform length (maximum length of the train),
- disposition of other technical facilities.

Factors which influence the circulation of railway vehicles from technological point of view:

- periodical repairs,
- periodical maintenance,
- interior cleaning,
- exterior cleaning,
- refuelling (diesel traction),
- filling water,
- emptying faecal tanks,
• management of technological acts and processes,
• regulation of technological acts and processes,
• supervision of technological acts and processes.

Factors which influence circulation of railway vehicles from economical point of view:

• quantitative indicators (grosstonkilometres, trainkilometres, vehiclekilometres),
• railway vehicle capacity,
• railway infrastructure fees,
• depreciation of railway vehicles,
• repairs and maintenance costs,
• costs for insurance of railway vehicles,
• traction energy consumption.

5. Optimization of railway vehicles circulation

There are many ways how to optimize railway vehicles circulation but transport companies in Slovakia and surrounding countries do it only empirically nowadays. In the paper, there is a description and application of some mathematical methods for optimization of railway vehicles circulation. Issues that are related to optimization of railway vehicles circulation could be described by a graph. The graph is a mathematical object, which consists of ordered pair of sets \( G = (V, E) \). First set is the set of vertices and other set is the set of edges, where each edge connects two vertices. In this case, vertices represent trains and edges represent possible switchings from one train to another. Railway vehicles circulation issues are specific from graph theory point of view, that the railway vehicle can switch from one train to another only in one direction therefore the graph is oriented. In many cases, vertices and edges represent values of some attributes. In the case of railway vehicles circulation, edges represent switching of railway vehicles from one train to another therefore each edge must have information about duration of each switching. It means that the graph is a ordered triplet of sets \( G = (V, E, d) \), where \( d \) is a function \( d: H \rightarrow \mathbb{R}^+ \) which assign the nonnegative real number to each edge and it represents the duration of the switching.

Switching of railway vehicles within individual operating days is possible to solve by flow network. Basic unit in regular railway passenger transport is a train \( t_i \). Every train is marked by unique number and its complete characteristics is stated in the train timetable, where are all stations and stops of this train and also its temporal position. Majority of trains are short from temporal point of view therefore one train cannot fulfill day-long operation of the railway vehicle. This is the reason, why each railway vehicle has got the sequence of more trains per one operating day. Train \( t_i \) foregoes train \( t_j \) in case, when there is on identical place (station or stop) enough time to \( t_j \) departure after \( t_i \) arrival. The difference between \( t_j \) departure and \( t_i \) arrival is temporal dissipation and it is marked as \( d_{ij} \) with its value in minutes.

There is a set of trains \( T = \{ t_1, t_2, \ldots , t_n \} \) and the main task is to schedule these trains into operating days so that every train would be just in one operating day. Number of operating days should be as small as possible because number of railway vehicles proportionally depends on number of operating days. This task can be solved by graph \( G \) with set of vertices \( V \) \( V_1 \cup V_2 \cup \{ s, m \} \), where \( V_1 \) is the set of all train arrivals from the set \( T \), \( V_2 \) is the set of all train departures from the set \( T \), \( s \) is the fictitious source and \( m \) is the fictitious mouth. Symbols \( v_i \) and \( w_j \) are signs for vertices of \( V_1 \) and \( V_2 \) sets, which represent train \( t_i \) and \( t_j \). Set of oriented edges \( E \) in the graph \( G \) consists of three types of edges.

First type includes oriented edges \( (s, v_i) \) where \( v_i \in V_1 \). Second type includes oriented edges \( (w_j, m) \) where \( w_j \in V_2 \). Third type includes oriented edges \( (v_i, w_j) \) in case when \( t_i \) foregoes \( t_j \) with \( d(v_i, w_j) \).

Network flow solution for the optimization of railway vehicles circulation is applied to real long-haul railway route from Zvolen to Košice according to train timetable.

The simplest solution is to use the same number of railway vehicles as number of trains in the timetable. It is possible to reduce the number of railway vehicles, when some edge \((v_i, w_j)\) of the graph \( G \) is used. It means that the railway vehicle switches from the train \( t_i \) to the train \( t_j \). Every other used edge means saving of another railway vehicle. Among used edges, there must not be any edges with the same vertex. It is an assignment of train departures to train arrivals and as many as possible train departures should have assigned some train arrival. This can be done by searching for the maximum flow in the network.
After the solution of operating days in railway vehicles circulation, sequence of these operating days must be determined. In the case study of the route Zvolen – Košice, basic temporal period for these trains in the timetable is 24 hours period. It means, that each train runs every 24 hours (everyday) repeatedly. Solution for operating days (one operating day = one 24 hours period) can be showed as the permutation \( P \) of \( \{i, j\} \) what shows switchings of railway vehicles from one train to another.

Example of permutation \( P = (930 \rightarrow 933 \rightarrow 936; D2: 931 \rightarrow 934 \rightarrow 937; D3: 932 \rightarrow 935) \).

Basic precondition is that every train in the timetable is implemented only once during the temporal period. Stated example of the permutation must be completed with switchings of railway vehicles between temporal periods (operating days). This can be solved similarly as solution for one operating day. Then it is necessary to change the permutation \( P \) into permutation \( P^* \). This adapted permutation is showed in oriented graph (fig. 2.), where vertices represent trains and oriented edges represent switchings of railway vehicles from one train to another.

Example of permutation \( P^* = (930 \rightarrow 931 \rightarrow 932 \rightarrow 933 \rightarrow 934 \rightarrow 935 \rightarrow 936 \rightarrow 937) \).

Table 1. Train overview on the route Zvolen – Košice

<table>
<thead>
<tr>
<th>TRAIN NUMBER</th>
<th>DEPARTURE</th>
<th>ARRIVAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>930</td>
<td>Košice</td>
<td>5:23</td>
</tr>
<tr>
<td>931</td>
<td>Zvolen</td>
<td>7:13</td>
</tr>
<tr>
<td>932</td>
<td>Košice</td>
<td>9:23</td>
</tr>
<tr>
<td>933</td>
<td>Zvolen</td>
<td>11:13</td>
</tr>
<tr>
<td>934</td>
<td>Košice</td>
<td>13:23</td>
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<tr>
<td>935</td>
<td>Zvolen</td>
<td>15:13</td>
</tr>
<tr>
<td>936</td>
<td>Košice</td>
<td>15:23</td>
</tr>
<tr>
<td>937</td>
<td>Zvolen</td>
<td>19:13</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8:49</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10:37</td>
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<tr>
<td></td>
<td></td>
<td>12:49</td>
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<td></td>
<td></td>
<td>14:37</td>
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<td></td>
<td></td>
<td>16:49</td>
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<tr>
<td></td>
<td></td>
<td>18:37</td>
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<tr>
<td></td>
<td></td>
<td>18:49</td>
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<tr>
<td></td>
<td></td>
<td>22:37</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Zvolen</td>
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<td>Košice</td>
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<td>Košice</td>
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<td></td>
<td></td>
<td>Košice</td>
</tr>
</tbody>
</table>

All edges in \( G \) have got capacity = 1. Resultant operating days: D1: 930 \(\rightarrow\) 933 \(\rightarrow\) 936; D2: 931 \(\rightarrow\) 934 \(\rightarrow\) 937; D3: 932 \(\rightarrow\) 935.

Fig. 1. a) digraph for the route; b) graph \( G \) with maximum flow.

Fig. 2. Graphical representation of permutation \( P^* \).
Example of permutation represents trains and oriented edges represent switchings of (operating days). This can be solved if the example of the permutation must be completed with switchings of railway vehicles between temporal periods. Operating day = one 24 hours period can be showed as the permutation determined. In the case study of the route Zvolen – Košice temporal period for these trains in the timetable is 24 hours period. It means, that each train runs every 24 hours (everyday) repeatedly. Solution for operating days (one temporal period) can be showed as the permutation (P).

Table 1. Train overview

<table>
<thead>
<tr>
<th>TRAIN NUMBER</th>
<th>931</th>
<th>932</th>
<th>933</th>
<th>934</th>
<th>935</th>
<th>936</th>
<th>937</th>
</tr>
</thead>
<tbody>
<tr>
<td>DEPARTURE</td>
<td>00</td>
<td>00</td>
<td>00</td>
<td>00</td>
<td>00</td>
<td>00</td>
<td>00</td>
</tr>
<tr>
<td>ARRIVAL</td>
<td>00</td>
<td>00</td>
<td>00</td>
<td>00</td>
<td>00</td>
<td>00</td>
<td>00</td>
</tr>
</tbody>
</table>

Fig. 1. a) digraph for the route Zvolen – Košice marked with its number.

Fig. 2. b) graph showed in oriented graph (fig. 1).

Table 2. Number of railway vehicles and their schedule

<table>
<thead>
<tr>
<th>RAILWAY VEHICLE</th>
<th>OPERATING DAY 1</th>
<th>OPERATING DAY 2</th>
<th>OPERATING DAY 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>930 → 933 → 936</td>
<td>931 → 934 → 937</td>
<td>932 → 935</td>
</tr>
<tr>
<td>2</td>
<td>931 → 934 → 937</td>
<td>932 → 935</td>
<td>930 → 933 → 936</td>
</tr>
<tr>
<td>3</td>
<td>932 → 935</td>
<td>930 → 933 → 936</td>
<td>931 → 934 → 937</td>
</tr>
</tbody>
</table>

Ultimate solution of assignment problem, for instance by “Hungarian method” based on König and Egerváry theorem, is searched in the matrix $D = (d_{ij})_{i=1,2,...,n}$ of $n$ elements. Any two of these elements are not located in the same row not even in the same column. Sum of these elements must be as minimal as possible.

Assignment problem is defined as the problem that deals with the optimal assignment of elements from the set, which consists of $n$ elements, to the same number of elements from other set. It is assumed that all right-side coefficients values $a_i$, $i = 1, 2, \ldots, n$ and $b_j, j = 1, 2, \ldots, n$ are equal 1. Matrix $X$ consists of variables $x_{ij}$, which value is equal 1 in case, when there is the $j$-elements assigned to $i$-element. When it is not true, $x_{ij}$ is equal 0. Coefficients $d_{ij}$ could be presented as temporal dissipations. Assignment problem can be defined this way:

$$
\min z(x) = \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} x_{ij},
$$

$$
\sum_{i=1}^{n} x_{ij} = 1, \quad j = 1, 2, \ldots, n,
$$

$$
\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1, 2, \ldots, n,
$$

$$
x_{ij} \in \{0, 1\}, \quad i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, n.
$$

Appropriate optimization of railway vehicles circulation in railway passenger transport is important task of transport technology. The aim is to minimize required number of railway vehicles and ensure maximum efficiency of their usage. Many limiting factors have got a significant impact to the task. In the table 3, there are temporal dissipations between arrivals and departures of each train on the route Zvolen – Košice. Every train is marked with its number. Values of temporal dissipations are written only for relevant trains where the arrival station of one train is the same as the departure station of other train. These values of temporal dissipations are written in minutes.

Table 3. Temporal dissipations between arrivals and departures of trains

<table>
<thead>
<tr>
<th>min</th>
<th>930</th>
<th>931</th>
<th>932</th>
<th>933</th>
<th>934</th>
<th>935</th>
<th>936</th>
<th>937</th>
</tr>
</thead>
<tbody>
<tr>
<td>930</td>
<td>1344</td>
<td>144</td>
<td>384</td>
<td>624</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>931</td>
<td>1126</td>
<td>1366</td>
<td>166</td>
<td>286</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>932</td>
<td>1104</td>
<td>1344</td>
<td>144</td>
<td>384</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>933</td>
<td>886</td>
<td>1126</td>
<td>1366</td>
<td>46</td>
<td></td>
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<tr>
<td>934</td>
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<td>1104</td>
<td>1344</td>
<td>144</td>
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<td>646</td>
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<td>1126</td>
<td>1246</td>
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<td>936</td>
<td>744</td>
<td>984</td>
<td>1224</td>
<td>24</td>
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<td>937</td>
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<td>886</td>
<td>1006</td>
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</tbody>
</table>

There is the software which can solve assignment problem easily and very fast. In the resultant table 4, assigning the arrival of one train to the departure of the other train is showed with value 1 in the relevant cell of the table. All values 0 in the table cells mean that there are not any assignments. Software solution shown in the table 4 shows the same solution as it was done by previous methods.
6. Conclusions

At least three railway vehicles are necessary to fulfil the train timetable on the route Zvolen – Košice according the timetable 2016. In some cases, more than one railway vehicle is required to one train because transport capacity must be higher. This issue can be also solved mathematically but firstly these selected trains have to be stated twice (in case that two railway vehicles are necessary) or more times. Technological acts and processes are also issues that must be taken into account, because repairs and maintenance, refuelling, water filling, cleaning or emptying faecal tanks are time-consuming and the railway vehicle cannot be used on the train during these processes. Simple solution is to include these technological acts and processes into some operating days.

Solving of railway vehicles circulation by some mathematical methods is a necessary step to its optimization. Mathematical methods are the basis for IT systems. It is necessary to know all algorithms of these IT systems and how they work for the correct using of them. Outputs of these IT systems must be correctly understood and interpreted therefore these IT systems would provide right and relevant solutions. It is a way, how to simplify and upgrade decision-making processes in transport companies and how to decrease the operating costs by optimal assignment of railway vehicles to trains.

Acknowledgement

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